Optical springs in Gravitational Wave Detectors

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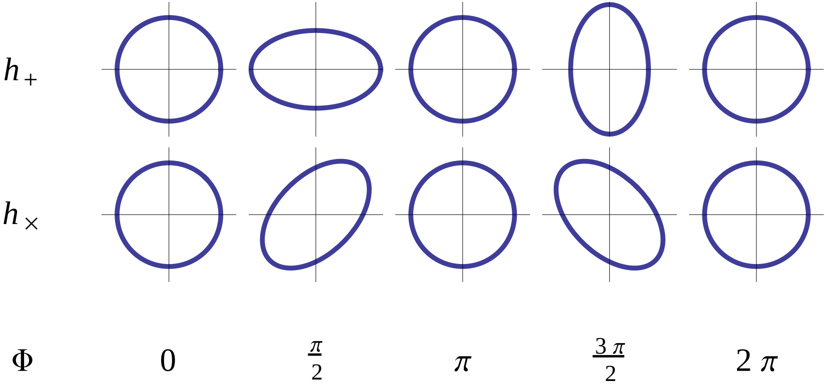


Top: plus polarisation; bottom: cross polarisation.

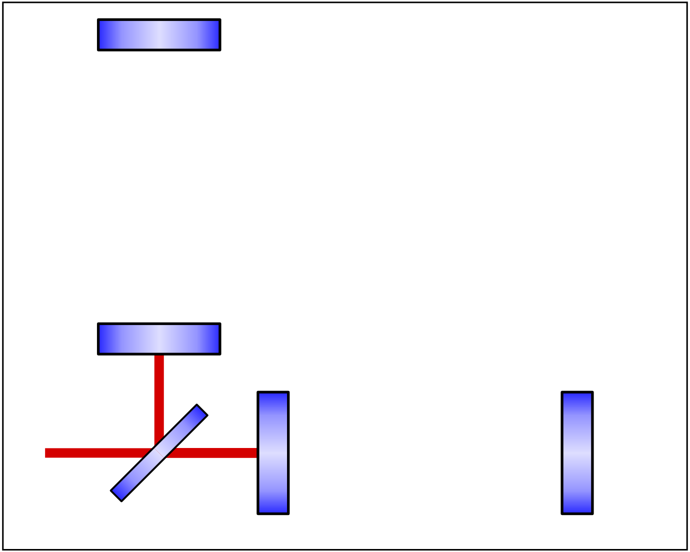
In optimistic scenario[1] two neutron stars coalesce in average once in 10^4 years, so in volume of radius $R = 10^{26}$ cm where there are 10^5 halaxies, we can expect 10 events a year. The amplitude of metric perturbation is about

$$h \sim 10^{-21 \div 22}.$$

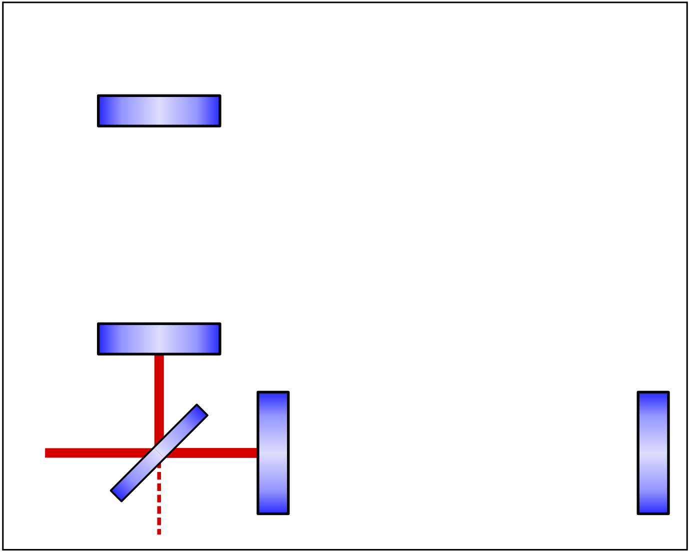
If two masses are separated by distance $L = 4$ km, the deviation of this distance is going to be equal

$$\Delta L_{\mathsf{grav}} = \frac{1}{2} h L \sim \; 2 \times 10^{-18 \div 19} \text{ (meters)}.$$

[1]Bethe H, Brown G, *Astron. J.*, **506**, 780 (1980)

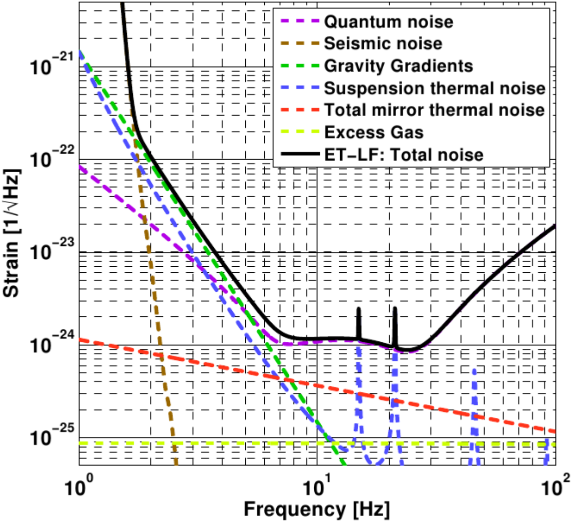


By default it is tuned in dark port regime



Gravitational wave disbalances the arms so some light leaks out

Laser Interferometer Gravitational Wave Observatory



Plotted is the square root of spectral density of all noises recalculated to dimensionless metric perturbation:

$$h(f) = \sqrt{S_{\frac{\Delta L}{L}}(f)}$$

In order to measure displacement, we consequatively measure position of test mass,

$$\delta x = x_\tau - x_0.$$

Measurement of position perturbs the momentum so

$$\Delta x_0 = \Delta_0; \quad \Delta p_0 \geqslant \frac{\hbar}{2\Delta_0}.$$

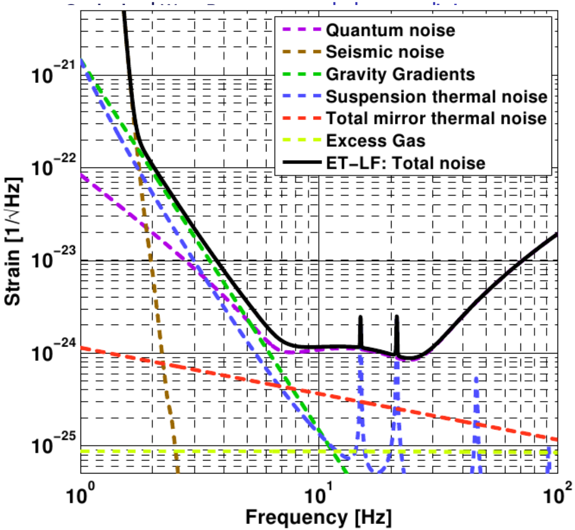
$$\Delta x_\tau \sim \Delta_\tau + \frac{\tau}{m}\Delta p_0 \geqslant \frac{\hbar\tau}{2m\Delta_0}.$$

$$\Delta(\delta x) = \Delta x_\tau + \Delta x_0 \geqslant \Delta_0 + \frac{\hbar\tau}{2m\Delta_0}.$$

In general case the measurement is provided by interaction hamiltonian

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{int}} = \hat{\mathcal{H}}_0 + \alpha \hat{Y} \hat{q}.$$

- $\hat{\mathcal{H}}_0$ — free evolution
- $\hat{Y}$ — observable of measuring device
- $\hat{q}$ — observable being measured



Plotted is the square root of spectral density of all noises recalculated to dimensionless metric perturbation:

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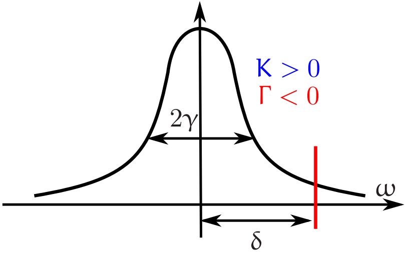
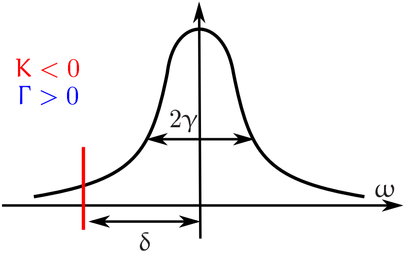
Electro-magnetic rigidity has been first noted by V.B.Braginsky [2]

Tuning onto the right slope



Positive rigidity and
negative damping (instability)

Tuning onto the left slope



Negative rigidity and
positive damping

[2] V.B. Braginsky, I.I. Minakova Bull. MSU III **1**, 83 (1964)

Use of optical rigidity is always accompanied by instability

$$m\ddot{x}(t) = F_{OR}(t) = -Kx(t - \tau_*) \approx -Kx(t) + K\tau_*\dot{x}(t).$$

Ways to avoid it

- Feedback[3]
- Additional Pump[4]

[3] A. Buonanno, Y. Chen, Phys. Rev.D, **65**, 042001 (2002)
[4] H. Rehbein, *et. al.*, Phys. Rev. D, **78**, 062003 (2008)

LIGO Livingston Site

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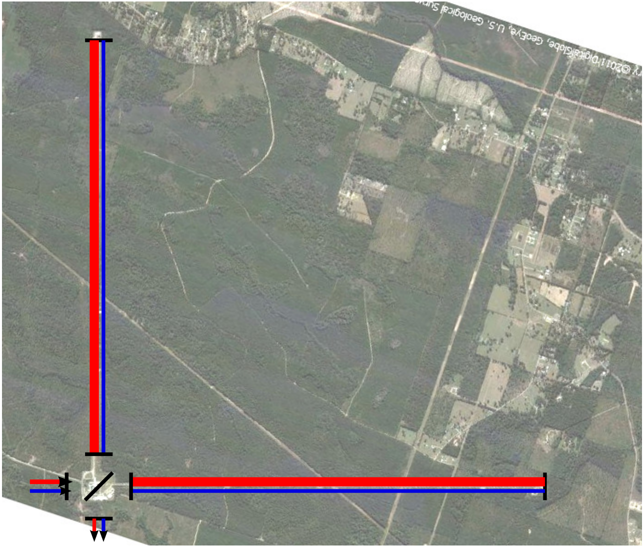
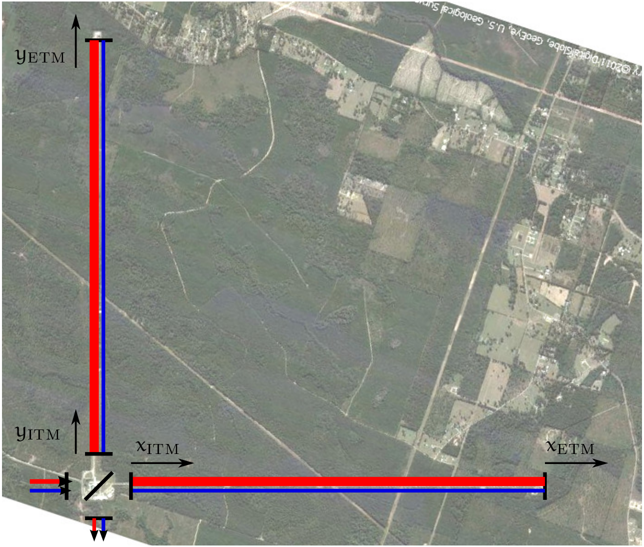
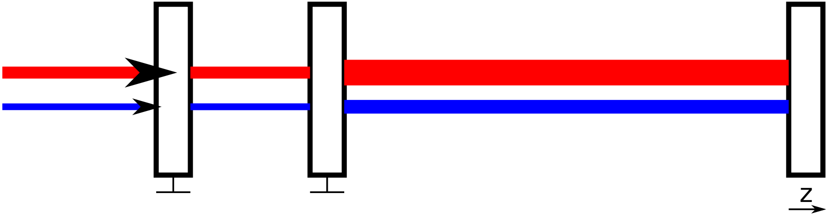


LIGO Livingston Site



LIGO Livingston Site



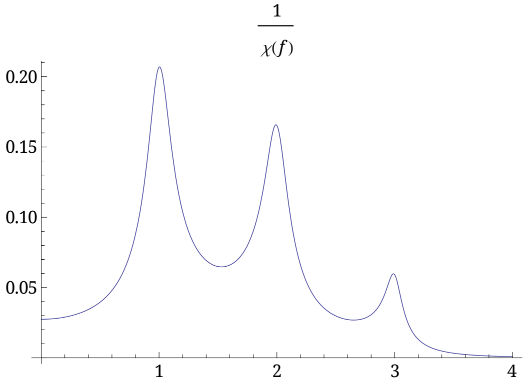
$$x = (x_{\mathrm{ETM}} - x_{\mathrm{ITM}}) - (y_{\mathrm{ETM}} - y_{\mathrm{ITM}}).$$

In spectral representation the equation of motion has the form

$$F(\Omega) = x(\Omega)\chi^{-1}(\Omega);$$

$$\chi^{-1}(\Omega) = -m\Omega^2 + K_1(\Omega) + K_2(\Omega).$$

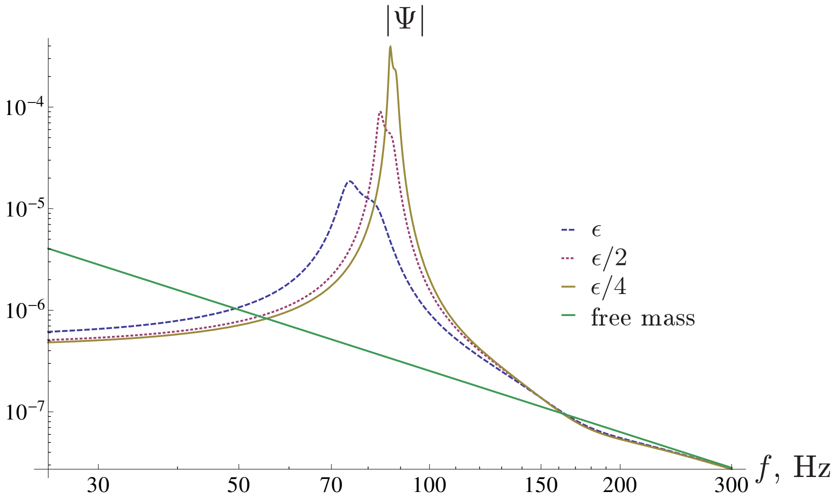
$$F(\Omega)\chi(\Omega) = x(\Omega)$$



Peaks correspond to roots of charachteristic equation:

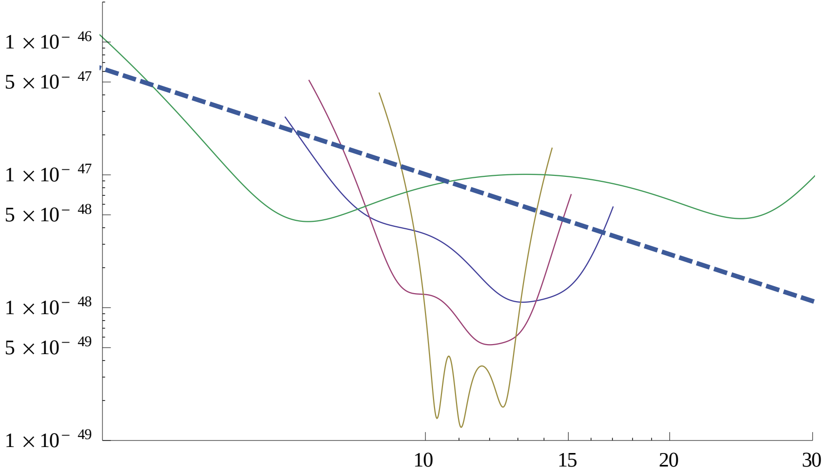
$$\chi^{-1}(\Omega_i) = -m\Omega_i^2 + K_1(\Omega_i) + K_2(\Omega_i) = 0.$$

Double resonance is narrow-band regime



Wider band is desired so we try to establish triple-resonance

Three close resonances



Resume

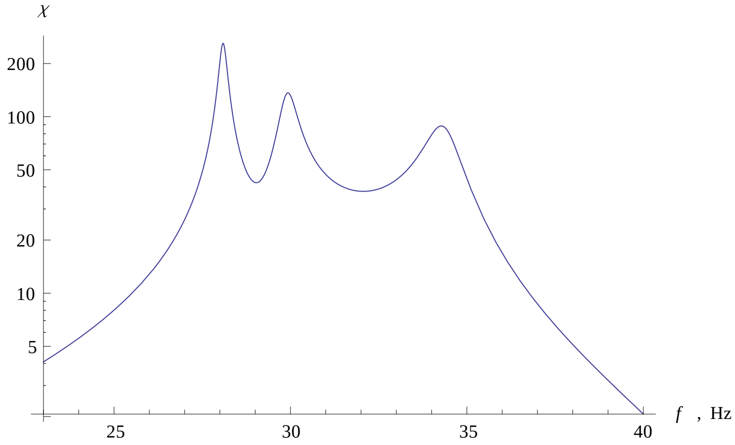
- optical spring allows to overcome the SQL
- double optical spring is itself stable (no need to apply feedback)

- double- and triple-resonance regimes are attractive ones
- a simple criterion is developed to estimate the pumps tuning needed to achieve the desired disposition of frequencies

- the regimes been investigated demonstrate high mechanical susceptibility and
- possibility to overcome the SQL in narrow (double-resonance) and wide (triple resonance) bands.

$$x(t) = \int_{-\infty}^{t} F(\xi)\chi(t-\xi)d\xi \quad \Leftrightarrow \quad x(\Omega) = \chi(\Omega)F(\Omega)$$

x — coordinate, F — force, χ — susceptibility



Each of peaks corresponds to eigen mode.

Conventional case

$$\chi^{-1} = -m\Omega^2 - 2i\gamma\Omega + m\omega_0^2;$$
$$E = E_p + E_k = m\omega_0^2 x_0^2 = kx_0^2.$$

So we guess

$$E \sim \mathrm{Re}\left(\chi^{-1}(\Omega) + m\Omega^2\right) x^2.$$

Optical rigidity

$$\chi^{-1} = -\mu\Omega^2 + K_1(\Omega) + K_2(\Omega);$$
$$S_E(\Omega) = \mathrm{Re}(K_1(\Omega) + K_2(\Omega))S_x;$$
$$S_x = S_F|\chi|^2.$$

Finally,

$$S_E(\Omega) = \mathrm{Re}(K_1 + K_2)|\chi|^2 S_F$$

$$E_i = \int_{\Omega_i - 3\Delta\Omega_i}^{\Omega_i + 3\Delta\Omega_i} \mathrm{Re}(K_1(\Omega) + K_2(\Omega))|\chi(\Omega)|^2 S_F(\Omega) d\Omega$$

Here Ω_i and $\Delta\Omega_i$ are real and imaginary parts of characteristic equation roots:

$$\chi^{-1}(\Omega_i + i\Delta\Omega_i) \equiv 0.$$

For three modes from first plot we obtain

$$E_1 \sim 1.3\hbar\Omega_1; \quad E_2 \sim 1.8\hbar\Omega_2; \quad E_3 \sim 1.1\hbar\Omega_3.$$

Condition on quantum behavior observation

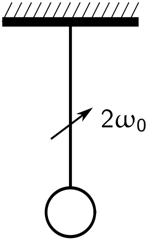
$$\frac{E_i}{\hbar\Omega_i Q_i} = \frac{E_i}{\hbar\Omega_i} \cdot \frac{2\Delta\Omega_i}{\Omega_i} < 1.$$

yields values

$$\frac{E_i}{\hbar\Omega_i Q_i} = 0.012; \quad \frac{E_2}{\hbar\Omega_2 Q_2} = 0.024; \quad \frac{E_3}{\hbar\Omega_3 Q_3} = 0.034.$$

We can expect each mode behaving as quantum object!

Mechanical oscillator



$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2(1 + \delta\cos(2\omega_0 t + \phi))x = f$$

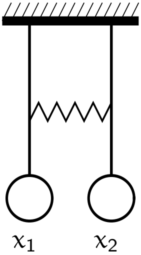
$$x(t) = A(t)\cos\omega_0 t + B(t)\sin\omega_0 t$$

$$S_A(\Omega) \sim \frac{S_f(\Omega + \omega_0)}{(\gamma + \epsilon)^2 + \Omega^2}$$

$$S_B(\Omega) \sim \frac{S_f(\Omega + \omega_0)}{(\gamma - \epsilon)^2 + \Omega^2}$$

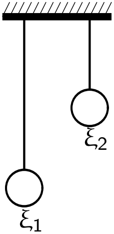
$\epsilon \sim \delta$ (modulation depth)

Conventional coordinates



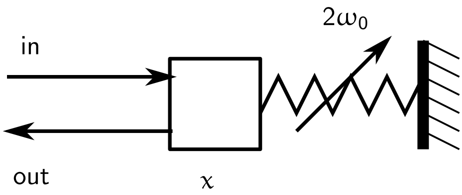
$$x_1(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t;$$
$$x_2(t) = A_1 k_1 \cos\omega_1 t + A_2 k_2 \cos\omega_2 t;$$

Eigen coordinates



$$\xi_1(t) = A_1 \cos\omega_1 t;$$
$$\xi_2(t) = A_2 \cos\omega_2 t;$$

Modulation of coupling spring constant results in modulation of eigen frequencies.
BUT: if ω_1 and ω_2 differ significantly the modulation will not affect ξ_1.



a — input field; b — output field

$$x(t) = X(t)e^{-i\omega_0 t} + X^\dagger(t)e^{i\omega_0 t} = A(t)\cos\omega_0 t + B(t)\sin\omega_0 t$$

A, B undergo squeezing

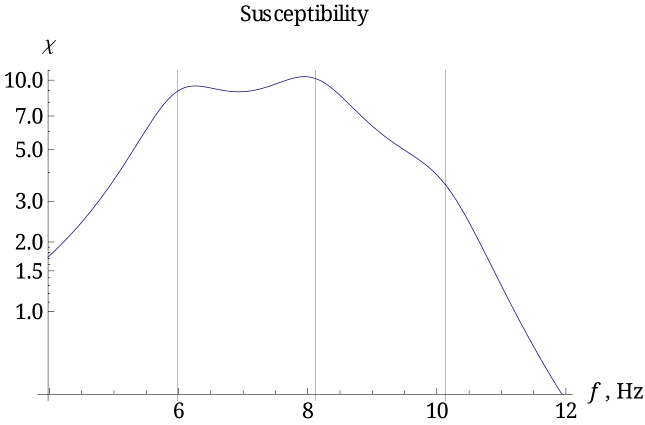
$$A(\Omega) = X(\Omega) + X^\dagger(\Omega); \quad B(\Omega) = (X(\Omega) - X^\dagger(-\Omega))/i$$

Phase quadrature of reflected wave:

$$b_2 = a_2 + \mathcal{C}x; \qquad \mathcal{C} \text{ — coupling constant}$$

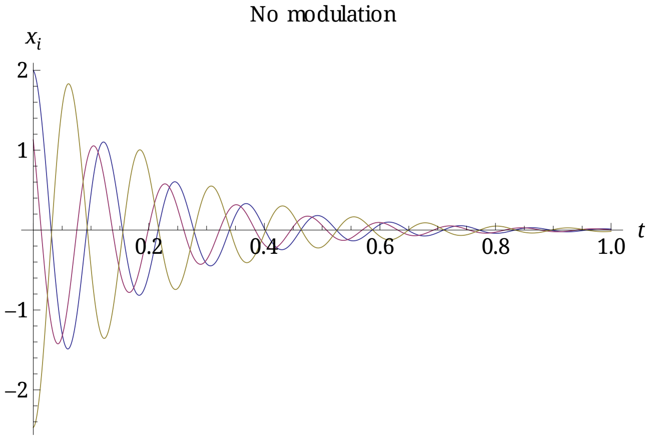
$$K(\Omega) \sim \text{Optical power}$$

Modulation of pump power allows modulation of rigidity

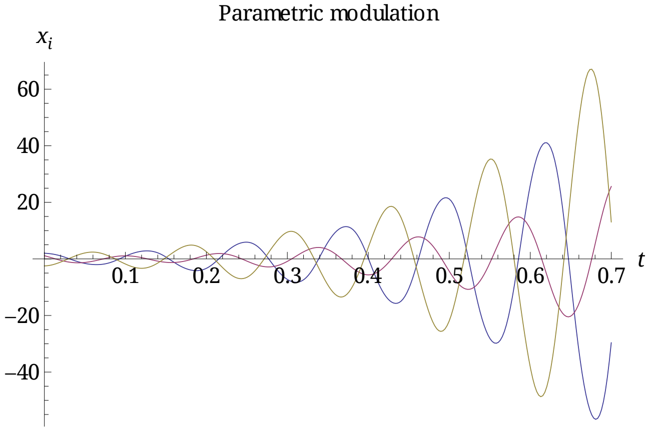


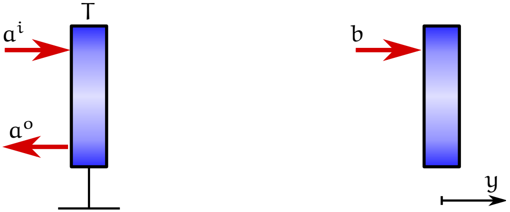
▸ Details

Initial conditions are fulfilled for one of modes, no modulation applied.



Initial conditions are fulfilled for one of modes, modulation is on.





The system is itself instable; for stability we apply feedback.

Details

The system evolution could be represented as a sum of harmonic oscillations:

$$\begin{pmatrix} b_1(t) \\ y(t) \end{pmatrix} = \sum_i \left[f_i \vec{v}_i e^{\lambda_i t} + f_i^+ \vec{v}_i^+ e^{\lambda_i^+ t} \right].$$

- f_i are modes amplitudes;
- λ_i are eigen frequencies (complex): $\lambda_i = -i\omega_i - \gamma_i$.
- $\vec{v}_i$ are vectors of forms of eigen modes.

If the evolution is free, f_i are constants depending on initial conditions.

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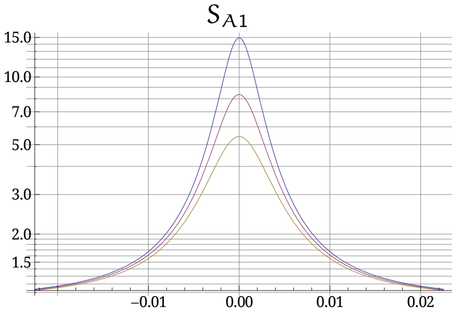
If the evolution is free, f_i are constants depending on initial conditions.
If we apply parametric modulation, $f_i(t)$ start depending on time.

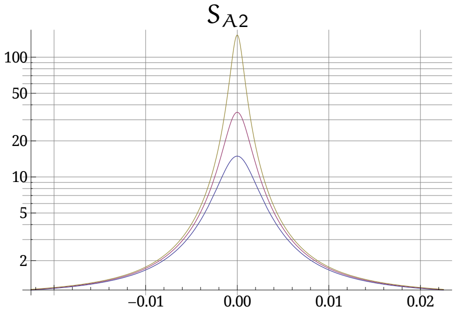
We measure phase quadrature of a^o:

$$a_2^o(\Omega) \equiv \frac{a^o(\Omega) - {a^o}^\dagger(-\Omega)}{i\sqrt{2}}.$$

Combination of a_2^o carries information about the squeezing:

$$A_{1,2}(\Omega) = \frac{a_2^o(p+\Omega) \pm a_2^o(p-\Omega)}{\sqrt{2}}$$





Increase of modulation results in increasing of squeezing.

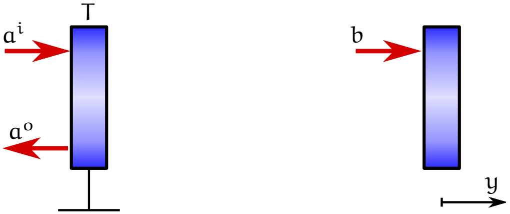
Thank you!

Appendices

$$\begin{cases} \ddot{x}_1 + 2\gamma_1\dot{x}_1 + \omega_1^2 x_1 + \lambda_1 z = 0, \\ \ddot{x}_2 + 2\gamma_2\dot{x}_2 + \omega_2^2 x_1 - \lambda_2 z = 0, \\ -\lambda_1 x_1 - \lambda_2 x_2 + \ddot{z} = 0. \end{cases}$$

$$\lambda_1 = \lambda_1^{(0)} \left(1 + 2|m| \cos(2pt + \phi + \phi)\right).$$

Back



The system of equations

$$\ddot{b}_1 + g\dot{b}_1 + 2b_1 + Ay =$$
$$= -g\left[\frac{g}{2}q_2 + \dot{q}_2 + \sqrt{2 - \frac{g}{4}}q_1\right];$$
$$\ddot{y} - Ab_1 + \alpha\dot{b}_1 = -\alpha\dot{q}_2.$$

When parametric modulation of pump is on, $A \to A(1 + 2|m|\cos(2pt + \phi))$.

Back