# Real life applications of Lukasiewicz-Pavelka logic

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## Part 1: Theory

Aristotelian logic:

| | |
|---|---|
| All human beings are mortal | $\forall x[(H(x) \Rightarrow M(x))]$ |
| Sokrates is a human being | $H(s)$ |
| Sokrates is mortal | $M(s)$ |

It took thousands of years before Aristotelian informal logic was expressed in a formal way, known as First-order Boolean Logic.

In 1960's Zadeh introdused

Fuzzy Logic:

| | |
|---|---|
| Red apples are ripe | |
| This apple is more or less red | |
| This apple is almost ripe | |

General fuzzy rule systems:

| |
|---|
| if $x$ is in $A_1$ and $y$ is in $B_1$ then $z$ is in $C_1$ |
| $\cdots$ |
| if $x$ is in $A_n$ and $y$ is in $B_n$ then $z$ is in $C_n$ |
| $x$ is in $A$ and $y$ is in $B$ therefore $z$ is in $C$ |

Here the sets $A, B, C$ etc are fuzzy; this means $x$ is in $A$ to a degree $\in [0, 1]$. For example an almost ripe apple is in the set of ripe apples to a degree 0.9.

The question arises: what is the mathematics of fuzzy logic? We aim to show that many – valued similarity plays a central role in fuzzy inference. In science the ambition is to minimize the set of axioms and maximize the set of the consequences of the axioms, so we present the following algebraic axioms that are necessary and sufficient to carry out fuzzy inference.

## Wajsberg algebra axioms

Let $L$ be a non–void set, $\mathbf{1}$ an element of $L$ and $\to$, $^*$ a binary and unary operation, respectively, defined on $L$ such that, for all $x, y, z \in L$, we have:

$$\mathbf{1} \to x = x, \quad (1)$$

$$(x \to y) \to [(y \to z) \to (x \to z)] = \mathbf{1}, \quad (2)$$

$$(x \to y) \to y = (y \to x) \to x, \quad (3)$$

$$(x^* \to y^*) \to (y \to x) = \mathbf{1}. \quad (4)$$

Then the system $L = \langle L, \to, ^*, \mathbf{1} \rangle$ is a Wajsberg algebra.

We use only four equational axioms to establish a rich structure: a Wajsberg algebra alias MV–algebra, which has a similar role in many–valued logic than Boolean algebras have in classical two valued logic; for example the operations $\odot$, $\oplus$ and $\rightarrow$ are the algebraic counterparts of the logical connectives and, or, implies in Łukasiewicz –Pavelka many–valued logic.

However, to be able to introduce fuzzy inference in an axiomatic way, we will still need two more axioms. Unfortunately, they are not equational. First consider

**Complete MV-algebra**

An MV–algebra $L$ is complete if it contains all suprema and infima, that is, for any subset $\{x_i : i \in \Gamma\} \subseteq L$, we assume that $\bigvee_{i\in\Gamma} x_i \in L$ and $\bigwedge_{i\in\Gamma} x_i \in L$, where $\Gamma$ is an index set.

## n-divisors and injective MV–algebras

An element $b$ of an MV-algebra $L$ is called an $n$–divisor of a non zero element $a$ of $L$ if

$$(a^* \oplus (n-1)b)^* = b \text{ and } nb = a,$$

where $0b = \mathbf{1}, 1b = b$ and $kb = (k-1)b \oplus b, k \in \mathbb{N}$. If all elements have $n$–divisors for all natural $n$, then $L$ is called divisible (the word has also another meaning). An MV-algebra $L$ is called injective if it is complete and divisible.

The six axioms of an injective MV-algebra are necessary and sufficient to construct fuzzy IF–THEN inrefence systems. A canonical example of an injective MV–algebra is the Łukasiewicz structure defined on the real unit interval $[0, 1]$: $1 = \mathbf{1}, x^* = 1 - x, x \to y = \min\{1, 1 - x + y\}$.

## Fuzzy similarity

Let $L$ be an injective MV–algebra and let $A$ be a non–void set. A fuzzy similarity $S$ on $A$ is such a binary fuzzy relation that, for each $x, y$, and $z$ in $A$,

- (i) $S(x,x) = \mathbf{1}$; everything is similar to itself,
- (ii) $S(x,y) = S(y,x)$; fuzzy similarity is a symmetric fuzzy relation,
- (iii) $S(x,y) \odot S(y,z) \leq S(x,z)$; fuzzy similarity is a weakly transitive fuzzy relation.

## Fuzzy subsets

Recall an $L$–valued fuzzy subset $X$ of $A$ is an ordered couple $(A, \mu_X)$, where the membership function $\mu_X : A \to L$ tells the degree to which an element $a \in A$ belongs to the fuzzy subset $X$.

Given a fuzzy subset $(A, \mu_X)$, define a fuzzy relation $S$ on $A$ by

$$S(a, b) = \mu_X(a) \leftrightarrow \mu_X(b), \text{ for any } a, b \in A. \tag{5}$$

This fuzzy relation is trivially symmetric, it is reflexive and transitive. Hence, any fuzzy set generates a fuzzy similarity, in fact, this is true for $L$ being any BL–algebra. Also notice that if $\mu_X(b) = \mathbf{1}$ then $S(a, b) = \mu_X(a)$.

**Proposition**

Consider $n$ injective MV–algebra $L$ valued fuzzy similarities $S_i, i = 1, \cdots, n$ on a set $A$. Then a fuzzy binary relation $S$ on $A$, defined by

$$S(x, y) = \frac{S_1(x, y)}{n} \oplus \cdots \oplus \frac{S_n(x, y)}{n}$$

is an $L$ valued fuzzy similarity on $A$.

**Corollary**

More generally, if $S_i, i = 1, \cdots, n$ are $n$ injective MV–algebra $L$ valued fuzzy similarities on a set $A$, then any weighted mean

$$SIM(x,y) = \frac{m_1 S_1(x,y)}{M} \oplus \cdots \oplus \frac{m_n S_n(x,y)}{M}, M = \sum_{i=1}^{n} m_i, m_i \in \mathbb{N}$$

is an $L$ valued fuzzy similarity on $A$, called total fuzzy similarity.

The idea of partial similarity is not new. Indeed, (by Niiniluoto) in 1843 J. S. Mill defined: If two objects $A$ and $B$ agree on $k$ attributes and disagree on $m$ attributes, then the number

$$sim(A,B) = \frac{k}{k+m}$$

can be taken to measure the degree of similarity or partial identity between $A$ and $B$. This *sim*–relation can be considered as an injective MV-algebra valued similarity.

## Injective MV–algebra valued Pavelka logic

There is an analogy between injective MV–algebras and Łukasiewicz–Pavelka logic on the one hand and Boolean algebras and Classical logic on the other hand. Indeed, we can define a logic language and interpret truth values semantically in an injective MV–algebra; if $\alpha$ is true at a degree *a* and $\beta$ is true at a degree *b* then $\alpha \mathtt{and} \beta$ is true at a degree $a \odot b$. Tautological degree of a formula $\alpha$ is the infimum of all such interpretations. We can also talk about fuzzy theories by fixing axioms and rules of inference. Provability degree of a formula $\alpha$ in a fuzzy theory is the supremum of degrees of its all possible proves.

Tautological degree of $\alpha$ and provability degree of $\alpha$ coincide.

The following Algorithm is based on a fact that the average of fuzzy similarities is a fuzzy similarity; this holds only in injective MV–algebras.

## Algorithm to Construct Fuzzy IF-THEN Inference I

Let us now return to our starting point, a fuzzy rule system

Rule 1: IF $x_1$ is in $A_{11}$ and $\cdots$ and $x_m$ is in $A_{1m}$ THEN $y$ is in $B_1$
Rule 2: IF $x_1$ is in $A_{21}$ and $\cdots$ and $x_m$ is in $A_{1m}$ THEN $y$ is in $B_2$
$\cdots$
Rule n: IF $x_1$ is in $A_{n1}$ and $\cdots$ and $x_m$ is in $A_{nm}$ THEN $y$ is in $B_n$

Here all $A_{ij}$ and $B_j$ are fuzzy subsets but can be crips actions, too. It is not necessary that the rule base is complete; some rule combinations can be missing without any difficulties. It is also possible that different IF–parts cause equal THEN–part, but it is not possible that a fixed IF–part causes two different THEN–parts.

We will not need any kind of defuzzification method – here the Algorithm differs from Sugeno or Mamdani fuzzy inference – instead of that everything is based on an experts knowledge and properties of injective MV-algebra valued similarity.

## Algorithm to Construct Fuzzy IF-THEN Inference II

Step 1: Create the dynamics of the inference system, i.e. define the IF–THEN rules and give shapes to the corresponding fuzzy sets.

Step 2: If necassary, give weights to various IF–parts to emphasize their importance.

Step 3: List the rules with respect to the mutual importance of their IF–parts.

Step 4: For each THEN–part, give a criteria on how to distinguish outputs with equal degree of membership.

A general framework for a fuzzy IF–THEN inference system is now ready. Step 3 and Step 4 are in place of defuzzification – to create such an inference system might be more laborious than Sugeno or Mamdani fuzzy inference, however, theoretical basis of the Algoritm is well established.

## Algorithm to Construct Fuzzy IF-THEN Inference III

Assume now we have an actual input Actual $= (X_1, \cdots, X_m)$.

A corresponding output $Y$ is counted in the following way.

(1): Consider each IF–part of each rule as a crisp case, that is $\mu_{A_{ij}}(x_j) = \mathbf{1}$, for $i = 1, \cdots, n$, $j = 1, \cdots, m$ holds.

(2): Compute the degree of similarity between Actual and the IF–part of each Rule $i$, $i = 1, \cdots, n$. Since

$$\mu_{A_{ij}}(X_i) \leftrightarrow \mu_{A_{ij}}(x_i) = \mu_{A_{ij}}(X_i) \leftrightarrow \mathbf{1} = \mu_{A_{ij}}(X_i),$$

we only need to calculate averages or weighted averages of membership degrees!

(3): Fire a $Y$ such that $\mu_{B_k}(Y) =$ Similarity(Actual, Rule $k$) corresponding to the greatest similarity degree between the input Actual and the IF–part of a Rule $k$. If such a maximal rule is not unique, use the preference list given in Step 3, and if there are several such outputs $Y$, use a creteria given in Step 4.

## If–Then rule systems as Pavelka's fuzzy theories

Note that each rule of the Algorithm corresponds to a non logical axiom of a form $\alpha \Rightarrow \beta$ of a fuzzy theory in injective MV–algebra valued Łukasiewicz –Pavelka logic. Moreover, computing the actual output can be viewed as an instance of using Generalized Modus Ponens

$$R_{GMP}: \quad \frac{\alpha, \alpha \Rightarrow \beta}{\beta} \quad , \quad \frac{a, b}{a \odot b}$$

where $\alpha$ corresponds to the IF–part of a Rule, $\beta$ corresponds to the THEN–part of the Rule, $a$ is the degree of similarity of Actual input and IF part of the Rule, and $b = 1$; the degree of truth of $\alpha \Rightarrow \beta$. This gives a theoretical many–valued logic based justification to fuzzy inference.

## Part 2: Applications

Total Fuzzy Similarity and the Algorithm – call it Total Fuzzy Similarity Algorithm – can be utilized in

- Classification and clustering tasks
- Constructing fuzzy IF–THEN inference systems
- Decision making problems

The following examples are implemented in real life applications. The aim here is to clarify the leading idea; we have linguistic rules that can be expressed by fuzzy sets, and certain comparison using the Algorithm is carried out.

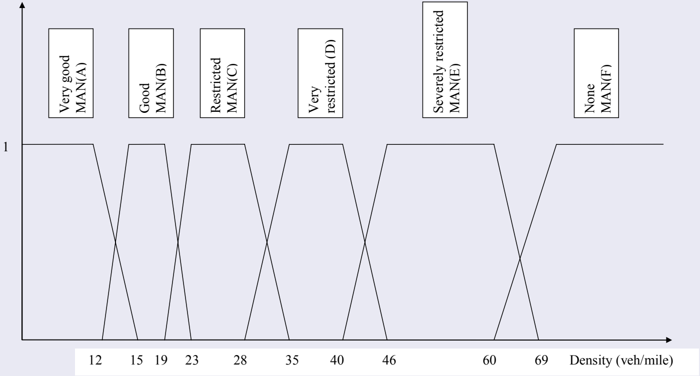
## Application 1 – Grouping hightway traffic fluency

According to U.S. Highway Capacity Manual , the fluency of a highway is divided to 6 classes LOS(A), $\cdots$, LOS(F), originally defined by vehicle density (veh/mile). Chackroborty and Kikuchi proposed the following linguistic division

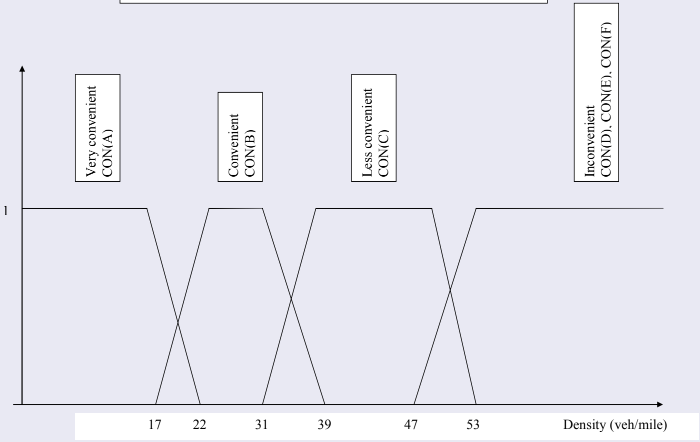
| LOS | Maneuver-ability (MAN) | Driving Con-venience (CON) | Freedom to Sel. Spd. (SSP) | Proximity to Oth. Veh. (PRV) |
|---|---|---|---|---|
| A | Very good | Very convenient | Absolute freedom | Very far |
| B | Good | Convenient | Free | Far |
| C | Restricted | Less convenient | Constrained | More or less far |
| D | Very Restricted | Inconvenient | More constrained | Close |
| E | Severely restricted | Inconvenient | None | Very close |
| F | None | Inconvenient | None | Bumper – to – Bumper |

and characterized by the following fuzzy sets:

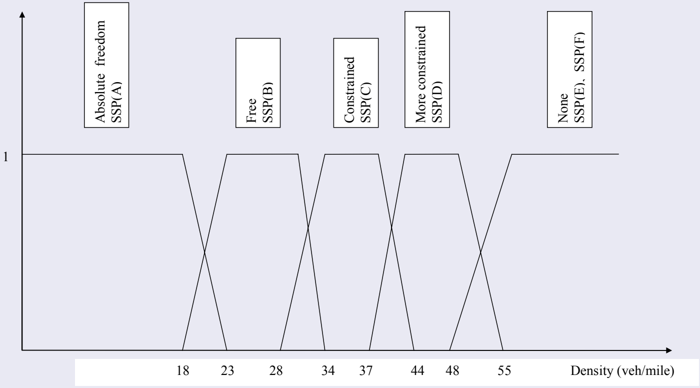
Fuzzy Sets for Linguistic values of Maneuverability

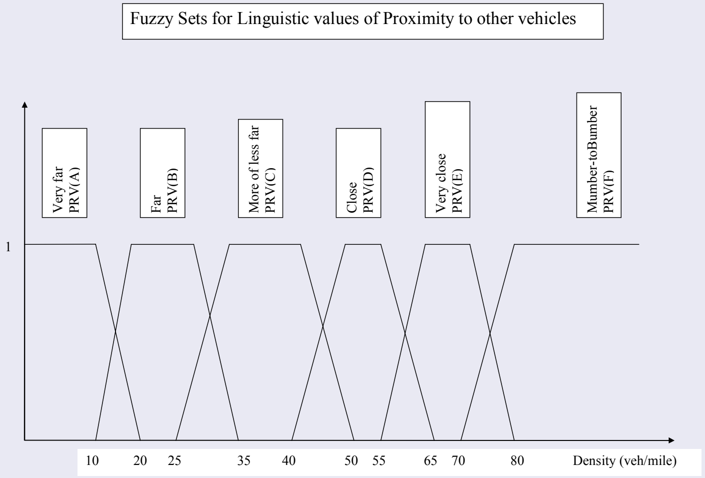


## Fuzzy Sets for Linguistic values of Driving convenience



Fuzzy Sets for Linguistic values of Freedom to select speed




Fuzzy Sets for Linguistic values of Proximity to other vehicles
Very far PRV(A)
Far PRV(B)
More of less far PRV(C)
Close PRV(D)
Very close PRV(E)
Mumber-toBumber PRV(F)
1
10
20
25
35
40
50
55
65
70
80
Density (veh/mile)

Chackroborty and Kikuchi propose a rather complicated way to determine the LOS(i) classes for an input value $x$ [veh/mile]. Indeed, they use fuzzy measures, fuzzy integrals and Sugeno's $\lambda$–weights.

By total fuzzy similarity method a traffic situation $x$ in class LOS($i$), $i = A, \cdots, F$, is easily calculated via

$$S_{LOS(i)}(x) = \frac{1}{4}[\mu_{MAN(i)}(x) + \mu_{CON(i)}(x) + \mu_{SSP(i)}(x) + \mu_{PRV(i)}(x)]$$

For example, if traffic flow $x$ is 20 vehicles/mile, the these equations yield the following table (membership degrees in fuzzy sets are obtained by ocular estimate from respective graphs)

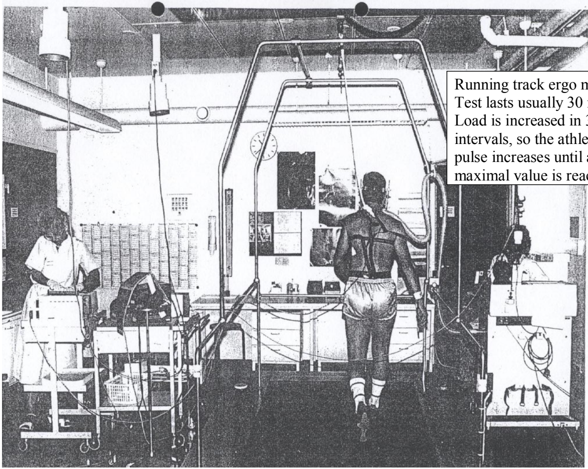
| i | MAN(i) | CON(i) | SSP(i) | PRV(i) | Total sim. degree |
|---|---|---|---|---|---|
| A | 0 | 0.4 | 0.6 | 0 | 0.25 |
| B | 0.6 | 0.6 | 0.4 | 1 | 0.65 |
| C | 0.6 | 0 | 0 | 0 | 0.10 |
| D | 0 | 0 | 0 | 0 | 0 |
| E | 0 | 0 | 0 | 0 | 0 |
| F | 0 | 0 | 0 | 0 | 0 |

Thus, the traffic situation 20 vehicles/mile belongs primary to class LOS(B), secondary to class LOS(A) and ternary to class LOS(C). It is worth noticing that we obtain the same result than Chackroborty and Kikuchi by the method they proposed.

## Application 2 – Determining Athlete's Thresholds.

A 100 meters sprinter has to run a short distance very fast, therefore, he has to have much training in the anaerobic zone where his pulse is close to maximal value, while a long distance runner needs endurance, thus, he needs training in the aerobic zone.

It is important for an athlete to let diagnose his aerobic and anaerobic thresholds regularly. These tests can be done e.g. on a running track ergo meter, see the following picture.



Running track ergo meter. Test lasts usually 30 min. Load is increased in 3 min intervals, so the athlete's pulse increases until almost maximal value is reached.

A test to determine a sportsman's aerobic and anaerobic thresholds is going on. The sportsman is wearing a mask that collects his respiration gases, continuous blood sample equipment is connected to his right forefinger and pulse is measured by belt around breast.

Aerobic and anaerobic thresholds are functions of

- blood lactate [mmol/l]
- ventilation $CO_2$ [l/min]
- $O_2$ uptake [%].

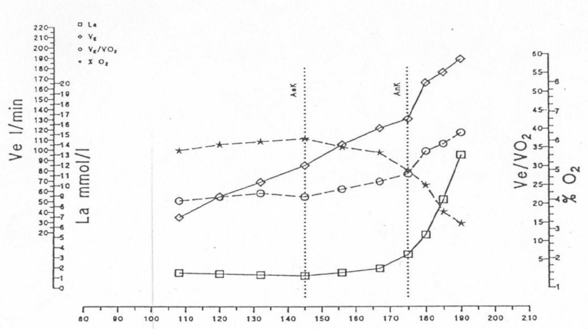
They, in turn, are functions of heartbeat [b/min].

A test starts with a 3 minutes warm–up, then pulse is around 100 b/min, and then the load is increased in every 3 minutes and blood lactate, ventilation $CO_2$, $O_2$ uptake and heartbeat are measured. A test lasts until volitional exhaustion (pulse near 200 b/min); this takes usually 20–30 minutes.

All measured data is collected on a test protocol, an example is presented on the next slide.

An aerobic and anaerobic test protocol

All measured values are plotted on one graph: Ventilation [l/min], lactate [mmol/l], $O_2$ uptake [%] and a relation Ventilation/ $O_2$ ventilation are all functions of heartbeat [b/min]. Based on such information and using certain vague rules (presented on the next slide), an experienced medical doctor or a coach is able to determine the aerobic threshold (Aek here 145 b/min) and anaerobic threshold (Ank here 175 b/min).




Pulse [beats/min]

The criteria to identify aerobic threshold are

• blood lactate has a very low value, blood lactate value starts to increase (most important criterion),

• ventilation is increasing, $O_2$ uptake [%] value is very high (the second most important criterion),

• pulse is about 40 b/min less than the maximal measured pulse, maximal tolerance being +/- 20 b/min.

Similarly, the criteria to identify anaerobic threshold are

• blood lactate value is rapidly increasing and is 3 mmol/l (most important criterion),

• ventilation is clearly increasing (the second most important criterion), $O_2$ uptake [%] value is decreasing (the third most important criterion),

• pulse is about 20 b/min less than the maximal measured pulse, maximal tolerance being +/- 5 b/min.

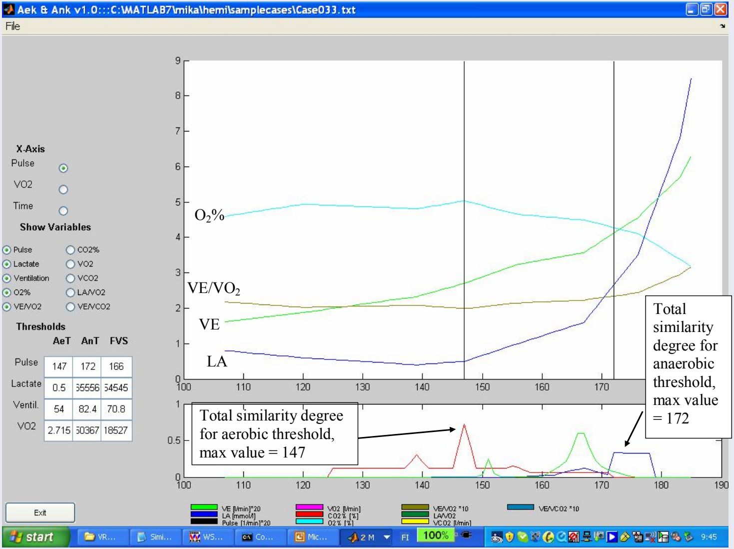
Clearly, all these criteria are expressible by fuzzy sets. They are context dependent, too. For example maximal pulse – 40 depends on a respective measurement. To mimic a specialist's action in determine an aerobic threshold, we need only one rule – expressible by a 5 component rule vector – namely

Blood lactate has a very low value
AND
blood lactate value starts to increase
AND
ventilation is increasing
AND
$O_2$ uptake [%] value is very high
AND
pulse is about 40 b/min less than the maximal measured pulse.

To determine an anaerobic threshold, a similar rule is needed.

The principal idea in automated threshold determination is to compare each measured input vector with the rule vector and search out the most similar one, i.e. apply total fuzzy similarity method. Mika Hempilä implemented this idea in his diploma work in which he

- consulting with medical experts, defined and implemented all the needed context dependent fuzzy sets,
- extended discrete rule vectors and input vectors to continuous valued vectors by using Spine functions,
- created a Matlab software which, after receiving an input vector – data of a threshold protocol – plots the corresponding graphs and reports where the most similar case is located; see an example on the next slide.



Aek & Ank v1.0:::C:\MATLAB7\mika\hemi\samplecases\Case033.txt
File
X-Axis
Pulse
VO2
Time
Show Variables
Pulse
Lactate
Ventilation
O2%
VE/VO2
CO2%
VO2
VCO2
LA/VO2
VE/VCO2
Thresholds
AeT AnT FVS
Pulse 147 172 166
Lactate 0.5
Ventil. 54 82.4 70.8
VO2 2.715
Exit
O2%
VE/VO2
VE
LA
Total similarity degree for aerobic threshold, max value = 147
Total similarity degree for anaerobic threshold, max value = 172
VE [l/min]*20
LA [mmol/l]
Pulse [1/min]*20
VO2 [l/min]
CO2% [%]
O2% [%]
VE/VO2 *10
LA/VO2
VCO2 [l/min]
VE/VCO2 *10

## Application 3 – Signalized isolated pedestrian crossing

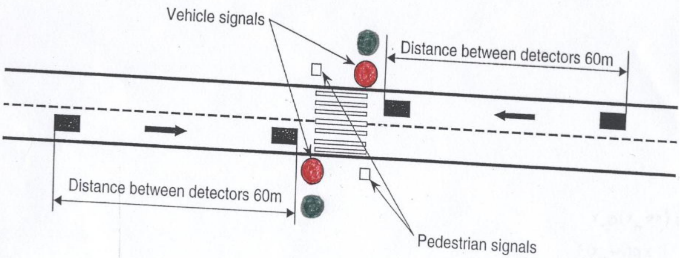
40% of traffics signals in Finland are in pedestrian crossings, for example near public buildings like schools.

Pedestrian aiming to cross the street pushes signal bottom. Detectors embedded in driveways recognize the amount of vehicles approaching the crosswalk, and caps between the vehicles.

Based on this data and simple rules that a traffic policeman would use, the task is automatically decide on vehicles' signal.

A fuzzy decision system for that purpose, based on total fuzzy similarity method, was introduced by Niittymäki and Turunen, and is implemented in several pedestrian crossings in Finnish cities.

Pedestrian aiming to cross the street pushes signal bottom. Detectors embedded in driveways recognize the amount of vehicles approaching the crosswalk, and caps between the vehicles. Based on this data and simple rules, the task is to automatically decide on vehicles' signal.

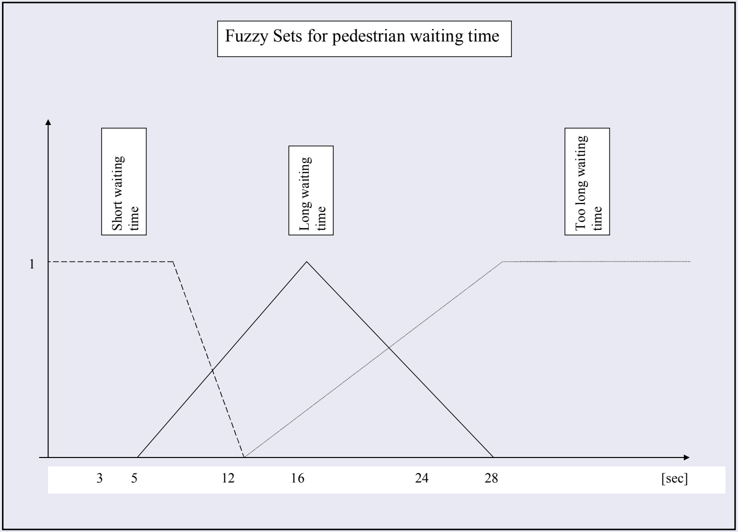



Layout of signalized pedestrian crossing

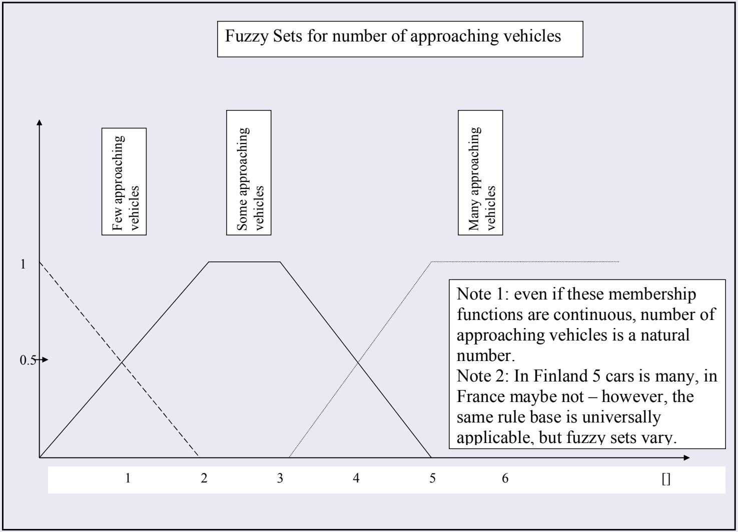
The rule base to define signal phase

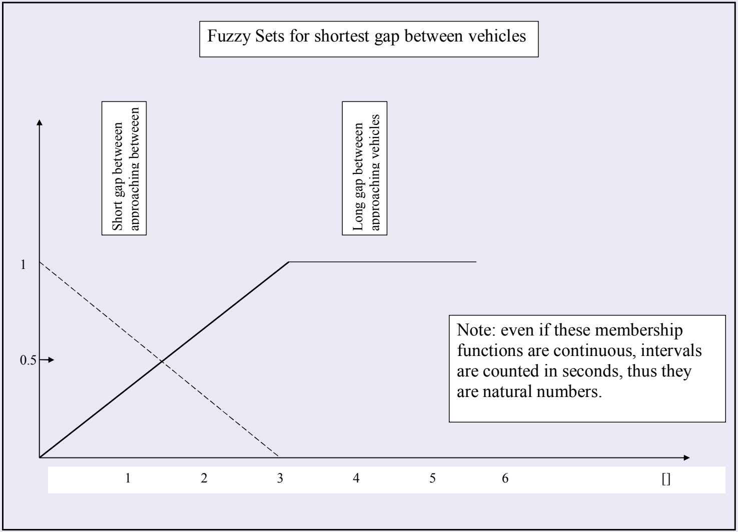
As long as there are no pedestrians, vehicles have green signal. If a pedestrian pushes a button, and no cars are approaching, the pedestrian will have immediately green signal. In case there are vehicles approaching and pedestrians waiting, then vehicles's green depends on the following factors:
• how long time have pedestrians been waiting for [a short/ long/ too long time]
• how many vehicles are approaching [few/ some/ many]
• what is the shortes gap [sec] between approaching vehicles [short/ large]
The decision is updated at intervals of one second. After a change, the selected phase lasts a fixed interval, and then the algorithm starts from beginning.

Experienced traffic engineers described the above fuzzy set as follows



Fuzzy Sets for pedestrian waiting time
Short waiting time
Long waiting time
Too long waiting time
1
3
5
12
16
24
28
[sec]


Fuzzy Sets for number of approaching vehicles
Few approaching vehicles
Some approaching vehicles
Many approaching vehicles
1
0.5
Note 1: even if these membership functions are continuous, number of approaching vehicles is a natural number.
Note 2: In Finland 5 cars is many, in France maybe not – however, the same rule base is universally applicable, but fuzzy sets vary.
1
2
3
4
5
6
[]



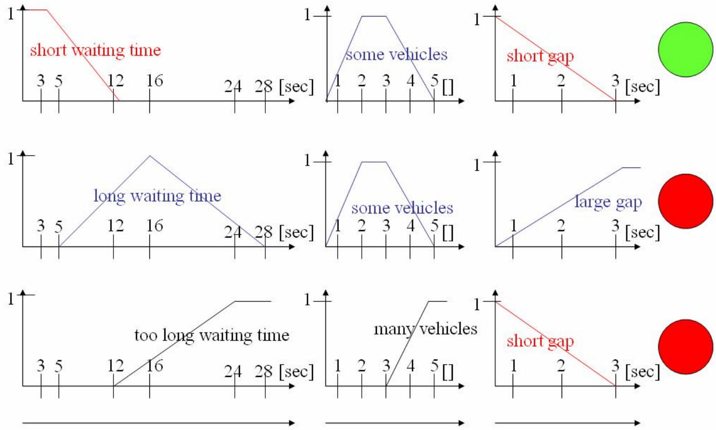
Fuzzy Sets for shortest gap between vehicles
Short gap betweeen approaching between
Long gap betweeen approaching vehicles
1
0.5
1
2
3
4
5
6
[]
Note: even if these membership functions are continuous, intervals are counted in seconds, thus they are natural numbers.

There are 18 rules in the fuzzy IF-THEN rule base. This corresponds to all possible combinations of the fuzzy sets thus, the rule base is complete. Vehicles green extension is prefered if there are several most similar rules to be fired.

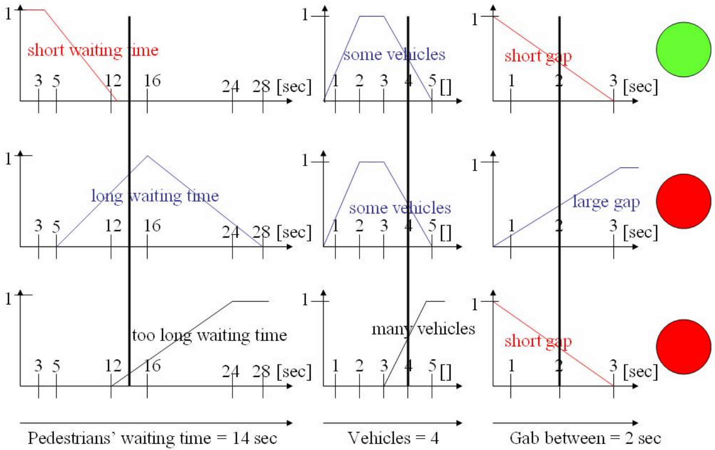
An example of a rule given by traffic engineers:
IF pedestrians' waiting time is short [weight = 1]
AND number of approaching vehicles is few [weight = 2]
AND shortest gab between approaching vehicles is short [weight = 1]
THEN extend vehicles green signal.

The output is always a crisp action; a red or green signal for vehicles. In a 1 – 1 situations it is green.

An example (assuming we would have only three rules)

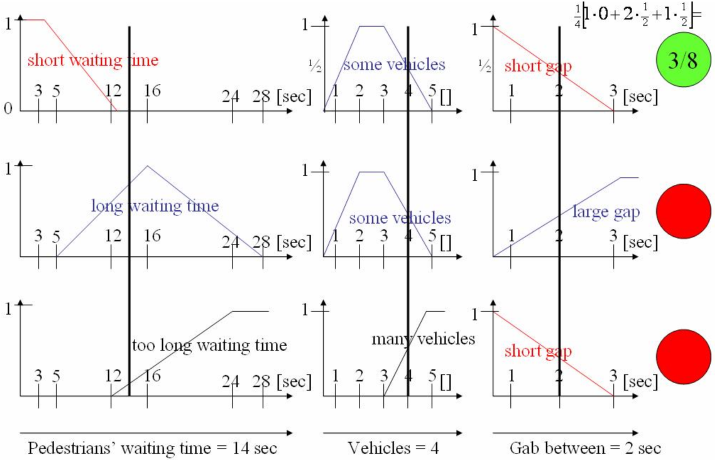


Assume the input vector – a traffic situation - is (14 sec, 4, 2 sec)

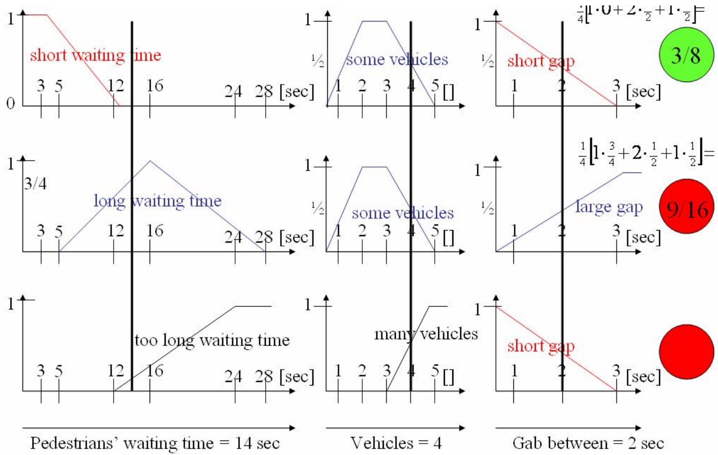



Which one of the three rules this input vector resembles the most?

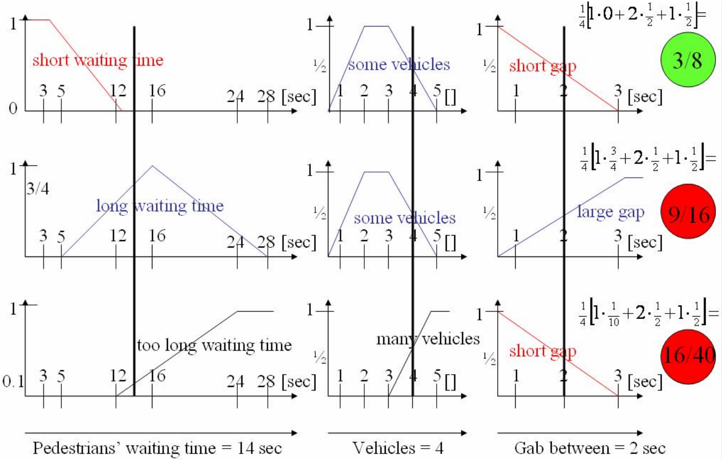
The degree of total similarity to the first rule and the input vector (14, 4, 2) is 0.375

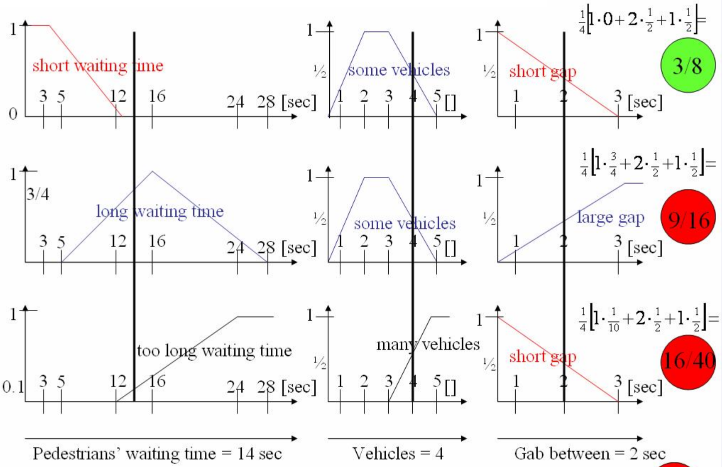


The degree of total similarity to the second rule and the input vector (14, 4, 2) is 0.5625

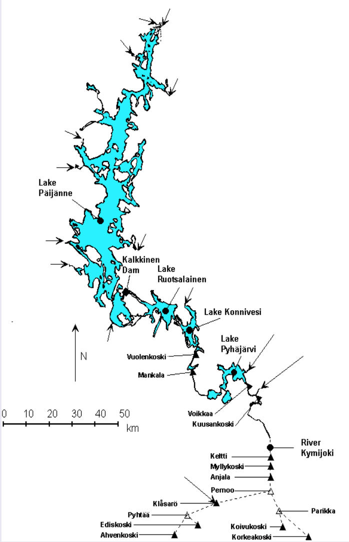


The degree of total similarity to the third rule and the input vector (14, 4, 2) is 0.4






The actual traffic situation is most similar to the 2. rule, thus...




## Application 6
## Real-Time Reservoir Operation

- Lake Päijänne is located in the Southern part of Finland, its water runs to the Golf of Finland via River Kymijoki.

- Each year Päijänne is frozen several months and lots of snow is accumulated.

- In spring floods caused by melting snow would be typical if Päijänne was not regulated; the water reference level is a function of date given by a law of Finnish parliament.

- Based e.g. on snow water equivalent, human experts are able to regulate several dams such that water level can be kept close to the reference level; a challenge is to create a formal control system to regulate water level of Lake Päijänne.

A control system based on Total Fuzzy Similarity Algorithm was created by Dudrovin, Jolma and Turunen.

The model consists of two real–time sub models; the first sub model sets up a reference water level (WREF) for each time step. Given this reference level, the observed water level (W), and the observed water inflow (I), the second sub model makes the decision on how much water should be released from the reservoir during the next time step.

For the snowmelt season, WREF value is dependent on the snow water equivalent (SWE) and can be inferred for each time step with the rules:

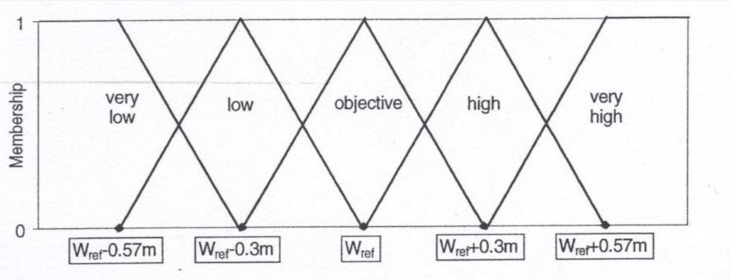
IF SWE is smaller than average/average/larger than average/much larger than average
THEN WREF is high/midle/low/very low.

In the second sub model, the rules have a form

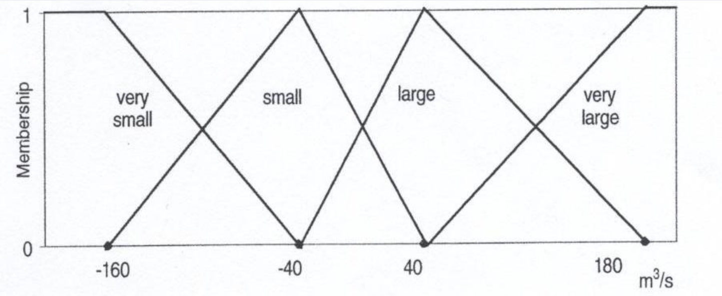
IF observed water level is very low/low/objective/high/very high
AND
observed water inflow is very small/small/large/very large
THEN
water release is exceptionally small/very small/small/quite small/quite large/large/very large/exceptionally large.

To calibrate the corresponding fuzzy set, a data of real control actions collected during 1975–1985 was used.

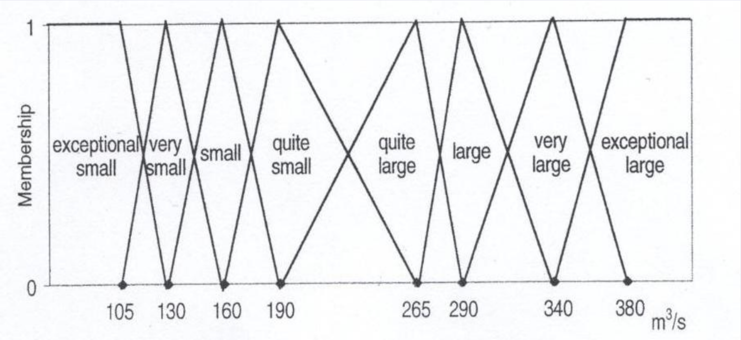
The membership functions for observed water level, observed water inflow and water release are presented on the next three slides.




Membership functions for observed water level W




Membership functions for water inflow I




Membership functions for water release (output)

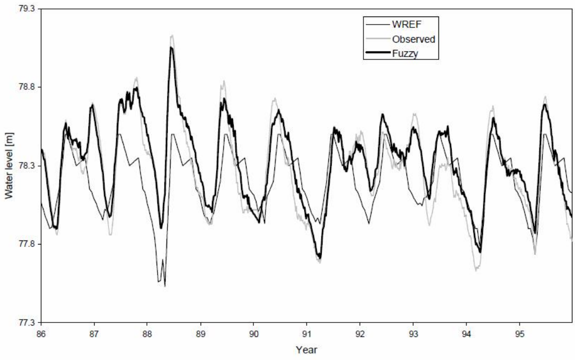
The model was tested using data from the years 1985–1996.

The Sugeno method – available in Matlab's Fuzzy logic Toolbox – was chosen for comparison against the Total Fuzzy Similarity Algorithm. With both methods the system was kept the same as much as possible. To apply the Sugeno method, the defuzzification was performed using a weighted average.

The performances of the two methods were almost indistinguishable. With the total fuzzy similarity method the water level targets during the summer were sometimes better fulfilled, but the release tended to fluctuate more, and the limitation on change in release was more relevant.

The model performance was generally good, but the first version of the model did not capture expert thinking in the most exceptional circumstances – later the model was completed by an extra subsystem to do the job.

Water reference level, observed water level and water level obtained by Total Fuzzy Similarity Control



Observed water release and water release ruled by Total Fuzzy Similarity Control