evropský sociální fond v ČR

EVROPSKÁ UNIE

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

OP Vzdělávání pro konkurenceschopnost

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Scientific stay

Universidad de Málaga

Jan Konečný

14. 4 - 15. 6. 2012

Research Visit – Jan Konečný – Malaga, Spain

Universidad de Málaga

- established 1972, almost 40000 students, 2000 researchers,
- new campus 1000 m^2
- 23 schools (50 departments)

Matemátika Aplicada

- 30 researchers
- various topics
- multiadjoint framework, functional dependencies, ...

Activities during the visit

Seminar

- on reduction size of CL.

Discussions on topics of joint research (with M. Ojeda-Aciego, J. Medina)

- comparison/merge of Radim's and Manuel's generalizations of FCA,
- almost closed sets,
- algorithm for computing a new lattice after change of priorities modelled by adjoint operations.

Joint research (with M. Ojeda-Aciego, J. Medina)

- Presented on CLA'12, Málaga, Fuengirola, Spain

Motivation

J. Medina and M. Ojeda-Aciego.
On multi-adjoint concept lattices based on heterogeneous conjunctors.
Fuzzy Sets and Systems, 2012.

- wide variety of concept-forming operators

R. Belohlavek and V. Vychodil.
Formal concept analysis and linguistic hedges.
Int. J. General Systems, 41(5):503–532, 2012.

- parametric way to reduce size of (fuzzy) concept lattice

We study

- What is ■ $\vee$ ■?
- What is ■ $\wedge$ ■?

Preliminaries – Linguistic (Truth-stressing) Hedges in FCA

Residuated lattice – algebra $\langle L, \wedge, \vee, \otimes, \rightarrow, \bot, \top \rangle$, s.t.

1. $\langle L, \wedge, \vee, \bot, \top \rangle$ – complete lattice, $\bot$ is the least element, $\top$ is the largest element.
2. $\langle L, \otimes, \top \rangle$ – commutative monoid
3. satisfies $x \otimes y \leq z$ iff $x \leq y \rightarrow z$ for each $x, y, z \in L$

Truth-stressing hedge – unary operation $*\colon L \rightarrow L$ satisfying

$$
\begin{aligned}
*(\top) &= \top \\
*(x) &\leq x \\
*(x \rightarrow y) &\leq *(x) \rightarrow *(y) \\
\left((x)\right) &= *(x)
\end{aligned}
$$

Meaning of $*(x)$: „It's *very true* that x"

L**-context** is a tuple $\langle A, B, R\rangle$

- $A, B \neq \emptyset$.
- R is a L-fuzzy relation $R\colon A \times B \to L$.

Formal concepts are pairs $\langle g, f\rangle$ satisfying

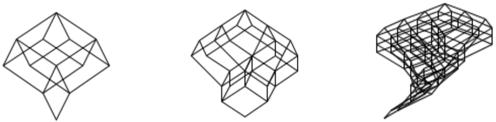
- $g \in L^B, f \in L^A$,
- $g = f^{\downarrow}, g^{\uparrow} = f$,

where

$$g^{\uparrow}(a) = \bigwedge_{x\in X} *_1(g(b)) \to R(a,b),$$

$$f^{\downarrow}(b) = \bigwedge_{y\in Y} *_2(f(a)) \to R(a,b).$$

The truth-stressing hedges serve as parameters to reduce size.



Preliminaries – Multi-adjoint concept lattices based on heterogeneous conjunctors

Given the posets $(P_1, \leq_1)$, $(P_2, \leq_2)$ and $(P, \leq)$, we say that P_1 and P_2 are **P-connected** if there exist non-decreasing mappings $\psi_1 \colon P_1 \to P$, $\phi_1 \colon P \to P_1$, $\psi_2 \colon P_2 \to P$ and $\phi_2 \colon P \to P_2$ satisfying that $\phi_1(\psi_1(x)) = x$, and $\phi_2(\psi_2(y)) = y$, for all $x \in P_1$, $y \in P_2$.

Multi-adjoint frame is a tuple

$$(L_1, L_2, P, \&_1, \swarrow^1, \nwarrow_1, \ldots, \&_n, \swarrow^n, \nwarrow_n)$$

where L_i are complete lattices and P is a poset, such that $(\&_i, \swarrow^i, \nwarrow_i)$ is an adjoint triple with respect to L_1, L_2, P
for all $i = 1, \ldots, n$; i.e.
$\&_i \colon L_1 \times L_2 \to P$, $\swarrow^i \colon P \times L_2 \to L_1$, $\nwarrow_i \colon P \times L_1 \to L_2$, such that

$$x \leq_1 z \swarrow^i y \quad \text{iff} \quad x \,\&_i\, y \leq z \quad \text{iff} \quad y \leq_2 z \nwarrow_i x,$$

where $x \in L_1$, $y \in L_2$ and $z \in P$.

- multi-adjoint frame is a generalization of residuated lattice

Let $(L_1, L_2, P, \&_1, \dots, \&_n)$ be a multi-adjoint frame.
Multi-adjoint context is a tuple (A, B, R, σ) s.t.

- $A, B \neq \emptyset$.
- R is a P-fuzzy relation $R\colon A \times B \to P$
- $\sigma\colon B \to \{1, \dots, n\}$ is a mapping which associates any element in B with some adjoint triple in the frame.

Formal concepts are pairs $\langle g, f \rangle$ satisfying

- $g \in L^B, f \in L^A$,
- $g = f^{\downarrow}, g^{\uparrow} = f$,

where

$$g^{\uparrow}(a) = \psi_1(\bigwedge_{1 b \in B} R(a, b) \swarrow^{\sigma(b)} \phi_2(g(b)))$$
$$f^{\downarrow}(b) = \psi_2(\bigwedge_{2 a \in A} R(a, b) \nwarrow_{\sigma(b)} \phi_1(f(a)))$$

Multi-adjoint concept lattice in MA-frame $\mathbf{A}$ is denoted $\mathfrak{M}_A$.

Let $(L, \leq, \top, \bot)$ be a complete lattice,
intensifying hedge is a mapping $*\colon L \to L$ s.t.

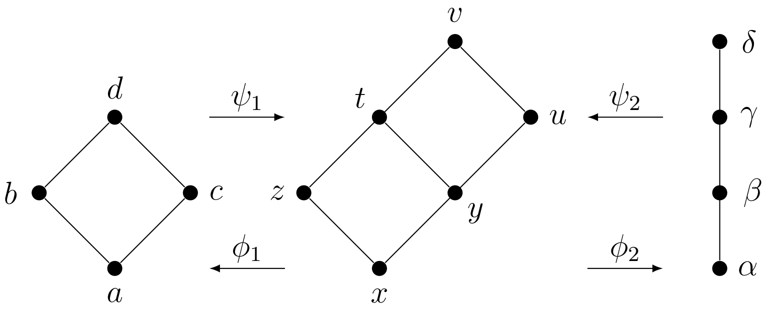
$$
\begin{gathered}
*(x) \leq x \\
x \leq y \text{ implies } *(x) \leq *(y) \\
((x)) = *(x)
\end{gathered}
$$

for each $x, y \in L$.

- in fact, it is a interior operator in L

Results – Reduction of Multi-Adjoint CL

- Reduction of concept lattice by a restriction of structures in the multi-adjoint frame.
  We restrict L_1, L_2 to $*_1(L_1)$, $*_2(L_2)$ to obtain a reduction.



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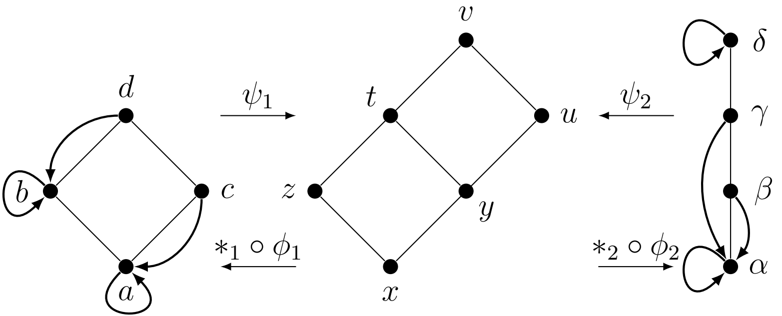


- adjoint triples $(\&_i, \swarrow^i, \nwarrow_i)$ are modified.

Results – Reduction of Multi-Adjoint CL

- Reduction of concept lattice by a restriction of structures in the multi-adjoint frame.
  We restrict L_1, L_2 to $*_1(L_1)$, $*_2(L_2)$ to obtain a reduction.

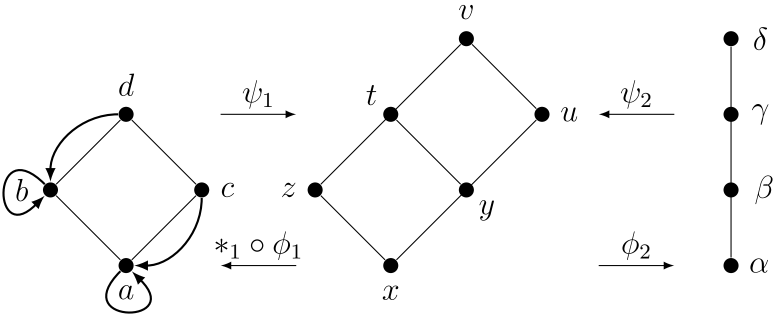
Let $\mathbf{A} = (L_1, L_2, P, \&_1, \dots, \&_n), \mathbf{A}' = (*_1(L_1), *_2(L_2), P, \&'_1, \dots, \&'_n)$ *be multi-adjoint frames, s.t.*

- $*_1$ *is an intensifying hedge in* L_1
- $*_2$ *is an intensifying hedge in* L_2
- $\&'_1, \dots, \&'_n$ *are restrictions of* $\&_1, \dots, \&_n$ *to* $*_1(L_1) \times *_2(L_2)$
- $\swarrow'^i, \nwarrow'_i$ *are residua of* $\&'_i$
- $\phi'_1 := *_1 \circ \phi_1, \phi'_2 := *_2 \circ \phi_2$

Then we have

$$|\mathfrak{M}_{\mathbf{A}'}| \leq |\mathfrak{M}_{\mathbf{A}}|.$$

- If we restrict just one of them we can say even more…



- If we restrict just one of them we can say even more...

Let $\mathbf{A} = (L_1, L_2, P, \&_1, \ldots, \&_n), \mathbf{A}' = (*_1(L_1), L_2, P, \&'_1, \ldots, \&'_n)$ *be multi-adjoint frames. s.t.*

- $*_1$ *is an intensifying hedge in* L_1
- $\&'_1, \ldots, \&'_n$ *are restrictions of* $\&_1, \ldots, \&_n$ *to* $*_1(L_1) \times L_2$
- $\phi'_1 = *_1 \circ \phi_1$,

Then,

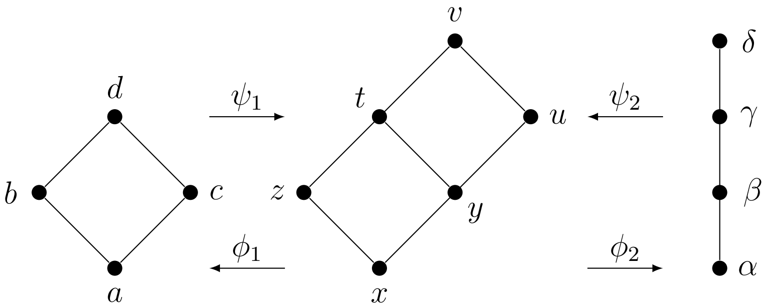
$$\mathrm{Ext}(\mathfrak{M}_{\mathbf{A}'}) \subseteq \mathrm{Ext}(\mathfrak{M}_{\mathbf{A}}).$$

($\mathrm{Ext}(\mathfrak{M})$ denotes the set of extents in $\mathfrak{M}$)

Little bit different approach:

- Now, we do not restrict the lattices;
  instead we just modify conjunctors $\&_i$ to $\&_i^*$ as follows:

$$x \&_i^* y := *_1(x) \&_i *_2(y)$$



Can we obtain the same as before?

Little bit different approach:

- Now, we do not restrict the lattices;
  instead we just modify conjunctors $\&_i$ to $\&_i^*$ as follows:

$$x \&_i^* y := *_1(x) \&_i *_2(y)$$

Let $\mathbf{A} = (L_1, L_2, P, \&_1, \dots, \&_n)$ *be a multi-adjoint frame;*
$*_1, *_2$ *be hedges on* L_1 *and* L_2.
Let $\mathbf{A}' = (*_1(L_1), *_2(L_2), P, \&_1', \dots, \&_n')$ *(as in previous theorems) be a multi-adjoint frame, s.t.*

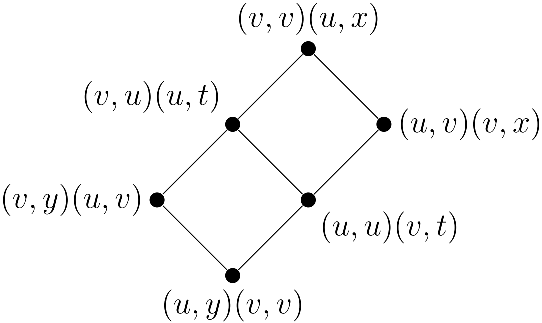
- $\&_1', \dots, \&_n'$ *are restrictions of* $\&_1, \dots, \&_n$ *to* $*_1(L_1) \times *_2(L_2)$
- $\phi_1' = *_1 \circ \phi_1, \phi_2' = *_2 \circ \phi_2$.

Let $\mathbf{A}^* = (L_1, L_2, P, \&_1^*, \dots, \&_n^*)$ *be a multi-adjoint frame, s.t.* $\&_i^*$ *satisfy* some technical conditions.

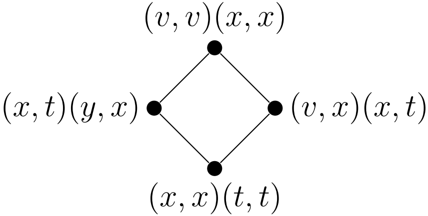
Then $(\mathfrak{M}_{A'}, \preccurlyeq')$ *and* $(\mathfrak{M}_{A^*}, \preccurlyeq^*)$ *are isomorphic.*

The $(\mathfrak{M}_{A'}, \preccurlyeq')$ and $(\mathfrak{M}_{A^*}, \preccurlyeq^*)$ are isomorphic.
They do not need to be equal (by illustration).

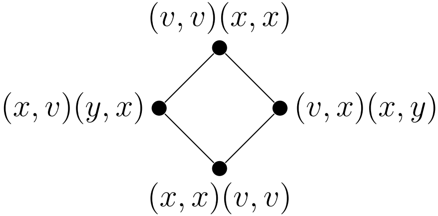
	1	2
1	u	v
2	v	y



$(\mathfrak{M}_{A'}, \preccurlyeq')$



$(\mathfrak{M}_{A^*}, \preccurlyeq^*)$



What if we used hedges analogous way as in (R. Belohlavek and V. Vychodil. Formal concept analysis and linguistic hedges. *Int. J. General Systems*, 41(5):503–532, 2012.)?

That is, if we define the concept-forming operators as follows

$$g^{\triangle}(a) = \psi_1 \bigwedge_{1 b \in B} R(a,b) \swarrow *_2(\phi_2(g(b))),$$
$$f^{\triangledown}(b) = \psi_2 \bigwedge_{2 a \in A} R(a,b) \nwarrow *_1(\phi_1(f(a))).$$

Concepts are in one-to-one correspondence with those from $(\mathfrak{M}_{A'}, \preccurlyeq')$ – reduction by restriction of L_1, L_2.

Main theorem must be stated differently.

The other results are rather technical. Read the paper :-)

Conclusions & Further Research

Conclusions

- merge of the two generalizations of FCA.
- parametric way to reduce size of multi-adjoint CL.

Further Research

- this seems to provide a good way how to handle different truth-degree structures for each attribute.
- the cases where the technical condition does not hold.

THANK YOU!