evropský sociální fond v ČR
EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
Univerzita Palackého v Olomouci
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Název projektu: Mezinárodní centrum pro informaci a neurčitost

Jak využít kvantovou informaci v optice…

Zdeněk Hradil
Department of Optics, Palacký University
Olomouc, Czech Rep.

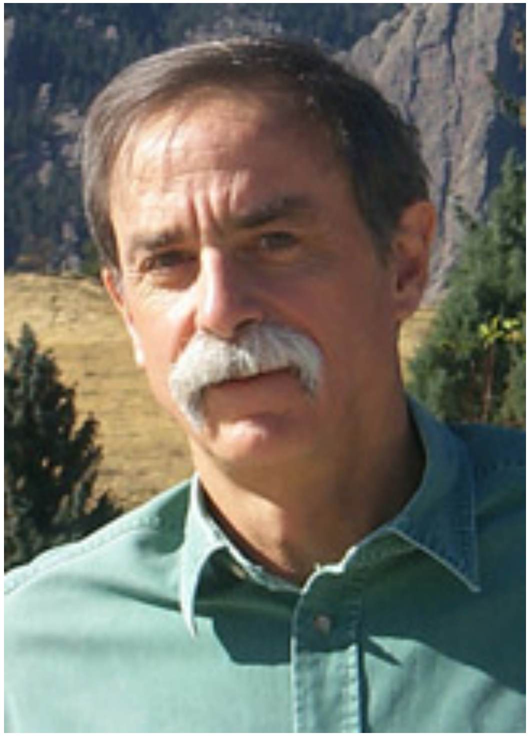
Spoluautoři: J. Řeháček, D. Mogilevtsev.
L. Sanchez-Soto, B.Englert, Y-S Teo, B. Stoklasa

Obsah

- Nobelova cena za fyziku 2012
- Optika: Optické zobrazování, Shack-Hartmannův sensor, rekonstrukce vlnoplochy, současné měření nekomutujících parametrů
- Čím se bavíme my v Olomouci – Tomografie založená na Radonově inverzi, MaxLik tomografie, …
- Co je to informace ? …
- Kvantová informace ve světě …

Kvantová informatika

Nobelova cena za fyziku 2012:
Serge Haroche and David J. Wineland Particle control in a quantum world



New Journal of Physics
The open access journal for physics

Optimal time-resolved photon number distribution reconstruction of a cavity field by maximum likelihood

C Sayrin[1], I Dotsenko[1], S Gleyzes[1], M Brune[1,3], J M Raimond[1] and S Haroche[1,2]
[1] Laboratoire Kastler-Brossel, ENS, UPMC-Paris 6, CNRS, 24 rue Lhomond, F-75005 Paris, France
[2] Collège de France, 11 place Marcelin Berthelot, F-75005 Paris, France
E-mail: michel.brune@lkb.ens.fr

[1] Paris M and Rehacek J (ed) 2004 *Quantum State Estimation* (*Lecture Notes in Physics* vol 659) (Berlin: Springer)

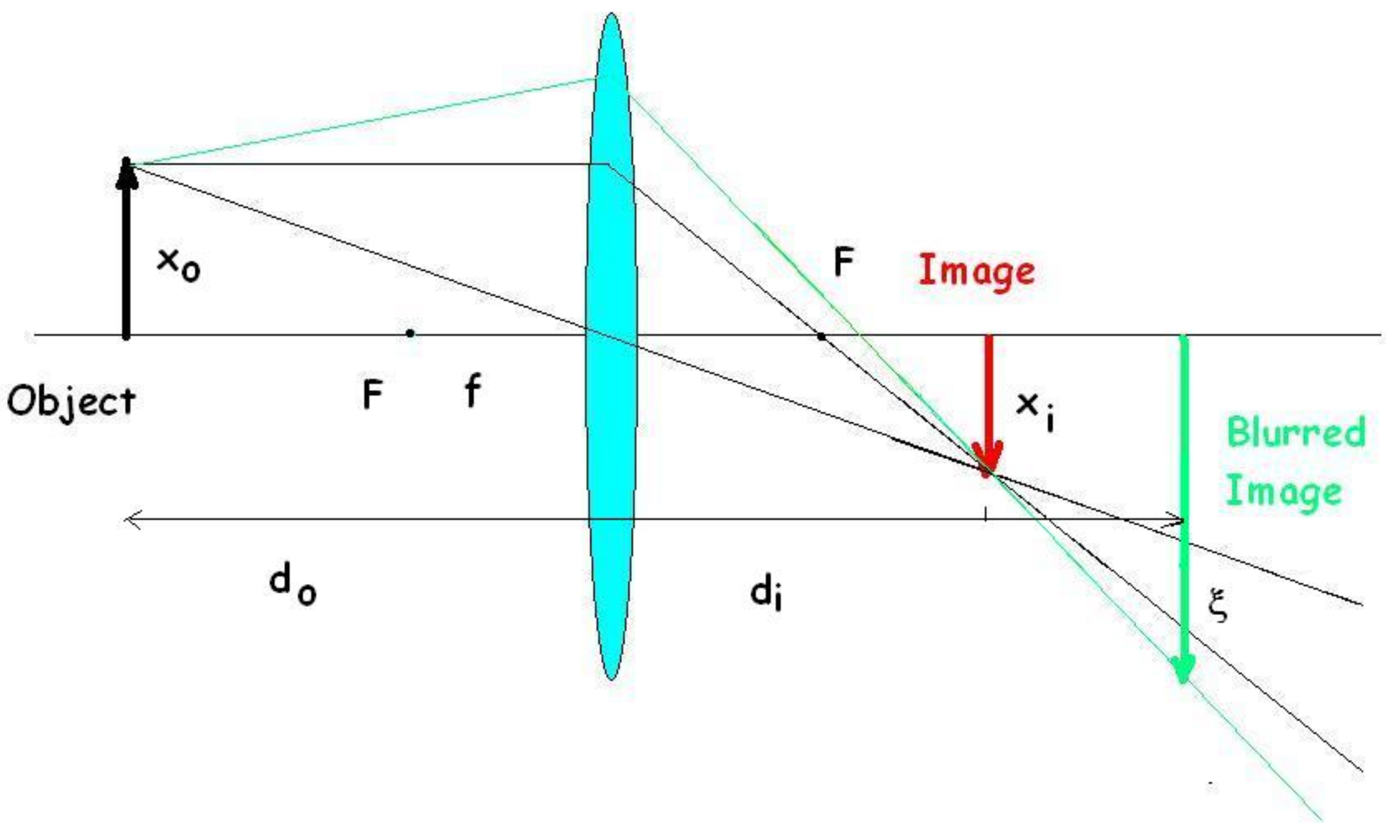
[13] Rehacek J, Hradil Z and Jezek M 2001 Iterative algorithm for reconstruction of entangled states *Phys. Rev.* A **63** 040303

Schroedingerovy kočky vs. Bellovy nerovnosti

Obsah

- Nobelova cena za fyziku 2012
- Optika klasicky i kvantově informaticky: Optické zobrazování, Shack-Hartmannův sensor, rekonstrukce vlnoplochy, současné měření nekomutujících parametrů
- Čím se bavíme my v Olomouci – Tomografie založená na Radonově inverzi, MaxLik tomografie, …
- Co je to informace a co je to Fisherova informace …
- Kvantová informace ve světě …

Optical Imaging: Lens equation in geometrical optics



Lens equation in geometrical optics:

$$1/d_o + 1/d_i = 1/f$$

For sharp image: $x_i = M\, x_o$,
magnification $M = d_i/d_o$

For the blurred image: $\xi = \alpha\, x_o + \beta\, p_o$
x_o … position of the ray
$p_o = 2\pi/\lambda\ \theta$ … direction of the ray

Meaning in quantum mechanics:

Rotated quadrature operator for $[x_o, p_o] = i\hbar$

See the analogy with the free evolution

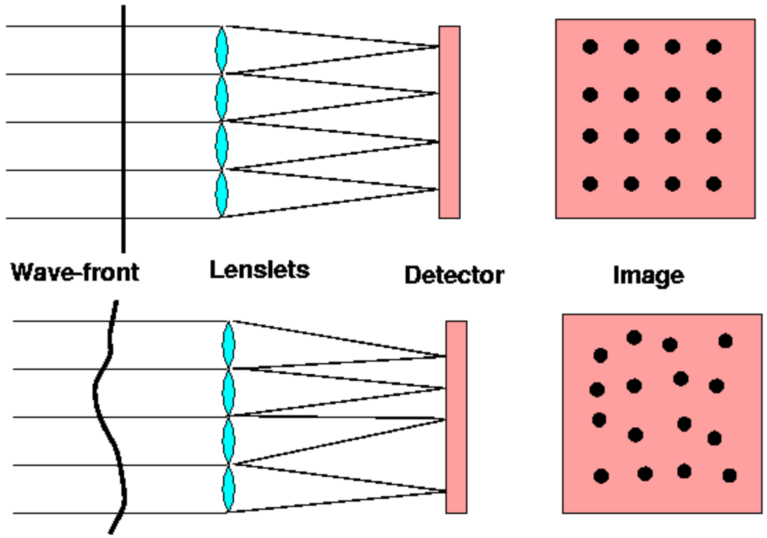
$$x(t) = x(0) + p(0)\, t/m$$

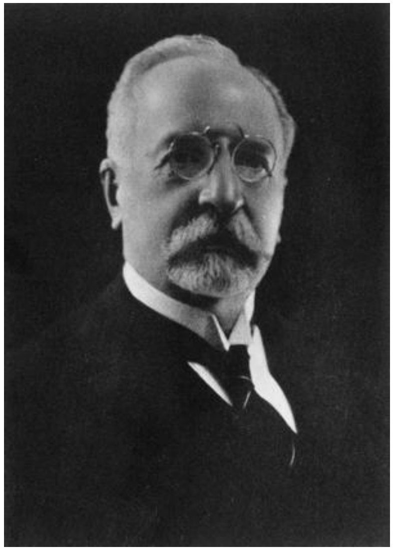
Klasická optika se snaží vytvořit obraz co možná nejpodobnější předmětu, drobné korekce jsou možné...

Digitální optika : všechno se dá dopočítat

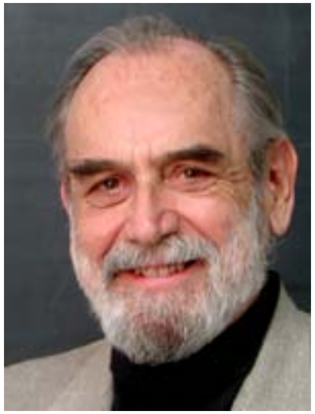
Kvantová tomografie: fundamentální statistická omezení plynoucí z neurčitosti dat (nekompletní informce)

Scanning of the optical field: Hartmann-Shack sensor



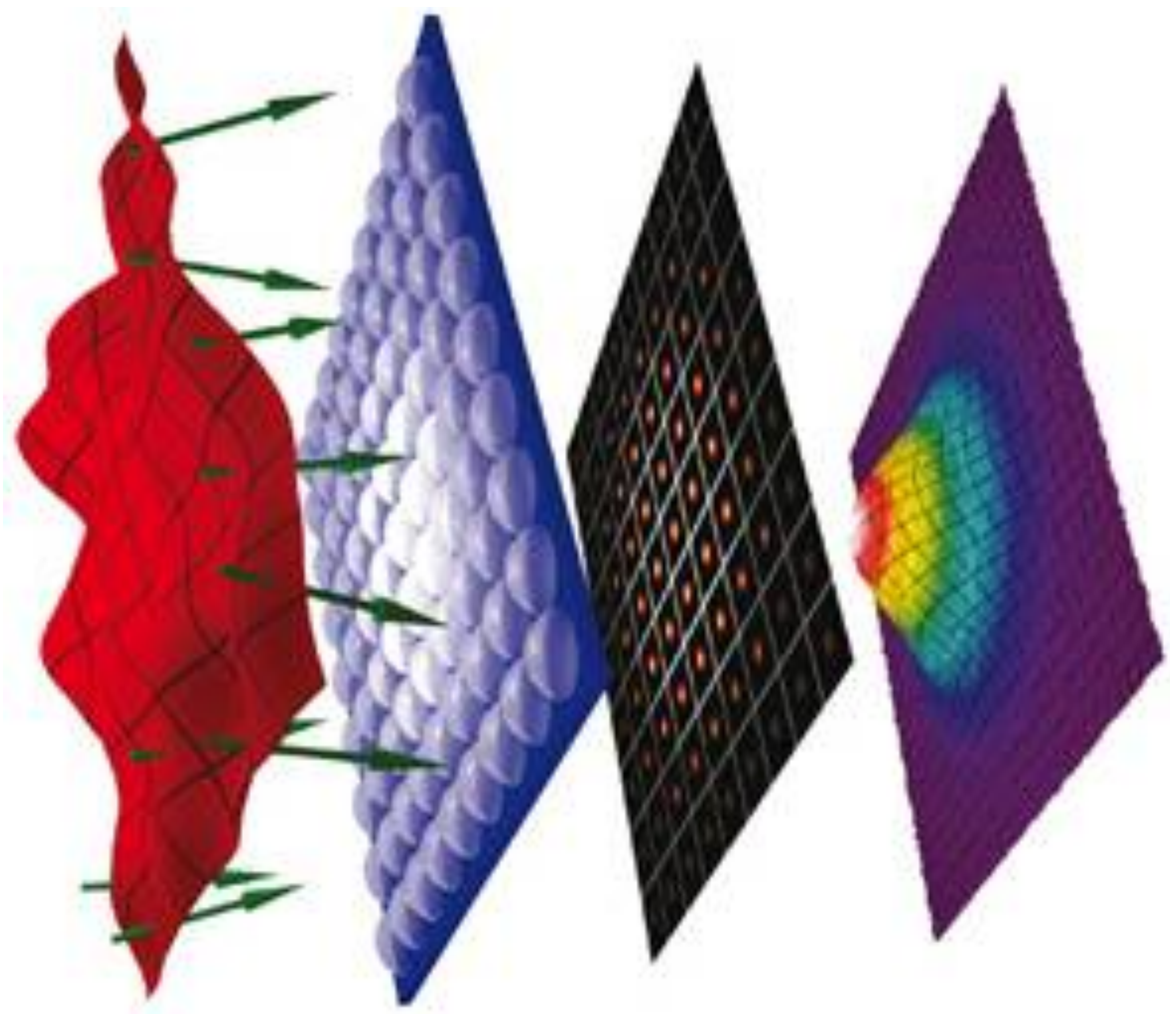


Johannes Hartmann
(1865-1936)



Roland Shack
(1970's)

Scheme of the wave-front reconstruction



Wave theory for HS sensor

- Detected amplitude:

$$\varphi_{det}(\xi)= \int dx'\, dq'\, \varphi(x')h(x'-q')A_i(q')\exp(i\, k\xi q'/f)$$

- Detected signal:

$$S_i(\xi) = \langle|\varphi_{det(}(\xi)|^2\rangle_{average}$$

$$= \int dx'\, dx'' \int dq'\, dq''\, Q(x',x'')\, h(x'-q')\, a(q',\xi)\, h^*(x''-q'')\, a^*(q'',\xi)$$

where Q… function of mutual coherence

$$a_i(q',\xi) = A_i(q')\exp(i\, k\xi q'/f)$$

- Quantum formulation in x-representation

$$S_i(\xi) = \langle a_{i\xi}|U^\dagger\, Q\, U|a_{i\xi}\rangle$$

$Q(x',x'')= \langle x'|Q|x''\rangle$, $h(x'-q')= \langle q'|U|x'\rangle$, $\langle x'|a_{i\xi}\rangle = a_i(q',\xi)$

HS sensor: Quantum Consequences

- Smooth Gaussian approximation of aperture function:

$$A_i(q') \approx \exp[-(q'-x_i)^2/4(\Delta x)^2]$$

- Detection= Projection into the minimum uncertainty states

$$a_{i,\xi} = \exp[-(q'-x_i)^2/4(\Delta x)^2 + i\, k\xi q'/f\,]$$

- Heisenberg uncertainty relations

$$\Delta x\, \Delta p \geq \hbar/2$$

- Generalized measurement of non-commuting variables x and p, (Arthurs, Kelly 1964)

$$\Delta X\, \Delta P \geq \hbar$$

See the excellent paper: S. Stenholm, *Simultaneous measurement of conjugate variables, Annals of Physics 218, 233-254 (1992).*

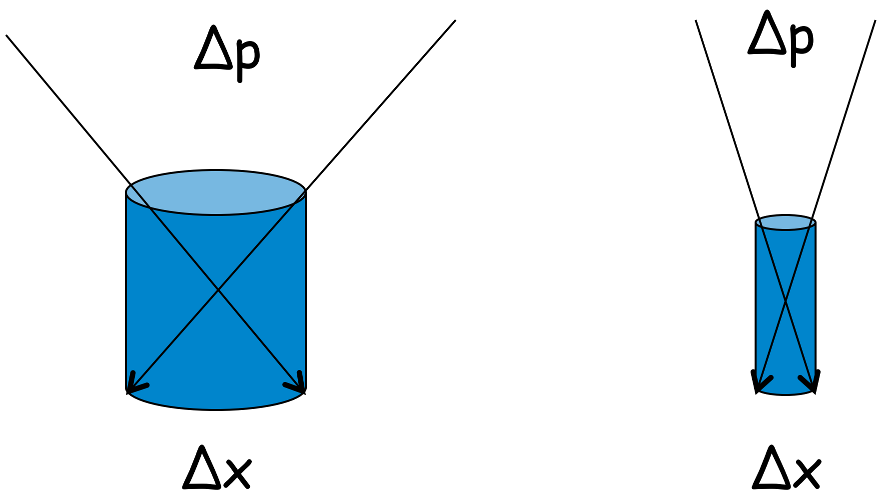
Further Quantum Consequences

- POVM corresponds to detection of annihilation operator

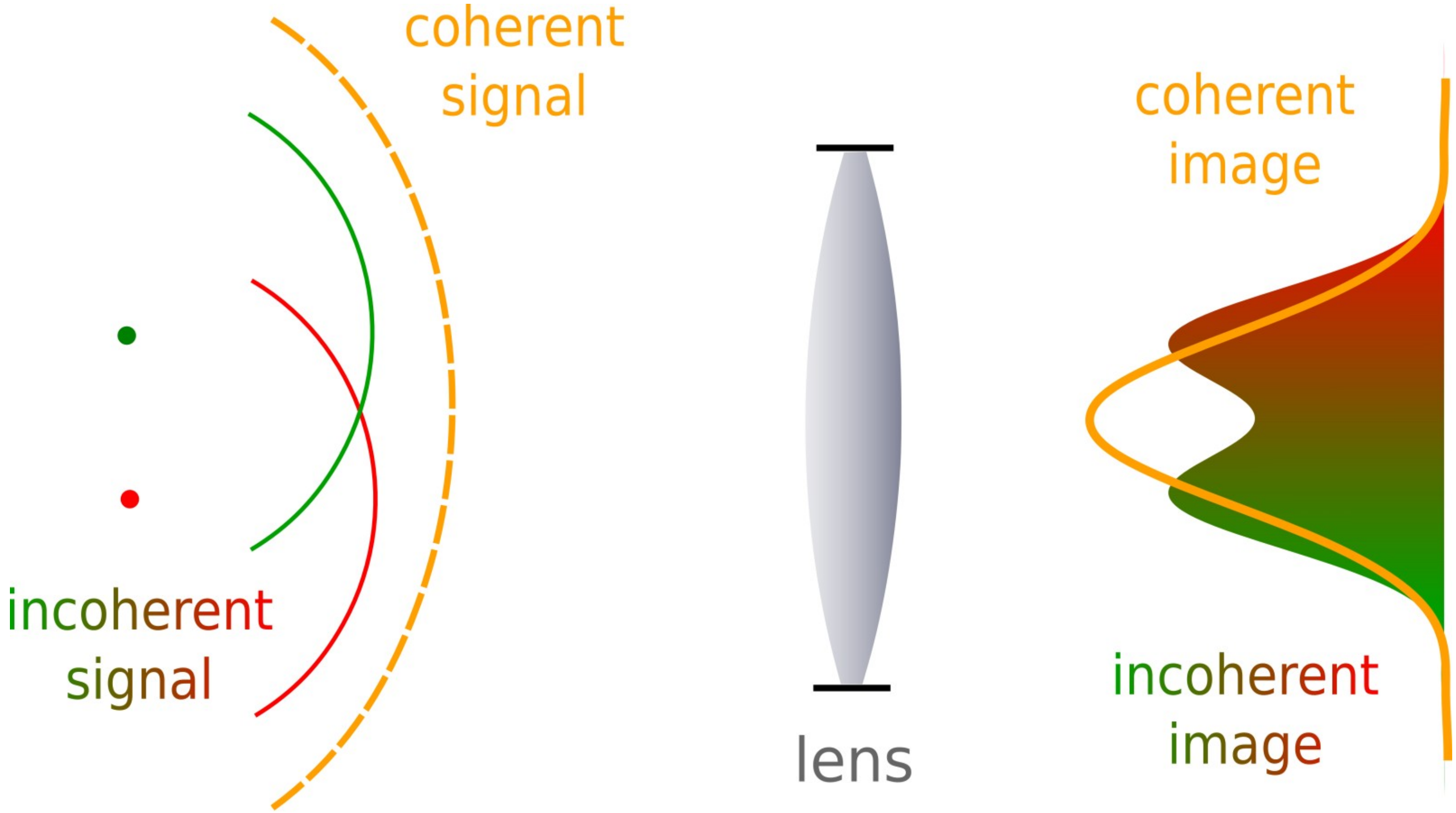
$$a = x + ip$$

$$1/\pi \int da^2 \; |a\rangle\langle a| = 1$$

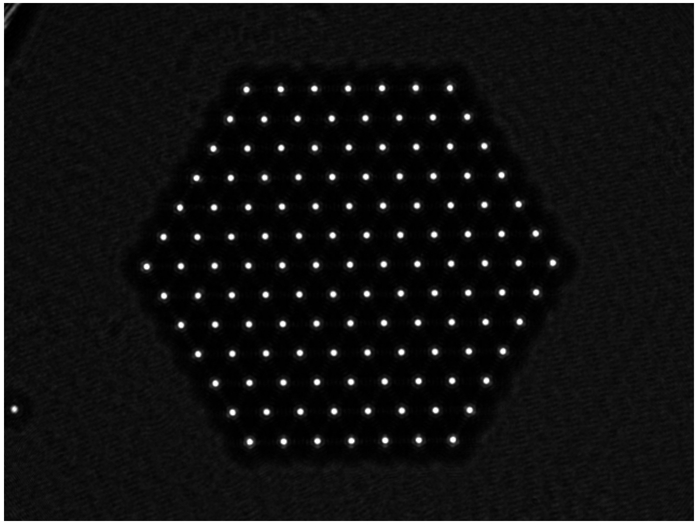
- Q-distribution (Husimi)

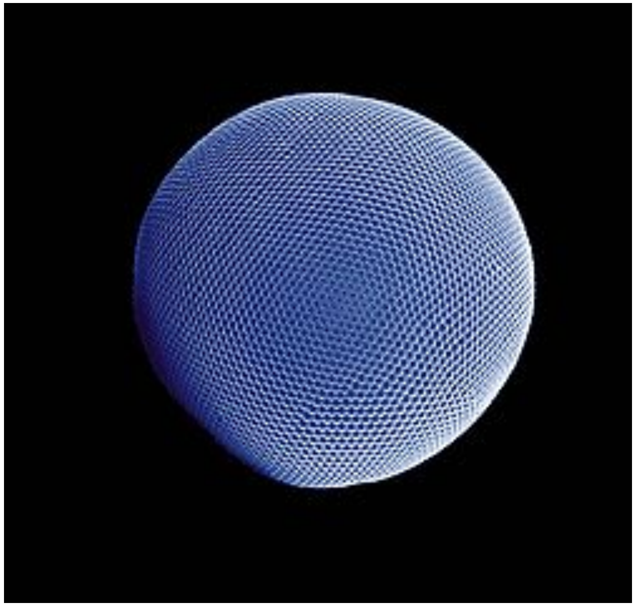


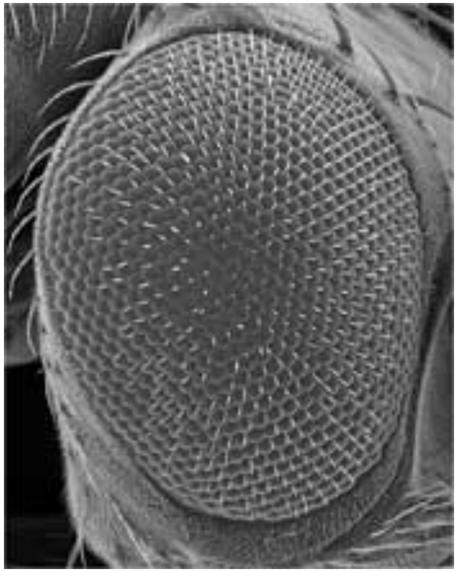
Detection of partially coherent signal



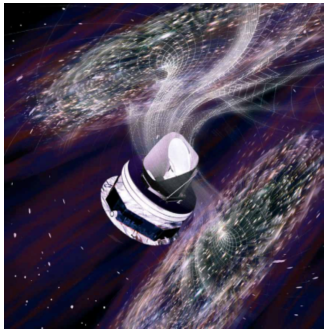
Hartmann-Shack sensor of the wavefront?

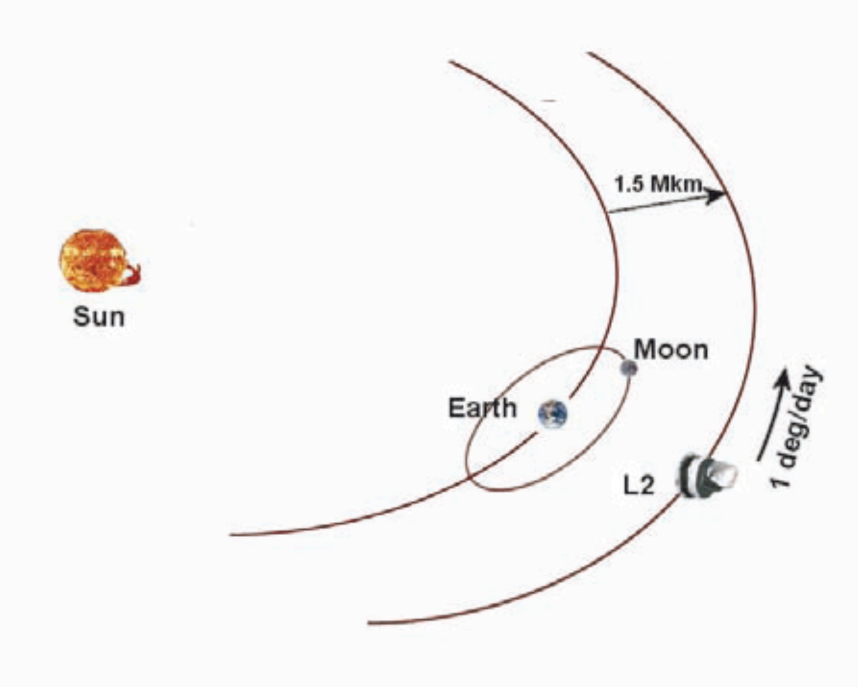


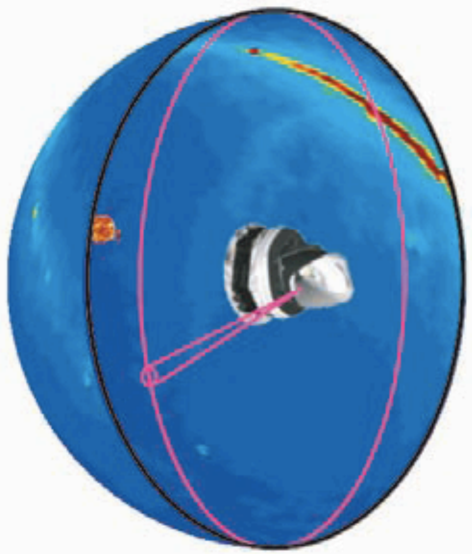




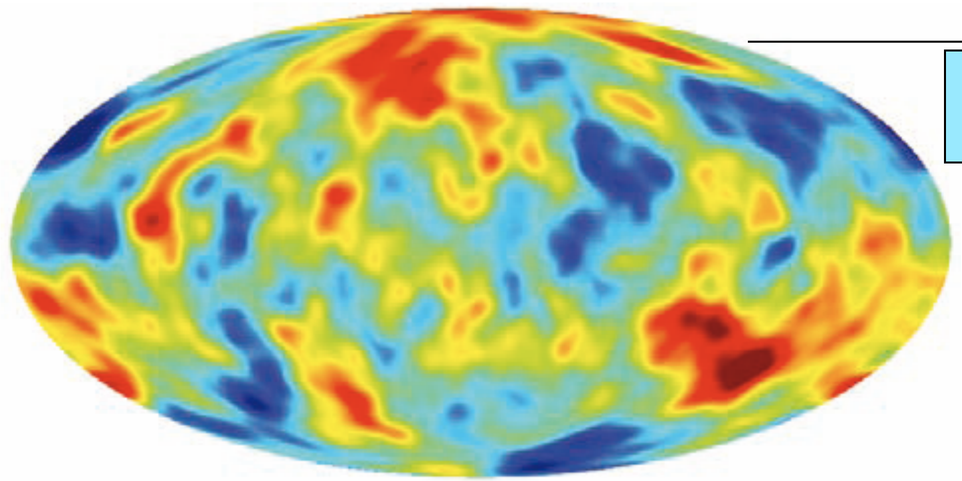
Planck mission of ESA: scanning of cosmic background radiation



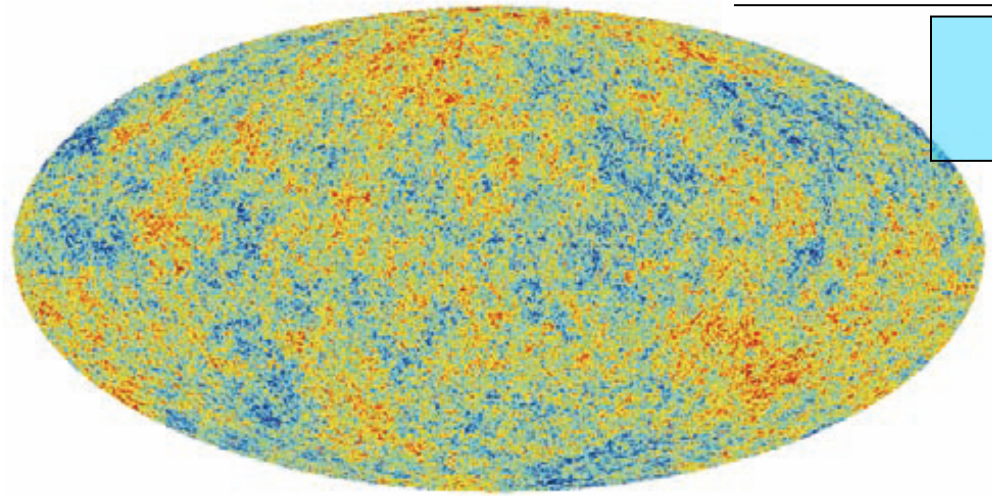




Temperature anisotropies



COBE-DMR resolution



Planck resolution

Linear inverse problems

ML estimation is excellent tool for solving linear inverse problems with constraints (= tomography)

$$I_j = \sum_k c_{jk}\,\mu_k$$

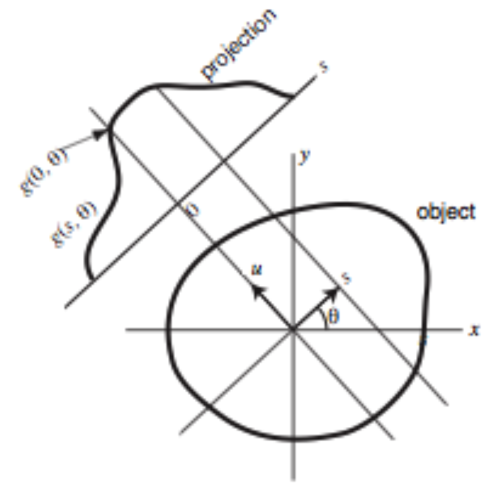
detected mean values I_j, $j = 1,2,\ldots M$
reconstructed signal μ_k $k = 1,2,\ldots N$

Over-determined problems **$M > N$**
Well defined problems **$M = N$**
Under-determined problems **$M < N$**

Tomography and Inverse Radon Transformation

Radon transformation

$$g(s,\theta) = \int dxdy f(x,y)\delta(x\cos\theta + y\sin\theta - s)$$



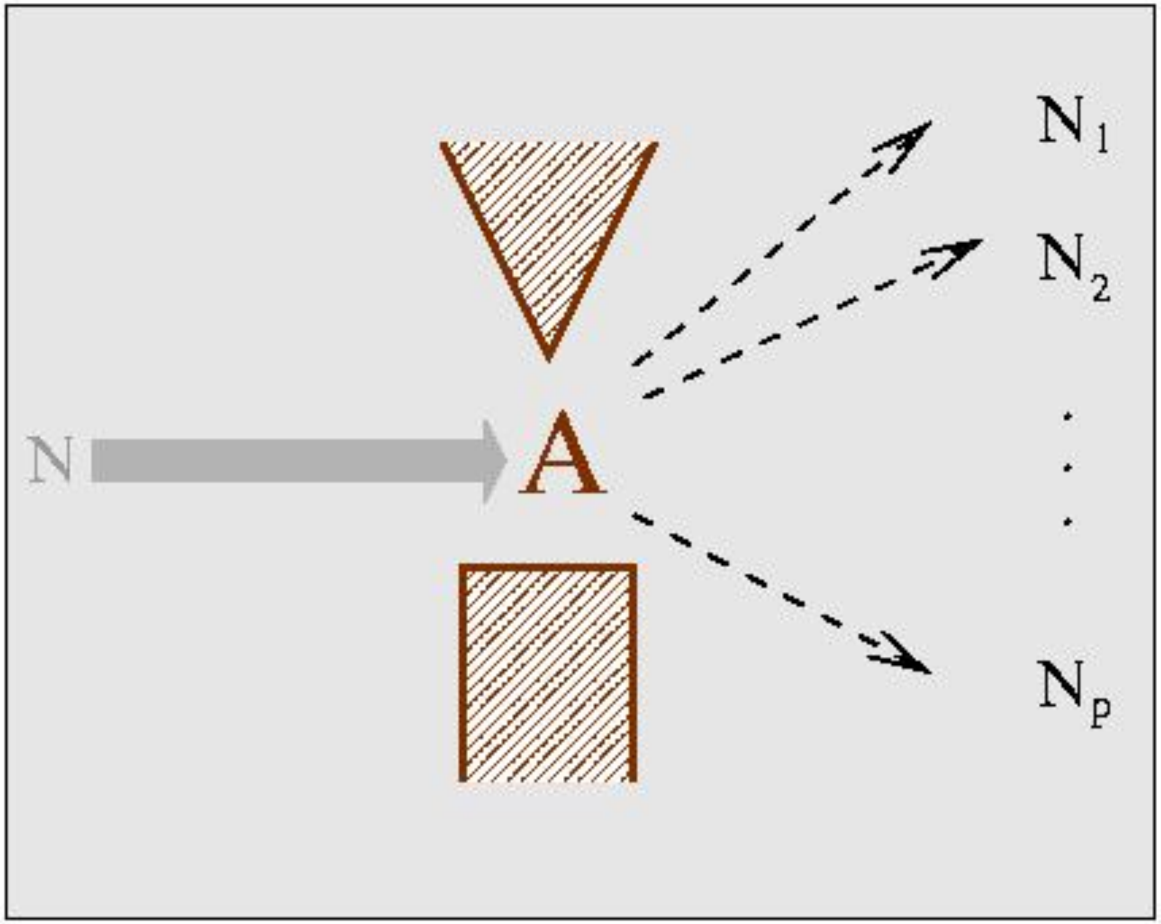
Projection theorem
(ray sum)

$$g(s,\theta) = \int_{-\infty}^{\infty} f(s\cos\theta - u\sin\theta, s\sin\theta + u\cos\theta)\, du$$

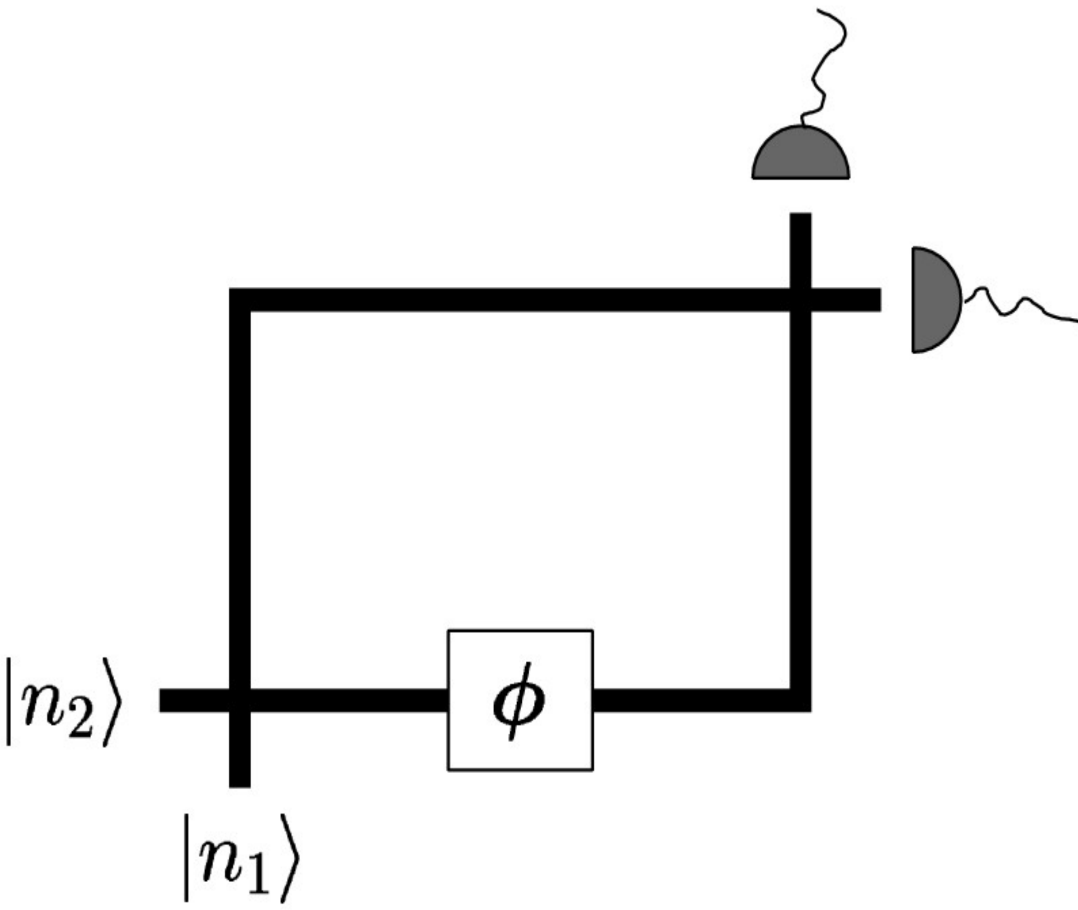
Inverse Radon transformation-
Fourier transformation method

$$G_\theta(\xi) = F(\xi\cos\theta, \xi\sin\theta)) \qquad f(x,y) = F^{-1}G_\theta$$

Von Neumann Measurement



Estimation Theory in Drawings



Necessary ingredients:

- Input signal
- Controllable transformation
- Feasible detection

Quantum Estimation Theory

Quantum Estimation Theory
= Quantum Theory + Estimation Theory

Some peculiarities:

•Quantum state ρ plays the role of c-number (matrix) with special constraints ($\rho \geq 0$)

•Quantum measurement must obey uncertainty principle

Maximum Likelihood Estimation (1922)

Sir Ronald Aylmer Fisher, FRS (17 February 1890 – 29 July 1962)
http://digital.library.adelaide.edu.au/coll/special/fisher/papers.html

- Maximum Likelihood (MaxLik) principle is not a rule that requires justification: Bet Always On the Highest Chance!
- Numerous applications in signal analysis, optics, geophysics, nuclear physics,…
- A. Witten, The application of ML estimator to tunnel detection, Inverse Problems 7(1991), 49.
- MaxLik analysis= pea plant experiment of G. Mendel was contrived (too good to be true, statistically ☺)

Maximum Likelihood Tomography

•Likelihood **L** quantifies the degree of belief in certain hypothesis under the condition of the given data.
•MaxLik principle selects the most likely configuration
•Information is updated according to the Bayes rule prior probability ➔ posterior probability

$$P(\rho|D) = P(D|\rho)\, p(\rho)\, [p(D)]^{-1}$$

Generic reconstruction scheme

Log-likelihood for generic measurement

$$\log L = \sum_i N_j \log p_j / (\sum_k p_k)$$

(probabilities are mutually normalized)

Equivalent formulation: estimation of parameters with Poissonian probabilities and unknown mean λ (constrained MaxLik by Fermi)

$$\log L = \sum_j N_j \log (\lambda\, p_j) - \lambda \sum_j p_j$$

Likelihood is convex functional defined on the convex manifold of density matrices

Information criteria and MaxLik tomography

“The most valuable commodity I know of is information, wouldn’t you agree?” (M. Douglas as tycoon Gordon Gekko in the movie Wall Street)

Good statistical models

Many random phenomena, such as those arising in biological and ecological applications, are extremely complex, potentially involving an endless assortment of variables and interactions, „good“ models are needed.An optimal statistical model is characterized by three fundamental attributes:

1. Parsimony (model simplicity)
2. Goodness-of-fit (conformity of the fitted model to the data at hand)
3. Generalizability (applicability of the fitted model to describe or predict new data)

Parsimony

•**Law of Parsimony: No more causes should be assumed than those that will account for the effect.**

More philosophy behind:

•Occam's Razor: "Plurality should not be posited without necessity." (Franciscan monk William of Ockham 1285–1349)

•"Everything should be made as simple as possible, but not simpler." (Albert Einstein, 1879–1955).

•"When you hear hoofbeats, think horses, not zebras." (popular adage from medical schools and residency programs)

•"Simplicity is the ultimate sophistication." (Leonardo da Vinci, 1452–1519).

•Laplace's Principle of Insufficient Reasoning: If there is no reason to prefer among several possibilities, than the best strategy is to consider them as equally likely and pick up the average.

All models are wrong, some are useful (George E. P. Box)

Akaike´s information criterion (AIC)

Akaike,IEEE Trans. Auto Control 19, 716 (1974)

Schwarz and Bayesian Information Criterion (BIC)

Schwarz, Annals of Stat. 6, 461 (1978)
Konishi, Ando, Imoto, Biometrica 91, 27 (2004)

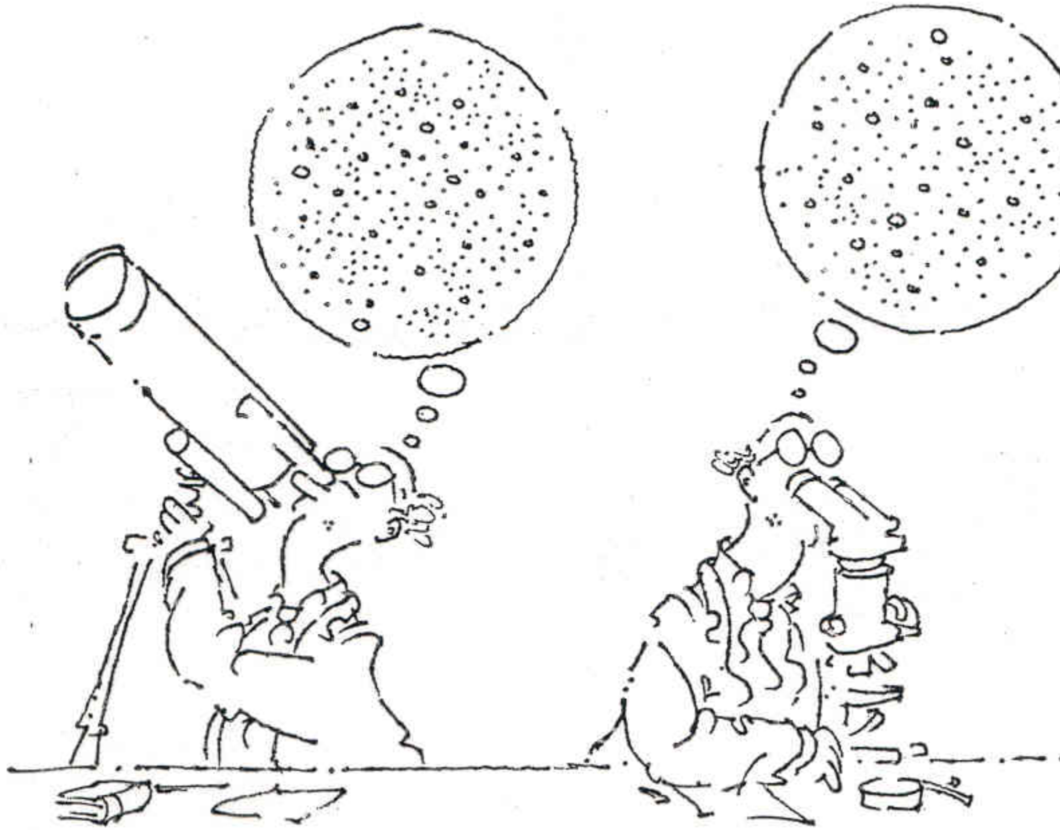
Entropy and quantification of ignorance

Yong Siah Teo, Huangjun Zhu, B-G Englert, J. Řeháček, Z. Hradil,

Quantum-State Reconstruction by Maximizing Likelihood and Entropy,**Phys. Rev. Lett. 107, 020404 (2011)**

Co všechno jsme už řešili …

- Phase estimation
- Transmission tomography
- Tomography of CP maps
- Reconstruction of photocount statistics
- Image reconstruction
- Vortex beam analysis
- Quantification of entanglement
- Reconstruction of neutron wave packet
- Reconstruction based on homodyne detection
- Full reconstruction based on on/off detection
- Reconstruction of coherent matrix



THE FIELDS INSTITUTE

Workshop on the Mathematical Methods of Quantum Tomography

February 19 – 22, 2013

FIELDS INSTITUTE

The objective in quantum tomography is to reconstruct the density matrix of a quantum state using "probes" (theoretical or experimental). The density matrix is a generalization of the quantum state vector that evolves following the Schrödinger equation of quantum mechanics. The solution to problems in this field are also of interest beyond the immediate quantum tomographic community as they are essential for a better understanding of quantum information processes and the prospective construction of a quantum computer. The objective of the Workshop is to gather experts and students working in the development and applications of mathematical tools related to quantum tomography and related problems.

ORGANIZERS

Hubert de Guise (Lakehead University)
Berthold-Georg Englert (National University of Singapore)
Zdenek Hradil (Palacky University)
Gerd Leuchs (Max Planck Institute for the Science of Light)
Luis L. Sanchez-Soto (Complutense University Madrid)

INVITED SPEAKERS

Gunnar Bjork (Royal Institute of Technology, Stockholm)
Agata Branczyk (University of Toronto)
Hubert de Guise (Lakehead University)
Ruynet L. de Matos Filho (Universidade Federal do Rio de Janeiro)
Aldo Delgado (Universidad de Concepción)
Rafal Demkowicz-Dobrzanski (University of Warsaw)
Berge Englert (National University of Singapore)
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The Fields Institute for Research in Mathematical Sciences
222 College Street, Toronto, ON M5T 3J1 Canada • Phone: (416) 348-9710 • Fax: (416) 348-9759 • www.fields.utoronto.ca