ENTANGLEMENT SHARING WITH SEPARABLE STATES

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INVESTMENTS IN EDUCATION DEVELOPMENT

Tripartite entanglement

Three qubits A, B and C, basis $\{|0\rangle, |1\rangle\}$.

- New type of nonlocality $\quad |GHZ\rangle = \frac{|000\rangle+|111\rangle}{\sqrt{2}}$ (GHZ 89')

- Two inequivalent separability classes (Dür et al PRA 00')

$$|GHZ\rangle,\ |W\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |010\rangle + |001\rangle)$$

- New types of bound entanglement

  (Dür et al PRA 00', Bennett et al PRL 99')

- First example of bound information (Acin et al PRL 04')

Entanglement classes

Three bipartite splittings $A-(BC)$, $B-(AC)$ and $C-(AB)$.

1. *Fully inseparable states* – entangled across all splittings, $|GHZ\rangle$.

2. *One-qubit biseparable states* – entangled across two splittings, $|\Psi_-\rangle_{AB}|0\rangle_C$.

3. *Two-qubit biseparable states* – entangled across one splitting.

4. *Three-qubit biseparable states* – separable across all splittings

   but $\widehat{\rho}_{ABC} \neq \sum_i p_i \widehat{\rho}_A^{(i)} \otimes \widehat{\rho}_B^{(i)} \otimes \widehat{\rho}_C^{(i)}$. $(*)$

5. *Fully separable states* – can be written as $(*)$, $|0\rangle_A|0\rangle_B|0\rangle_C$.

(Dür et al PRA 99')

Applications:

- Quantum secret sharing (Hillery et al PRA 99', Cleve et al PRL 99').
- Gate construction (Gottesman et al Nature 99')
- Assisted teleportation (Karlsson et al PRA 98')
- Telecloning (Murao et al PRA 99')

All utilize pure fully inseparable states, mostly $|GHZ\rangle$.

- Entanglement distribution by separable ancilla. (Cubitt et al PRL 03')

Utilizes mixed partially entangled states.

Is there another application of mixed partially entangled states?

Continuous variables

Systems with $\dim\mathcal{H} = \infty$.

E.g.: linear harmonic oscillator, $\widehat{H} = \left(\widehat{x}^2 + \widehat{p}^2\right)/2$,

$\widehat{x}, \widehat{p}$, $[\widehat{x}, \widehat{p}] = i$ canonically conjugate variables (continuous spectra).

Realization: mode of electromagnetic field,

$\widehat{x}, \widehat{p}$ – position and momentum quadrature operators.

Wigner function

N modes, phase space $x_A, p_A, \ldots, x_N, p_N$;

$$\hat{\rho} \quad \rightarrow \quad W(\mathbf{r}) = \frac{1}{(2\pi)^N} \int e^{i\mathbf{x}'^T \cdot \mathbf{p}} \left\langle \mathbf{x} - \frac{\mathbf{x}'}{2} \right| \hat{\rho} \left| \mathbf{x} + \frac{\mathbf{x}'}{2} \right\rangle d^N \mathbf{x}',$$

$\mathbf{r} = (x_A, p_A, \ldots, x_N, p_N)^T.$

Gaussian states:

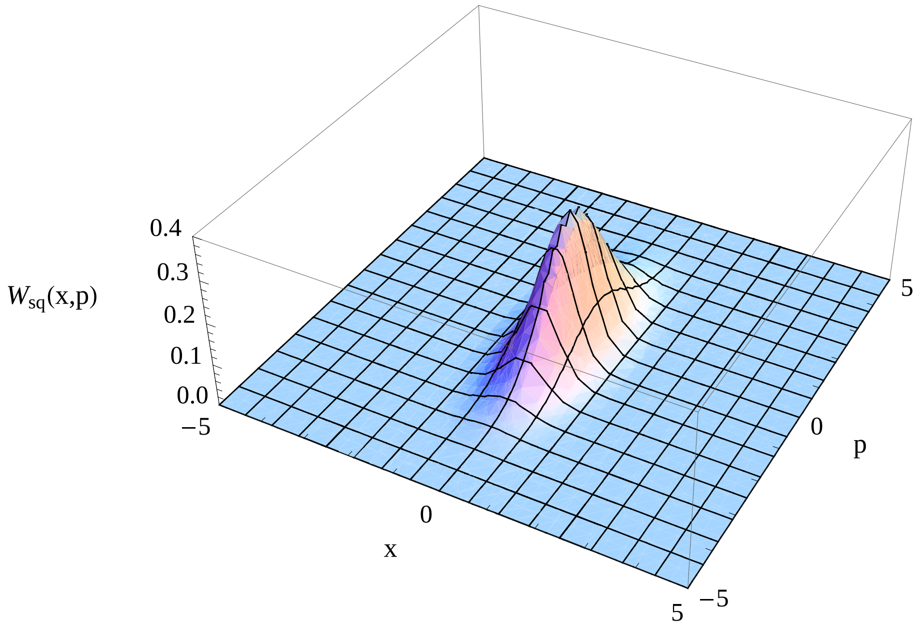
$$W(\mathbf{r}) = \frac{e^{-(\mathbf{r}-\mathbf{d})^T \gamma^{-1} (\mathbf{r}-\mathbf{d})}}{\pi^N \sqrt{\mathrm{det}\gamma}},$$

$\mathbf{d} = \langle \hat{\mathbf{r}} \rangle$-displacement, γ – covariance matrix (CM),

$$\gamma_{ij} = \langle \Delta\hat{r}_i \Delta\hat{r}_j + \Delta\hat{r}_j \Delta\hat{r}_i \rangle, \quad \Delta\hat{r}_i = \hat{r}_i - \langle \hat{r}_i \rangle,$$

$\hat{\mathbf{r}} = (\hat{x}_A, \hat{p}_A, \ldots, \hat{x}_N, \hat{p}_N)^T.$

Example: squeezed state $|r\rangle = e^{\frac{r}{2}\left[\widehat{a}^2 - \left(\widehat{a}^\dagger\right)^2\right]}|0\rangle$, r − squeezing parameter.



$$\langle(\Delta\widehat{x})^2\rangle = \frac{e^{-2r}}{2}, \quad \langle(\Delta\widehat{p})^2\rangle = \frac{e^{2r}}{2}.$$

Physical approximation of $|x=0\rangle$ ($|p=0\rangle$ for $-r$), $\widehat{x}|x\rangle = x|x\rangle$, $\widehat{p}|p\rangle = p|p\rangle$.

Entanglement criteria

1 × 1-mode criterion: PPT criterion with symplectic eigenvalues (Vidal et al PRA 02')

A is entangled with B for $\sigma_{AB} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$, if

$$\mu = \sqrt{\frac{\det A + \det B - 2\det C - \sqrt{(\det A + \det B - 2\det C)^2 - 4\det \sigma_{AB}}}{2}} < 1.$$

1 × 2-mode criterion: PPT criterion with symplectic invariants (Serafini PRL 06')

X is entangled with (YZ) for γ_{XYZ} if

$$\Sigma_X = \Pi_{j=1}^{3}(\mu_j^2 - 1) = I_3 - I_2 + I_1 - 1 < 0,$$

where

$$\det(\Omega\gamma_{XYZ}^{(T_X)} - qI) = q^6 + I_1 q^4 + I_2 q^2 + I_3,$$

$$\gamma_{XYZ}^{(T_X)} \equiv \sigma_z^{(X)} \oplus I^{(Y)} \oplus I^{(Z)} \gamma_{XYZ} \sigma_z^{(X)} \oplus I^{(Y)} \oplus I^{(Z)},$$

$$\Omega = \bigoplus_{i=1}^{3} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Two-mode Gaussian entanglement

Correspondence: $|0\rangle, |1\rangle \leftrightarrow \{|x\rangle\}_{x\in\mathbb{R}}$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle) \leftrightarrow \{|p\rangle\}_{p\in\mathbb{R}}$$

$$CNOT \leftrightarrow \text{beam splitter} \quad \left(\hat{B}(1/\sqrt{2}), \frac{\hat{x}_A \pm \hat{x}_B}{\sqrt{2}}, \frac{\hat{p}_A \pm \hat{p}_B}{\sqrt{2}}\right)$$

$$\hat{U}_{CNOT}|+\rangle|0\rangle = \frac{(|00\rangle + |11\rangle)}{\sqrt{2}} \leftrightarrow \hat{B}(1/\sqrt{2})|p=0\rangle|x=0\rangle \propto |EPR\rangle.$$

EPR state of modes A and B:

$$|EPR\rangle_{AB} = \int dx|x,x\rangle_{AB} = \int dp|p,-p\rangle_{AB}.$$

$$\hat{x}_A - \hat{x}_B = 0, \quad \hat{p}_A + \hat{p}_B = 0.$$

Physical approximation: $|x=0\rangle \rightarrow |r\rangle$, $|p=0\rangle \rightarrow |-r\rangle$

Two-mode squeezed vacuum (TMSV)

$$|TMSV\rangle_{AB} = \hat{B}_{AB}(1/\sqrt{2})|r\rangle_A|-r\rangle_B \quad \rightarrow \quad |EPR\rangle_{AB} \quad \text{for} \quad r \rightarrow \infty.$$

Pure entangled state.

Three-mode Gaussian entanglement

1. Fully inseparable (class 1) CV GHZ state:

$$\hat{B}_{BC}(1/\sqrt{2})\hat{B}_{AB}(1/\sqrt{3})|p=0\rangle_A|x=0\rangle_B|x=0\rangle_C \propto \int |x,x,x\rangle dx$$

$$\hat{x}_i - \hat{x}_j = 0 \;\; (i,j = A,B,C), \quad \hat{p}_A + \hat{p}_B + \hat{p}_C = 0.$$

Physical approximation: infinite $r \rightarrow$ finite r.

(van Loock et al PRL 00')

2. One-mode biseparable state (class 2): $|0\rangle_A|TMSV\rangle_{BC}$

or mixed $\gamma_{\alpha\beta} = \gamma_{AB}^{(TMSV)} \oplus I_C + \alpha p_1 p_1^T + \beta p_2 p_2^T$, $\alpha = \beta = 0.1$, $p_1 = (0,1,0,1,1,2)^T$, $p_2 = (1,0,-1,0,0,1)^T$.

3. Two-mode biseparable state (class 3): only mixed, e.g., $\gamma_{\frac{1}{2}\frac{1}{2}}$.

4. Three-mode biseparable states (class 4): only mixed, e.g.,

γ_{11} or $\gamma_{GHZ} + \delta I$, $\delta = \sqrt{\cosh^2(2r) + \frac{4}{3}\sqrt{2}\sinh(2r)} - \cosh(2r)$.

5. Fully separable states (class 5): $|0\rangle_A |0\rangle_B |0\rangle_C$ or mixed $\gamma_{GHZ} + \delta I$, $\delta \geq 1$.

(Giedke et al PRA 01')

Most applications use fully inseparable states, e.g.,

- Teleportation network (van Loock et al PRL 00').
- Quantum secret sharing (Tyc et al PRA 02').

Application of mixed partial entanglement:

- Entanglement distribution by separable ancilla.

(Mišta et al PRA 09')

Step 1: Fully separable state (class 5).

$\gamma_{A,C} = \text{diag}(e^{\pm 2r}, e^{\mp 2r}), \gamma_B = I$.

$\hat{p}_A \to \hat{p}_A - \frac{u}{\sqrt{2}}, \hat{x}_C \to \hat{x}_C + \frac{v}{\sqrt{2}}, \hat{x}_B \to \hat{x}_B + v, \hat{p}_B \to \hat{p}_B + u, \langle {u^2 \atop v^2} \rangle = 2x.$

Four displacements – nightmare!

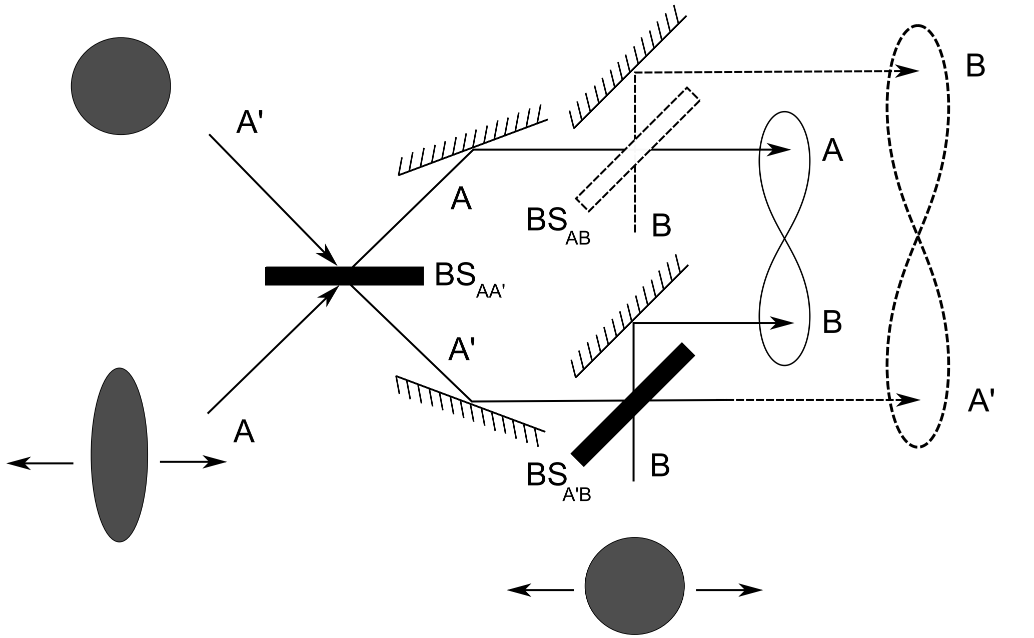
Step 2: $BS_{AC} \to B - (AC)$ a $C - (AB)$ separable, $A - (BC)$ entangled (class 3).

Step 3: $BS_{BC} \to A - B$ entangled, $C - (AB)$ separable (class 2).

Interference of displaced modes B and C – nightmare!

Is there something more simple?

Entanglement sharing with separable states



Step 1: $\gamma_A = \mathrm{diag}[e^{-2(r-\varepsilon)}, e^{2r}]$, $\gamma_{A'} = \gamma_B = I$, $r \geq 0, \varepsilon \geq 0$.

$x_A \to x_A + \bar{x}$, $\quad x_B \to x_B - \bar{x}$, $\langle \bar{x}^2 \rangle = (1 - e^{-2r})/2$.

$\downarrow$

Fully separable three-mode Gaussian state (class 5).

Step 2: Beam splitter (BS) on A and A'

$$\downarrow$$

$B-(AA')$ separability, $\Sigma_A = \Sigma_{A'} = 8e^{\varepsilon - r}\sinh(\varepsilon - r)\sinh^2(r)$
$A-(A'B)$ and $A'-(AB)$ entanglement for $r > \varepsilon$.

$$\downarrow$$

One-mode biseparable state (class 2).
Also no two-mode entanglement (mixed).

Step 3:

1. A' sent to Bob, BS on A' and B entangles A with B.

2. A sent to Bob, BS on A and B entangles A' with B.

$$\downarrow$$

Alice entangles with Bob in both cases

for $r > r_\mathrm{e} = \frac{1}{2}\ln\left[\frac{11e^{2\varepsilon}+8\sqrt{2}-13+\sqrt{(11e^{2\varepsilon}+8\sqrt{2}-13)^2+4e^{2\varepsilon}(8\sqrt{2}-1)}}{2(8\sqrt{2}-1)}\right]$.
$\tilde{\Sigma}_{A'} = \tilde{\tilde{\Sigma}}_A = \frac{\Sigma_A}{4} \Rightarrow$ Fully inseparable (class 1) for $r > \varepsilon$.

Entanglement sharing

- Classical secret sharing (Blakely 79', Shamir 79')

  Alice: m − message, r − random string $\rightarrow b = m \oplus r \mod 2$

  $\swarrow$ $\searrow$

  b to Bob $\qquad$ r to Clare

  $\searrow$ $\swarrow$

  $m = b \oplus r \mod 2$

- Quantum secret sharing

  1. Classical message (Hillery et al PRA 99')

  A, B and C measure $|GHZ\rangle$ in two complementary bases and all announce the basis. B and C cannot individually determine the results of A but they can do it together.

2. Quantum state (QSS) (Cleve et al PRL 99')
A quantum state is split into several shares such that it can be reconstructed only from certain subsets of shares, whereas it cannot be reconstructed from the remaining subsets.

- Entanglement sharing (Choi et al quant-ph 12')
Dealer uses QSS to split one part of a maximally entangled state into several shares. Entanglement with the dealer can be reconstructed only from certain subsets of shares, whereas it cannot be reconstructed from the remaining subsets.

- Entanglement sharing with separable states
Alice creates one share A' (A) by splitting one half of a separable state, whereas Bob holds the separable share B. Bob can establish entanglement with Alice's share A (A') only if he has both his share B and the share A' (A).

Gaussian localizable entanglement

Gaussian measurement on $B \longrightarrow$ conditional state $\tilde{\rho}_{AA'}$.

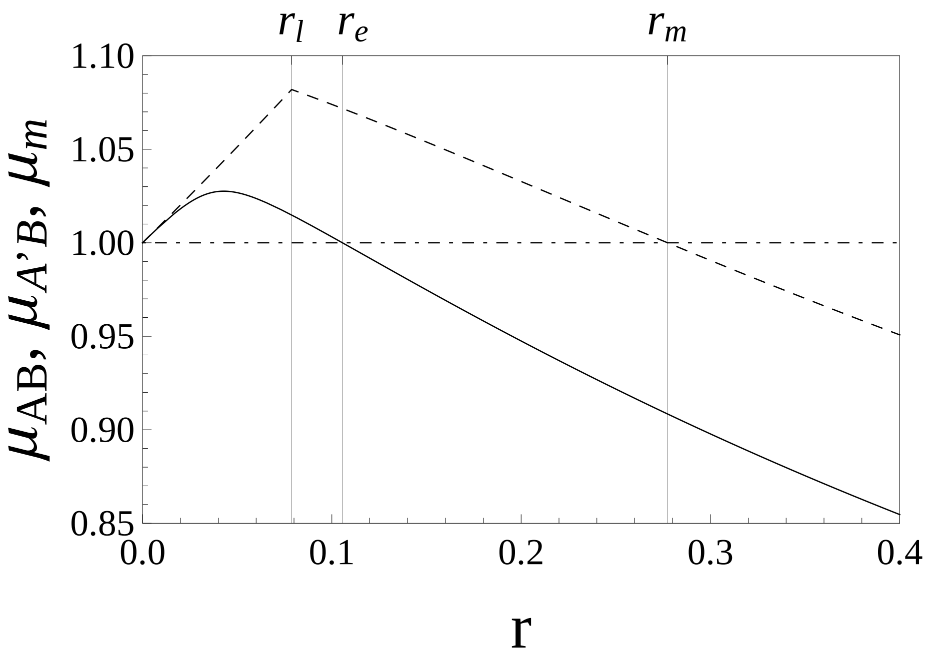
For which measurement $\tilde{\rho}_{AA'}$ contains maximum entanglement? (Fiurášek et al PRA 07',Mišta et al PRA 08')

Maximize logarithmic negativity $\max[0, -\log_2(\mu)]$, where μ is lower symplectic eigenvalue of the $\tilde{\rho}_{AA'}^{T_A}$.

For state from step 2 measurement of $\{|x\rangle\}_{x\in\mathbb{R}}$ is optimal and entanglement can be localized if

$$r > r_{\mathrm{m}} = \frac{1}{2}\ln\left[e^{2\varepsilon} + \sqrt{e^{2\varepsilon}\left(e^{2\varepsilon} - 1\right)}\right].$$

Entanglement localizability in ES

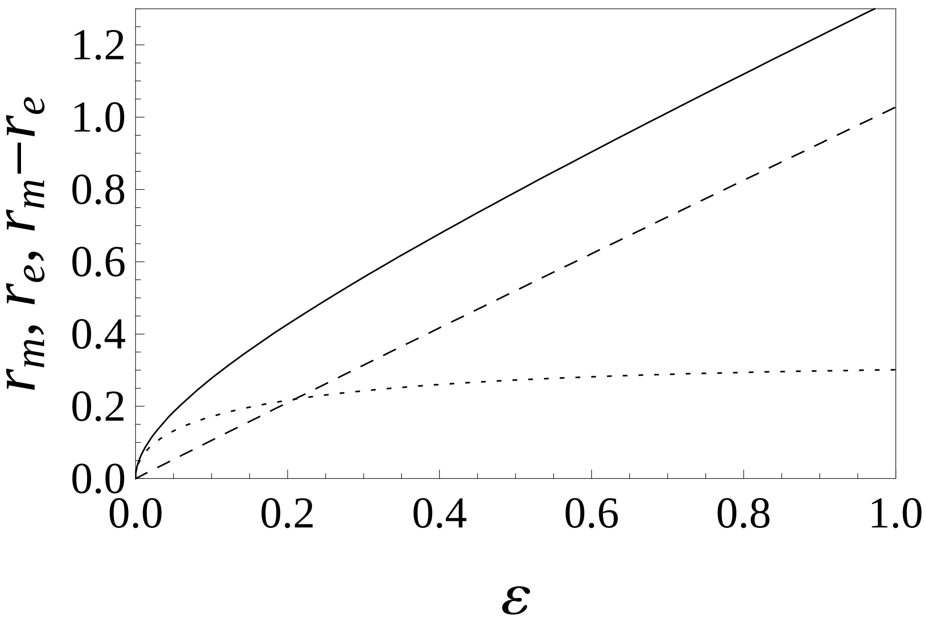


Solid curve – lower symplectic eigenvalue μ_{AB} ($\mu_{A'B}$) of the partial transpose of the states of modes A and B (A' and B) in step 3.

Dashed curve – symplectic eigenvalue μ_{m} corresponding to maximum localizable entanglement.

$\varepsilon = 0.1$, gap for $r_{\mathrm{m}} = 0.28 \geq r > r_{\mathrm{e}} = 0.11$.

Localizability gap



r_m – solid curve, r_e – dashed curve, $(r_\mathrm{m} - r_\mathrm{e})$ – dotted curve.

Signatures of bound entanglement

Three-mode bound entanglement (BE):

1. The state cannot be created by LOCC.

2. Any two parties cannot distill singlets by LOCC even with the help of the third party.

Two- and three-mode biseparable states (class 3 and 4) are BE.

Are there also one-mode biseparable BE states?

One-mode biseparable state (class 2) from ES satisfies 1. and 2. if we are restricted to single copy and Gaussian measurements on mode B – nontrivial necessary prerequisite for BE.

Conclusion

- New application of tripartite partially entangled states.
- Candidate for one-mode biseparable bound entangled state.
- Symmetrical protocol?

Thank you!