Fuzzy Logic as a Theory of Vagueness

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Part I: Theories of Vagueness

vague predicates:

1. borderline cases
   tall vs $\geq$1800mm in height
2. blurred boundaries
   Frege: if we represent concepts in extension by areas on a plane, then vague concepts do not have sharp boundaries, but rather fade off into the background.
3. sorites susceptibility
   bald vs $\geq$10 hairs

Conceptual map of theories of vagueness

origin: the classical semantic/model-theoretic picture.

locate theories of vagueness according to which component(s) of the classical picture they deny classical semantic picture:

1. Interpretations of the language have the following features:
   1.1 They employ a two-element Boolean algebra of truth values (which serve both as the truth values of sentences, and as the values of the characteristic functions of sets).
   1.2 The interpretation function (which assigns truth values to simple sentences, elements of the domain to names, and *n*-ary relations on the domain to *n*-ary predicates) is total, and the characteristic functions of sets are total.
   1.3 The truth values of compound wffs built up from simpler wffs by means of the connectives $\vee$, $\wedge$, $\neg$ etc. are determined in a recursive fashion from the truth values of their components.
2. Each discourse has a unique intended interpretation.

Epistemicism: Accept all

Semantically, is tall is exactly like is $\geq$1.8m in height.

- each has a crisp set as its meaning (= extension on the unique intended interpretation).
- every claim *a* is tall is either true or false.

Difference is epistemic: cannot know where cutoff is.

Williamson: "ignorance is the real essence of the phenomenon ostensively identified as vagueness" approach to sorites:

Mistake?

inductive premise if x is tall, so is x' is false.

Why compelling?

We are inclined to think inductive premise is true because we cannot know where the cutoff is and so we (mistakenly) think there is no cutoff.

location problem

What determines the location of the cutoff?

Language is a human artefact; use determines meaning:

Lewis: "Surely it is our use of language that somehow determines meaning"

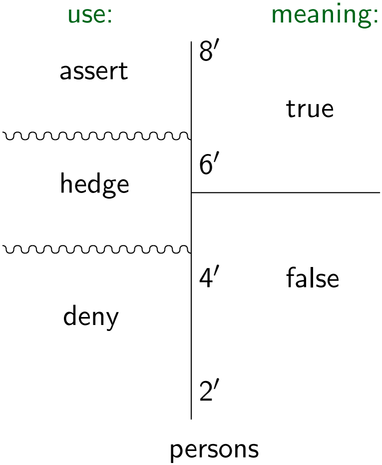
Williamson: "Words mean what they do because we use them as we do".

plausible principle connecting meaning and use:

(MU) The claim Pa is

- true iff most competent speakers would confidently assent if presented with a in normal conditions and asked whether it was P, and
- false iff most competent speakers would confidently dissent if presented with a in normal conditions and asked whether it was P.

epistemicist must reject (MU):


use:
meaning:
assert
hedge
deny
8′
6′
4′
2′
true
false
persons

Additional Truth Values: Deny 1.1

replace classical Boolean algebra of truth values with a different algebra with more values

many-valued logic:

- use algebra to define truth conditions of connectives etc.

many-valued set theory:

- set = characteristic function (range is the set of truth values)
- use algebra to define operations on sets ($\cup$, $\cap$ etc.)

simplest case: three values

still many choices:

α	β	$\neg\alpha$	$\alpha \wedge \beta$	$\alpha \vee \beta$	$\alpha \rightarrow \beta$	$\alpha \leftrightarrow \beta$
1	1	0	1	1	1	1
1	*		*	*	*	*
1	0		0	1	0	0
*	1	*	*	*	*	*
*	*		*	*	*	*
*	0		*	*	*	*
0	1	1	0	1	1	0
0	*		*	*	*	*
0	0		0	0	1	1

Bočvar

α	β	$\neg\alpha$	$\alpha \wedge \beta$	$\alpha \vee \beta$	$\alpha \rightarrow \beta$	$\alpha \leftrightarrow \beta$
1	1	0	1	1	1	1
1	*		*	1	*	*
1	0		0	1	0	0
*	1	*	*	1	1	*
*	*		*	*	*	*
*	0		0	*	*	*
0	1	1	0	1	1	0
0	*		0	*	1	*
0	0		0	0	1	1

Kleene (strong)

α	β	$\neg\alpha$	$\alpha \wedge \beta$	$\alpha \vee \beta$	$\alpha \rightarrow \beta$	$\alpha \leftrightarrow \beta$
1	1	0	1	1	1	1
1	*		*	1	*	*
1	0		0	1	0	0
*	1	*	*	1	1	*
*	*		*	*	1	1
*	0		0	*	*	*
0	1	1	0	1	1	0
0	*		0	*	1	*
0	0		0	0	1	1

Łukasiewicz

- allows parallelism between meaning and use:
  assert cases = true cases; deny = false; hedge = third value.
- typical response to sorites:
  some of the conditionals $Px \rightarrow Px'$ are not true.
  We are taken in because none of them is false
  (that would require true antecedent and false consequent).

Chief problem: the jolt problem:

Imposes a sharp semantic jump between the clear cases and the borderline cases
(and between the borderline cases and the clear non-cases).

Does not allow for a gradual change in applicability of the predicate as we go along the series—
even though there is a gradual change in the objects in the series in respects relevant to the application of the predicate.

- leads naturally to fuzzy views: values are all the reals in [0,1].

many choices:

$$\begin{aligned} \neg x &= 1 - x \\ x \wedge y &= \min(x, y) \\ x \vee y &= \max(x, y) \\ \alpha \to \beta &\equiv \neg\alpha \vee \beta \end{aligned}$$

Zadeh/Kleene logic

t-norm fuzzy logics

conjunction: a t-norm

conditional: residuum of the t-norm:

$$x \rightarrow y = \max\{z : x \wedge z \leq y\}$$

negation: precomplement of the conditional:

$$\neg x = x \rightarrow 0$$

$$
\begin{array}{rcl}
x \wedge y & = & y \wedge x \\
(x \wedge y) \wedge z & = & x \wedge (y \wedge z) \\
x_1 \leq x_2 & \Rightarrow & x_1 \wedge y \leq x_2 \wedge y \\
y_1 \leq y_2 & \Rightarrow & x \wedge y_1 \leq x \wedge y_2 \\
1 \wedge x & = & x \\
0 \wedge x & = & 0
\end{array}
$$

Conditions on t-norms

$$
\begin{aligned}
x \wedge y &= \max(0, x + y - 1) \\
x \rightarrow y &= \begin{cases} 1 & \text{if } x \leq y \\ 1 - x + y & \text{if } x > y \end{cases} \\
\neg x &= 1 - x
\end{aligned}
$$

Łukasiewicz logic

$$x \wedge y = \min(x, y)$$

$$x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{if } x > y \end{cases}$$

$$\neg x = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Gödel logic

$$
\begin{aligned}
x \wedge y &= x \cdot y \\
x \rightarrow y &= \begin{cases} 1 & \text{if } x \leq y \\ y/x & \text{if } x > y \end{cases} \\
\neg x &= \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Product logic

Truth Gaps: Deny 1.2

- leave the classical Boolean algebra of truth values untouched
- allow the part of the interpretation function that assigns a truth value to each simple sentence to be partial
- allow the characteristic functions of sets to be partial

gives truth value gaps for simple sentences, and gappy sets; hence also truth gaps for atomic sentences.

Now need to define truth conditions for the connectives (and operations on sets).

Do this via truth tables.

For each option in the three-valued case (regarding operations on truth values) there is an analogous gappy truth table.

Supervaluationism: Deny 1.3

can be combined with gaps or third values; we use gaps here.

say that a classical interpretation extends a partial interpretation if it just fills the gaps:

- it agrees with all the assignments the partial interpretation (and the partial characteristic functions therein) make
- and just makes assignments where they assign nothing.

Now assign truth values to sentences in the partial base model $\mathfrak{M}^p$ as follows:

- a sentence gets the value 1 (0) on $\mathfrak{M}^p$ iff it gets the value 1 (0) on every classical interpretation that extends $\mathfrak{M}^p$

NB some sentences will get no value.

The assignment of truth values is not truth-functional.

E.g. suppose p and q have no value on the base model.

Then $\neg p$ also has no value.
(Some extensions make p true, others make p false.)

But $p \vee q$ has no value, while $p \vee \neg p$ has value 1.

To get a theory of vagueness add an extra twist:

restrict attention to admissible extensions.

base model corresponds to our usage of vague predicates (the gaps are where we hedge).

An admissible extension corresponds to a way of precisifying all vague predicates in a mutually compatible way.

- e.g. might be permissible to precisify red in such a way that this autumn leaf is red, and also permissible to precisify orange in such a way that the leaf is orange:
  but we cannot precisify both terms together in such a way that the leaf counts as both red and orange.

solution to sorites:

inductive premise: there is no cutoff
$\forall n(Pn \to Pn')$, $\neg\exists n(Pn \wedge \neg Pn')$

the inductive premise is false
(on each admissible extension, there is a cutoff)

the cutoff is in a different place on each extension, so for each n the sentence the cutoff is at n $Pn \wedge \neg Pn'$ is not true (it lacks a value).

We mistakenly infer from the non-truth of the cutoff is at n for each n to the truth of the inductive premise there is no cutoff.

missing witness problem:

Some have taken this precisely to be an objection to supervaluationism:

- it allows existentials to be true even though no instance is true
- it allows universals to be false even though no instance is false.

(Likewise: a disjunction can be true without either disjunct being true, and a conjunction can be false without either conjunct being false.)

Plurivaluationism: Deny 2

Accept all parts of the classical picture apart from claim 2.

Usage constrains acceptable interpretations:

- e.g. we apply the name Helen Clark to a certain person, so on acceptable interpretations, this person is the referent of that name;
- we apply the word apple to certain objects, so on acceptable interpretations, these objects are in the extension of that predicate; etc.

Classical view: the set of acceptable interpretations is a singleton, which contains the intended (aka correct etc.) interpretation.

Plurivaluationism: there are multiple (equally) acceptable interpretations.

Classical picture: true simpliciter = true on intended model.

Plurivaluationism: a class of acceptable interpretations.

- If a sentence is true on all of them, in an obvious sense it does not matter that we have many acceptable interpretations and not just one: we can just talk as if the sentence is true.
- If a sentence is false on all the acceptable interpretations, then again it does not make any difference that we have many acceptable interpretations and not just one, and we can just say that the sentence is false.
- If a sentence is true on some acceptable interpretations and false on others, then it does matter that we have many acceptable interpretations and not just one: we can say neither that it is true (simpliciter), nor that it is false (simpliciter).

This view should not be — but in the literature it has been — conflated with supervaluationism:

Varzi:

Broadly speaking, ~~supervaluationism~~ plurivaluationism tells us two things. The first is that the semantics of our language is not fully determinate, and that statements in this language are open to a variety of interpretations each of which is compatible with our ordinary linguistic practices. The second thing is that when the multiplicity of interpretations turns out to be irrelevant, we should ignore it. If what we say is true under all the admissible interpretations of our words, then there is no need to bother being more precise.

Lewis:

I regard vagueness as semantic indecision: where we speak vaguely, we have not troubled to settle which of some range of precise meanings our words are meant to express.

supervaluationism: classical models are admissible extensions of a unique intended non-classical base model (they are not intended/acceptable interpretations).

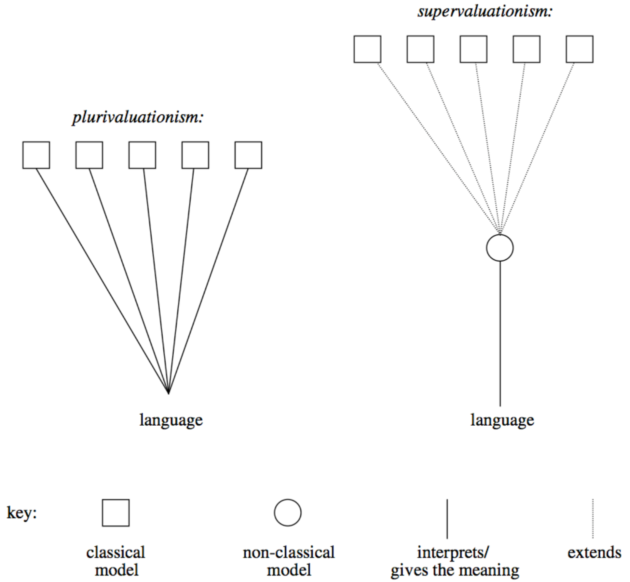
- The partial base model represents the actual meaning of the vague discourse;
- the classical extensions represent how it could be precisified.

Each vague claim is in a unique (nonclassical) semantic state.

plurivaluationism: classical models are acceptable interpretations:

- equally good representations of the actual meaning of the vague discourse.

Each vague claim is in multiple (classical) semantic states.


supervaluationism:
plurivaluationism:
language
language
key:
classical model
non-classical model
interprets/ gives the meaning
extends

solution to sorites:

inductive premise false on all acceptable interpretations—hence can be said to be false (simpliciter).

but no point n is such that the cutoff is at n can be said to be true (simpliciter)

hence we (mistakenly) think the inductive premise is true.

no missing witness problem:

unlike supervaluationism, no level of semantic reality at which existential claim is true without a true witness (or universal false without a false instance):

- each acceptable model is classical
- there is no extra semantic machinery, beyond the classical models: only a level of talk
  (at which we can claim there is a cutoff although we cannot claim the cutoff is at n for any n)

location problem:

the epistemicist's location problem is solved by plurivaluationism:

- usage does not fix a unique intended model (with a unique cutoff therein)
- it leaves open many acceptable interpretations, each with its own, different cutoff

The distinctions that remain:

- true on all acceptable models / true on none / true on some and false on others

do correspond precisely to usage.

By construction, semantic facts cannot outrun usage facts.

Part II: The Nature of Vagueness

Surface characterisation vs Fundamental definition

A: What is water?

B: It's the 'watery stuff': the clear liquid that falls from the clouds and fills rivers and lakes...

A: No, I mean, what is water?

B: H_2O.

Part II: The Nature of Vagueness

Surface characterisation vs Fundamental definition

A: What is water?

B: It's the 'watery stuff': the clear liquid that falls from the clouds and fills rivers and lakes. . .

A: No, I mean, what is water?

B: H_2O.

A: What is a vague predicate?

B: One that admits of borderline cases, draws blurred boundaries and generates Sorites paradoxes.

A: No, I mean, what is a vague predicate?

B: Good question!

we have three piecemeal characterisations of vagueness.

would be desirable to have a fundamental definition of vagueness, in terms of which we can explain why vague predicates have these three features.

fundamental definition should be:

1. true
2. useful (e.g. not circular, not vague, not defining the obscure in terms of the more obscure)
3. fundamental

need not be theory-neutral
(to avoid talking past one another — surface characterisation must be theory-neutral)

Candidate Definitions: 1. borderline cases

not fundamental

1. If x is less than four feet in height, then x is schort is true.
2. If x is more than six feet in height, then x is schort is false.

   [If x is between four and six feet in height, then x is schort is neither true nor false.]

borderline cases but no blurred boundaries: not vague

2. blurred boundaries

fundamental — but not sufficiently precise.

suggestive, but not fully useful.

Too vague for a fundamental definition.

3. Sorites susceptibility

not fundamental:

missing explanation argument:

vague predicates generate Sorites paradoxes
because they are vague.

4. semantic indeterminacy
(of the sort involved in plurivaluationism)

Braun and Sider:

> *Like many, we think that vagueness occurs when there exist multiple equally good candidates to be the meaning of a given linguistic expression. . . Vagueness is a type of semantic indeterminacy.*

not fundamental: e.g. gavagai (Quine) and mass (Field):
even assuming they exhibit semantic indeterminacy (rabbits vs undetached rabbit parts; rest mass vs relativistic mass),
they are not vague.

5. tolerance

vague = tolerant

predicate F is tolerant iff for any objects a and b:

Tolerance if a and b are very similar in F-relevant respects, then 'Fa' and 'Fb' are identical in respect of truth.

problem: tolerance + Sorites series ⇒ contradiction

Proposed Definition: Vagueness as Closeness

predicate F is vague iff for any objects a and b:

Closeness if a and b are very similar in F-relevant respects, then 'Fa' and 'Fb' are very close in respect of truth.

tolerance: very small differences between objects in F-relevant respects make no difference to the application of the predicate F.

closeness: very small differences between objects in F-relevant respects make at most a very small difference to the application of the predicate F.

closeness $\neq$ continuity

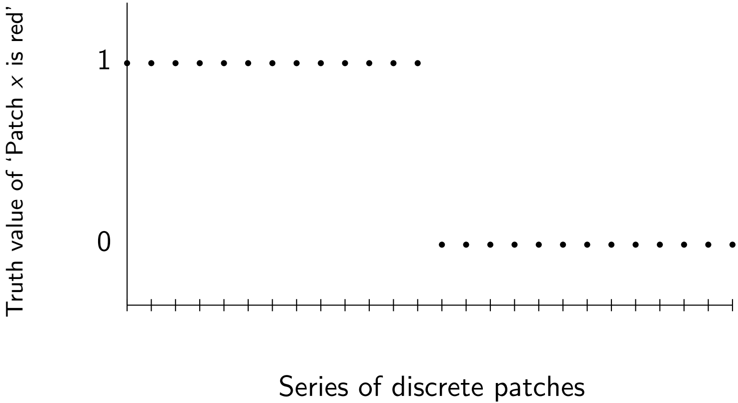
Related to, but not the same as, the idea that vague predicates are those whose extensions (thought of as functions from objects to truth values) are continuous.

Closeness employs a notion of absolute similarity:
very similar/close.

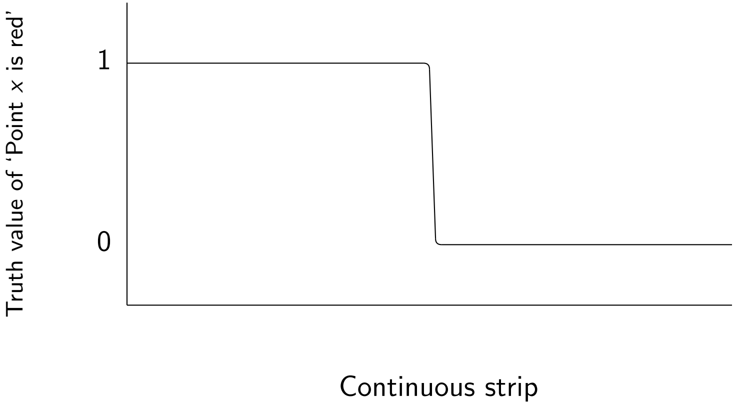
The informal statement of continuity employs such a notion:
a small change in the input produces at most a small change in the output.

But this drops out of the formal definition:
for any positive sized target area in the codomain (whether or not we would ordinarily regard it as ‘small’) we can find a positive sized launch area in the domain (which again need not be ‘small’ in the ordinary absolute sense) such that everything sent by the function from that launch area lands in that target area.

Redd: continuous but not vague



Rred: continuous but not vague



Advantages of the Closeness Definition
1. tolerance intuitions without incoherence

arguments for Tolerance also support Closeness

- vague predicates used in contexts of casual observation, on basis of rough and ready judgement (heap)
- social importance attached to application of predicate (child, adult)
- vague predicates learned by ostension so distinctions must be memorable (red, green, blue)

and Closeness generates no contradictions

2. Explains surface characterisations
a) blurred boundaries

consider all possible objects, structured by relationships of similarity in F-relevant respects

If F conforms to Closeness, its extension cannot consist in a sharp line between the F's and the non-F's

rather, F-ness must gradually fade away as one travels further from the definite F objects.

E.g. consider a rainbow and the predicate is red:

- if it conforms to Closeness, it cannot cut a sharp band out of the rainbow: its boundary must be blurry.

b) borderline cases

Consider a predicate F which conforms to Closeness, and a Sorites series $x_1, \ldots, x_n$ for F.

Fx_1 is true and Fx_n is false

given Closeness, it cannot be that there is an i such that Fx_i is true and Fx_{i+1} is false

so there must be sentences Fx_i which are neither true nor false

the corresponding objects x_i are borderline cases for F.

c) Sorites paradoxes

1. The first object in the series is F.
2. For any object in the series (except the last), if it is F, then so is the next object.
3. Therefore the last object in the series is F.

mistake: valid only if premise 2 expresses Tolerance (not Closeness)

compelling: Tolerance is a useful approximation to Closeness in normal situations

3. Higher-order vagueness

vagueness = borderline cases

problem: sharp boundaries to the borderline region

response: "higher-order vagueness"

- borderline borderline cases
- borderline borderline borderline cases; etc.

better approach

vagueness = closeness

captures in one definition the idea of gradual transition from the cases where the predicate clearly applies to the cases where it clearly does not apply (no jolts) as essential to vagueness (not a bolt-on option)

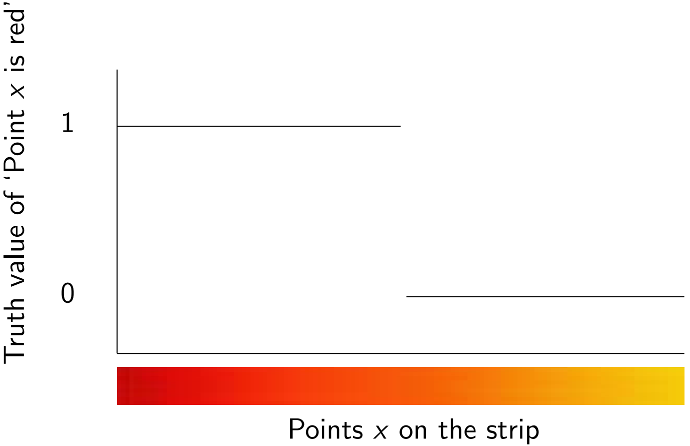
Accommodating Vagueness

Q: Which types of theory of vagueness can allow for the existence of predicates that satisfy the Closeness definition?

A: Only theories that countenance degrees of truth.

Given the existence of a Sorites series for the predicate F, there is no way to accommodate the claim that F conforms to Closeness without accepting the idea that truth comes in degrees.

Epistemicism



Not the problem: there is a last red point (or a first non-red point)

The real problem: there is a jump point (jolt) in the characteristic function of the predicate 'red' even though the points on the strip vary gradually in 'red'-relevant respects. So Closeness is violated.

A possible response:

The correct definition of vagueness is not Closeness but:

JA-Closeness if a and b are very similar in F-relevant respects, then 'Fa' and 'Fb' are very similar in respect of ~~truth~~ justified assertibility.

Problem: JA-Closeness does not explain blurred boundaries or Sorites susceptibility.

blurred boundaries:

Suppose someone asserts cutoff for baldness is 400 hairs.

Intuitively, this is problematic.

JA-Closeness: it's a pragmatic/epistemic problem:
assertion couldn't be justified.

Closeness: it's an alethic problem:
assertion couldn't be true.

To see it's an alethic problem, imagine someone guessing.
Intuitively, this is just as bad.

But no violation of pragmatic/epistemic norm this time.

So the problem must be alethic.

JA-Closeness gives us sharp but unknowable boundaries.
(They would still be guessable.)

Intuitively that's not real vagueness: not genuinely blurry boundaries.

Contrast:

- The jump from baldness to non-baldness comes at hair 400.
- The least upper bound of velocities reached by polar bears on 11th January 2004 was 31.35 kilometres per hour.

Sorites-susceptibility:

Suppose we have a predicate F that conforms to JA-Closeness, and a Sorites series for F.

Will the corresponding Sorites paradox be compelling?

We have no reason to think so.

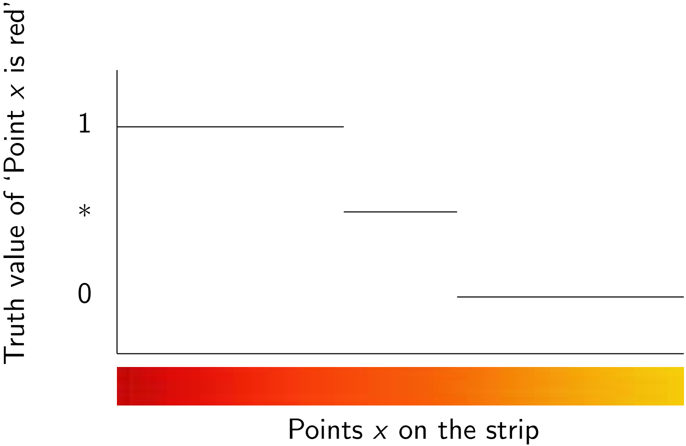
We know that for any object in the series, to whatever extent it is warranted to assert that it is F, it is to a very similar extent warranted to assert that the next object is F.

No reason to conclude that the Sorites premise is true: that for any object in the series, if it is F, then so is the next object:

for all we have said it might be that F has sharp but unknowable boundaries, in which case the sorites premise is plainly false.

Additional Truth Values and Gaps

Three values is not sufficient (same applies to gaps):



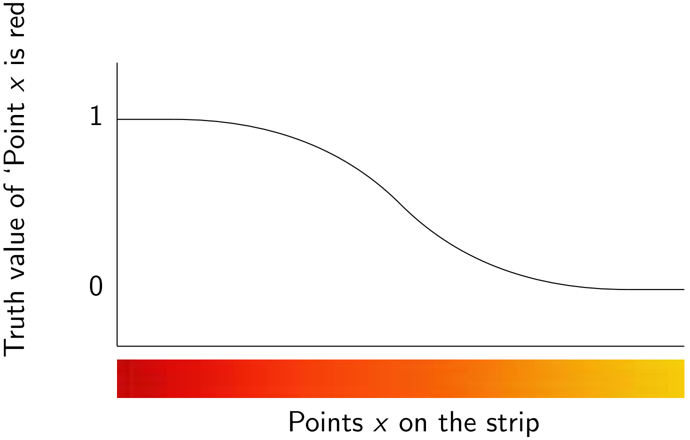
Truth and Falsity are not similar at all in respect of truth.

So nothing can be very close to both of them.

So Closeness violated at at least one of two points.

Given the natural assumption that the third truth status is symmetric with respect to Truth and Falsity, Closeness is violated at both points.

Fuzzy values are sufficient:



Here there are no jump points (points at which there is a big change in truth value between Fa and Fb even though there is only a small change in F-relevant respects between a and b).

There is still a last red point (or first non-red point): but that was never the problem.

Supervaluationism

no better than truth-functional three-valued/gap views

a true sentence and a sentence with no truth value are not similar in respect of truth.

having the same value on almost all admissible extensions means being similar in respects relevant to truth, not in respect of truth.

Plurivaluationism

on every acceptable interpretation, F will violate Closeness (because they are all classical)

if Closeness is violated on every acceptable interpretation, then it is violated everywhere.

there are only the acceptable interpretations, so there is nowhere else for Closeness to be accommodated

Part III: Fuzzy Plurivaluationism

have seen why we should want a theory of vagueness that employs degrees of truth

but fuzzy theory faces a major objection that needs to be solved before we can accept it as our theory of vagueness

The Problem: Higher-Order Vagueness/Artificial Precision

> [Fuzzy logic] imposes artificial precision... [T]hough one is not obliged to require that a predicate either definitely applies or definitely does not apply, one is obliged to require that a predicate definitely applies to such-and-such, rather than to such-and-such other, degree (e.g. that a man 5 ft 10 in tall belongs to tall to degree 0.6 rather than 0.5) Haack 1979

One immediate objection which presents itself to [the fuzzy] line of approach is the extremely artificial nature of the attaching of precise numerical values to sentences like 73 is a large number or Picasso's Guernica is beautiful. In fact, it seems plausible to say that the nature of vague predicates precludes attaching precise numerical values just as much as it precludes attaching precise classical truth values. Urquhart 1986

[T]he degree theorist's assignments impose precision in a form that is just as unacceptable as a classical true/false assignment. In so far as a degree theory avoids determinacy over whether *a* is *F*, the objection here is that it does so by enforcing determinacy over the degree to which *a* is *F*. All predications of is red will receive a unique, exact value, but it seems inappropriate to associate our vague predicate red with any particular exact function from objects to degrees of truth. For a start, what could determine which is the correct function, settling that my coat is red to degree 0.322 rather than 0.321? Keefe 1998

Also: Copeland 1997, Goguen 1968–9, Lakoff 1973, Machina 1976, Rolf 1984, Schwartz 1990, Tye 1995, Williamson 1994. . .

(NB includes both proponents and opponents of degrees of truth)

The problem in a nutshell

It is artificial/implausible/inappropriate to associate each vague predicate in natural language with a function that assigns one particular fuzzy truth value (real number between 0 and 1) with each object (the object's degree of possession of that property).

It is artificial/implausible/inappropriate to associate each sentence in natural language which predicates a vague property of an object with one particular fuzzy truth value (the sentence's degree of truth).

Proposed Solutions

1. Fuzzy epistemicism
2. Fuzzy metalanguage
3. Blurry sets
4. Fuzzy plurivaluationism

Fuzzy epistemicism

Statements such as Bob is tall do indeed have unique fuzzy truth values (e.g. 0.4).

However in general we cannot know what these values are.

That is why it seems (falsely) to us as though these statements do not have unique fuzzy truth values.

Cf. Machina 1976, Copeland 1997, Keefe 1998. . .

Fuzzy metalanguage

If a vague language requires a continuum-valued semantics, that should apply in particular to a vague meta-language. The vague meta-language will in turn have a vague meta-meta-language, with a continuum-valued semantics, and so on all the way up the hierarchy of meta-languages. Williamson 1994

Cf. also Cook 2002, Edgington 1997, Field 1974, Horgan 1994, Keefe 2000, McGee and McLaughlin 1995, Rolf 1984, Sainsbury 1990, Tye 1990, 1994, 1995, 1996, Varzi 2001, Williamson 1994, 2003...

The idea is this:

1. Present a semantics for vague language which assigns vague sentences real numbers as truth values
2. then say that the metalanguage in which these assignments were made is itself subject to a semantics of the same sort.

On this view, statements of the form

The degree of truth of Bob is tall is 0.4

need not be simply True or False:
they may themselves have intermediate degrees of truth.

So rather than exactly one sentence of the form

The degree of truth of Bob is tall is x

being True and the others False,
many of them might be true to various degrees.

Thus there is a sense in which sentences in natural language that predicate vague properties of objects are not each assigned just one particular fuzzy truth value.

Blurry Sets

Smith 2004 ‘Vagueness and Blurry Sets’ (JPL 33, pp.165–235).

The truth values of this system are *DF*’s (degree functions).

Each *DF* is a function $f : [0,1]^* \to [0,1]$

$[0,1]^*$ is the set of words on the alphabet [0,1]

(i.e. the set of all finite sequences of elements of [0,1], including the empty sequence $\langle\rangle$).

Suppose f is the truth value of (B): Bob is tall.

$f(\langle\rangle)$ is a number in $[0, 1]$. This number is a first approximation to Bob's degree of tallness/the degree of truth of (B).

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If $f(\langle 0.3, 0.4\rangle) = 0.5$, then it is 0.5 true that it is 0.4 true that Bob is tall to degree 0.3. The assignments to all sequences of length 2 together constitute a third level of approximation to Bob's degree of tallness/the degree of truth of (B).

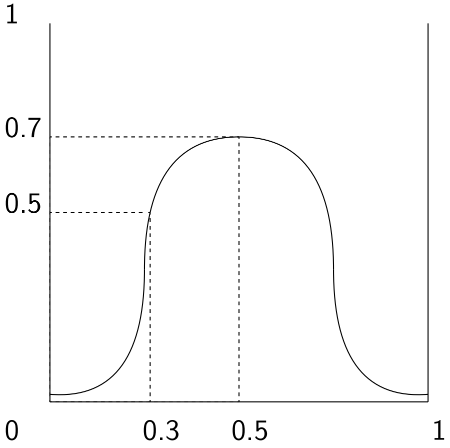
$\vdots$

In addition:

The assignments made by f to sequences of length 1 determine a function $f_{\langle\rangle} : [0,1] \rightarrow [0,1]$.

This can be seen as encoding a density function.

We require that its centre of mass is $f(\langle\rangle)$.



Bob's degree of tallness: second approximation

Likewise:

The assignments made by f to sequences $\langle a, x \rangle$ of length 2 whose first member is a determine a function $f_{\langle a \rangle} : [0, 1] \to [0, 1]$.

This can be seen as encoding a density function.

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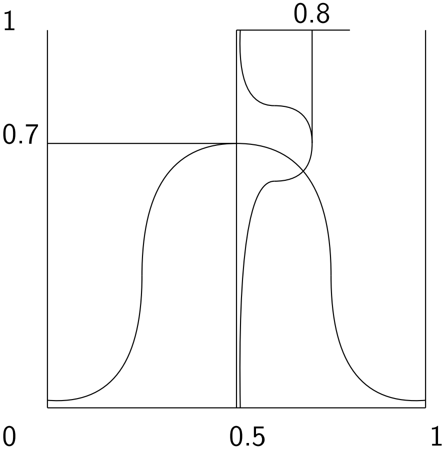

Bob's degree of tallness: third approximation (part view)

And so on. . .

Image: We can picture a degree as a region of varying shades of grey spread between 0 and 1 on the real line.

If you focus on any point in this region, you see that what appeared to be a point of a particular shade of grey is in fact just the centre of a further such grey region.

The same thing happens if you focus on a point in this further region, and so on.

The region is blurry all the way down: no matter how much you increase the magnification, it will not come into sharp focus.

On this view, as on the fuzzy metalanguage view, statements of the form

The degree of truth of Bob is tall is 0.4

need not be simply True or False:
they may themselves have intermediate degrees of truth.

So rather than exactly one sentence of the form

The degree of truth of Bob is tall is x

being True and the others False,
many of them might be true to various degrees.

Thus there is a sense in which sentences in natural language which predicate vague properties of objects are not each assigned just one particular fuzzy truth value.

NB On both views, we have a hierarchy of statements, none of which tells us the full and final story of the degree of truth of Bob is tall.

However there is a crucial difference between the two views:

Fuzzy metalanguage: A hierarchy of assignments of simple truth values.

Blurry sets: A single assignment of a complex truth value which has an internal hierarchical structure. Each vague sentence is assigned a unique degree function as its truth value. These assignments can be described in a classical metalanguage.

Fuzzy plurivaluationism

Smith 2008 *Vagueness and Degrees of Truth* (OUP).

Recall:

Orthodox classical (monovaluationist) picture:

When I utter Bob is tall, I say something, I make a claim:

that this guy (the referent of Bob in the intended model) has that property (the extension of is tall in the intended model).

What I say is true on some models and false on others.

It is true (simpliciter) if it is true on the intended model.

Classical plurivaluationist picture:

employs only classical models.

but instead of supposing that each discourse has one intended model, it allows that a discourse may have multiple acceptable models.

When I utter Bob is tall, I say many things at once:
one claim for each acceptable model.

I mean each of these things equally: there is not a unique meaning.

Semantic indeterminacy — or equally, semantic plurality.

But if all the claims I make are true/false, we can pretend (talk as if) I made only one claim, which is true/false.

Image: If all the shotgun pellets go through the bullseye, we can talk as if there was just one bullet, which went through.

Fuzzy plurivaluationist picture:

Just like classical plurivaluationism except that the models are fuzzy, not classical.

Upshot: there is not one uniquely correct assignment of truth value to Bob is tall. There are multiple, equally-correct assignments: one in each acceptable model.

Choosing a Solution

Four solutions on the table: which is the right one?

We can rule out the fuzzy metalanguage view on methodological grounds.

Our models are typical purely exact constructions, and we use ordinary exact logic and set theory freely in their development. . . It is hard to see how we can study our subject at all rigorously without such assumptions. Goguen 1968–9

We understand fuzzy model theory as standard mathematics, presented in the usual precise language of mathematics.

If you say at the end of presenting fuzzy model theory that the language in which you made your presentation was governed by the very semantics you just presented, then we do not really understand your presentation after all.

Formal semantic treatments of vague languages—many-valued logics, supervaluations and the like—are characteristically framed in a meta-language that is conceived as precise. Thus one cannot say in the precise meta-language what utterances in the vague object-language say, for to do so one must speak vaguely; one can only make precise remarks about those vague utterances. Since the expressive limitations of such a meta-language render it incapable of giving the meanings of object-language utterances, it can hardly be regarded as adequate for a genuine semantic treatment of the object-language. . . . the formality of the semantics [comes] at the cost of giving up the central task of genuine semantics: saying what utterances of the object language mean. Williamson 1994

What does it mean to give the semantics/meaning of Bob is tall?

1. Make a (different) claim which has the same content as Bob is tall
2. Give an account of the semantic relations between parts of the sentence Bob is tall and parts of the world, and of how these combine to determine the truth status of the whole sentence.

Contra Williamson, 2 is the task of genuine semantics.
(1 is the task of translation.)

And 2 requires a precise metalanguage.

That still leaves three solutions on the table:

1. Fuzzy epistemicism
2. Blurry sets
3. Fuzzy plurivaluationism

How do we decide which is the right one?

We need a clearer idea of the true nature and source of the problem.

Haack: no diagnosis.

Urquhart: the nature of vague predicates precludes attaching precise numerical values.

Keefe: what could determine which is the correct function, settling that my coat is red to degree 0.322 rather than 0.321?

Nature of Vagueness

Before we can say whether something — e.g. assignment of a unique fuzzy truth value to each vague sentence — conflicts with the nature of vagueness, we need a fundamental definition of vagueness.

We now have one: vagueness = Closeness

Determination of Meaning

The Problem of the Intended Interpretation:

1. Facts of type T do not determine a unique intended interpretation of discourse D.
2. No facts of any type other than T are relevant to determining the intended interpretation of D.
3. From 1 and 2: All the facts together do not determine a unique intended interpretation of D.
4. It cannot be a primitive fact—i.e. a fact not determined by other facts—that some interpretation $\mathfrak{M}$ is the unique intended interpretation of D.
5. From 3 and 4: It is not a fact at all that D has a unique intended interpretation.

Examples

- Quine on the indeterminacy of translation:

  Type T includes all and only the publicly accessible facts concerning what people say in what circumstances.

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- Quine on the indeterminacy of translation:

  Type T includes all and only the publicly accessible facts concerning what people say in what circumstances.
- Kripkenstein's sceptical puzzle:

  Type T also includes dispositional facts, and private mental facts.

Vagueness

Type T includes:

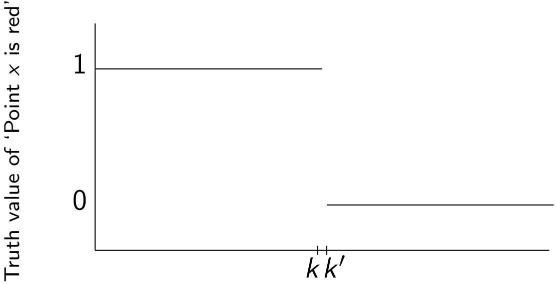
- All the facts as to what speakers of D actually say and write, including the circumstances in which these things are said and written, and any causal relations obtaining between speakers and their environment.
- All the facts as to what speakers of D are disposed to say and write in all kinds of possible circumstances.
- All the facts concerning the eligibility as referents of objects and sets.
- All the facts concerning the simplicity or complexity of the candidate interpretations.

Any theory of vagueness must cohere with this picture of how meaning is determined.

If the theory says that a vague predicate has a meaning of such-and-such a kind (e.g. a function from objects to classical truth values, or a function from objects to fuzzy truth values), we should be able to satisfy ourselves that the type T facts could indeed determine such a meaning for actual vague predicates.

The classical view

Does it conflict with the nature of vagueness? Yes!



k and k' are very close in respects relevant to the application of is red, but k is red and k' is red are not close in respect of truth.

Does it encounter the problem of the intended interpretation? Yes!

It seems that the type T facts do not suffice to pick out a particular height dividing the tall from the non-tall, etc.

Does it encounter the problem of the intended interpretation? Yes!

It seems that the type T facts do not suffice to pick out a particular height dividing the tall from the non-tall, etc.

Williamson: the classical view is not (logically) incompatible with the view that use determines meaning.

True: but we have no idea how use could determine a unique meaning.

So the classical view faces two problems:

1. The existence of a sharp drop-off from True to False in a sorites series: conflicts with the nature of vagueness.

   I call this the jolt problem.

So the classical view faces two problems:

1. The existence of a sharp drop-off from True to False in a sorites series: conflicts with the nature of vagueness.

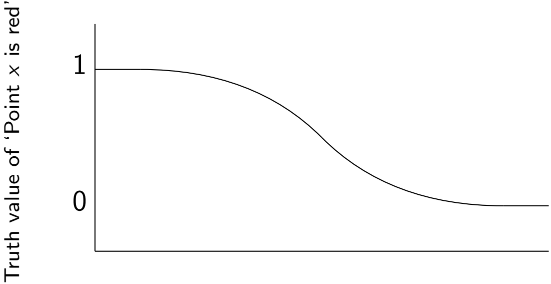
   I call this the jolt problem.

2. The location of the drop-off: conflicts with our best views about how meaning is determined.

   I call this the location problem.

The basic fuzzy view

Does it conflict with the nature of vagueness? No!



No jolt problem

Does it encounter the problem of the intended interpretation? Yes!

It seems that the type T facts do not suffice to pick out a particular function from objects to fuzzy truth values representing the extension of is tall, etc.

So there is a location problem.

This is the higher-order vagueness/artificial precision problem.

Return to the proposed solutions

Keeping in mind:

- The basic fuzzy view already solves the jolt problem. Assigning a unique degree of truth to each vague sentence does not conflict with the nature of vagueness.

But:

- The basic fuzzy view does face the location problem. Assigning a unique degree of truth to each vague sentence does conflict with our best views about the determination of meaning.

Fuzzy epistemicism

Does not solve the location problem.

The problem is how there could be a unique function which is the extension of is tall, given that our usage (etc.) does not suffice to pick out a unique such function.

Saying that we do not know which function it is just misses the point of the problem.

Blurry sets

Does not solve the location problem.

The type T facts do not suffice to pick out a unique fuzzy set (function from objects to fuzzy truth values) as the extension of is tall.

Likewise, the type T facts do not suffice to pick out a unique blurry set (function from objects to degree functions) as the extension of is tall.

Fuzzy plurivaluationism

Solves the location problem:

Indeed, it is the minimal solution to the problem: it accepts as its starting point the very idea which comprises the problem.

Fuzzy plurivaluationism

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Indeed, it is the minimal solution to the problem: it accepts as its starting point the very idea which comprises the problem.

The problem (artificial precision): The type T facts do not suffice to pick out a unique fuzzy set as the extension of is tall, etc — in general, a unique intended interpretation of vague discourse.

The solution (fuzzy plurivaluationism):

An acceptable interpretation is one which is not ruled out as incorrect by the type T facts.

In light of the problem, there is not a unique acceptable interpretation of vague discourse — i.e. one intended interpretation.

A fortiori, fuzzy plurivaluationism is correct: for it is precisely the view that there is no unique intended interpretation of vague discourse.

Instead, there are many equally correct interpretations — the acceptable ones.

Thus Bob is tall does not have a uniquely correct degree of truth. It is assigned many different degrees of truth—one on each acceptable interpretation—and none of these is more correct than any of the others.

This is the desired result. That this was not the case on the original fuzzy view was precisely the problem with which we started.

Summary and conclusion

We looked at existing theories of vagueness.

We then turned to the question of a fundamental definition of vagueness. We proposed and argued for Closeness.

We argued on this basis that the correct theory of vagueness must utilise degrees of truth: or else it will not be able to accept that there are any vague predicates (that have associated Sorites series).

The basic fuzzy theory of vagueness — fuzzy monovaluationism — faces the artificial precision problem. We argued for fuzzy plurivaluationism as the solution.

Overall the conclusion is that fuzzy plurivaluationism is the best theory of vagueness.