

Partial suppression of nonadiabatic transitions

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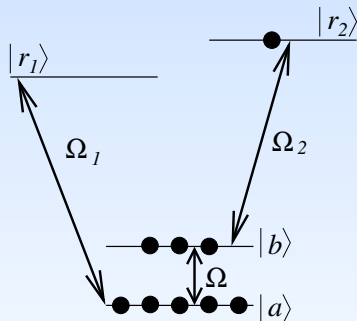
INVESTMENTS IN EDUCATION DEVELOPMENT

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Partial suppression of nonadiabatic transitions

Outlook

- Adiabatic processes, Berry compensation
- Partial compensation of nonadiabatic transitions
- Examples: interacting spins, expanding potential well, atoms with Rydberg blockade
- Discussion: further prospects of the method



Partial suppression of nonadiabatic transitions

Dynamic vs. adiabatic transport



Adiabatic processes and Berry compensation

Evolution under time-dependent Hamiltonian

$$i\hbar \frac{d}{dt} |\Psi\rangle = H_0(t) |\Psi\rangle \quad (1)$$

expand

$$|\Psi(t)\rangle = \sum_n a_n(t) e^{i\theta_n(t)} |n(t)\rangle \quad (2)$$

with instantaneous eigenstates

$$H_0(t) |n(t)\rangle = E_n(t) |n(t)\rangle \quad (3)$$

$$\theta_n(t) = - \int_0^t \frac{E_n(t')}{\hbar} dt' \quad (4)$$

Adiabatic processes and Berry compensation

Evolution under time-dependent Hamiltonian

$$i\hbar \sum_n \dot{a}_n e^{i\theta_n} |n\rangle + \sum_n a_n E_n e^{i\theta_n} |n\rangle + i\hbar \sum_n a_n e^{i\theta_n} |\dot{n}\rangle = \sum_n a_n E_n e^{i\theta_n} |n\rangle \quad (5)$$

multiplying with $\langle k|$:

$$\dot{a}_k = - \sum_n a_n e^{i(\theta_n - \theta_k)} \langle k | \dot{n} \rangle \quad (6)$$

Adiabatic processes and Berry compensation

Evolution under time-dependent Hamiltonian

Time derivative of $H_0|n\rangle = E_n|n\rangle$:

$$\dot{H}_0|n\rangle + H_0|\dot{n}\rangle = \dot{E}_n|n\rangle + E_n|\dot{n}\rangle \quad (7)$$

Multiply with $\langle k|$ with $k \neq n$:

$$\langle k|\dot{H}_0|n\rangle + \langle k|H_0|\dot{n}\rangle = E_n\langle k|\dot{n}\rangle, \quad (8)$$

$$\langle k|\dot{H}_0|n\rangle + E_k\langle k|\dot{n}\rangle = E_n\langle k|\dot{n}\rangle, \quad (9)$$

$$\langle k|\dot{n}\rangle = \frac{\langle k|\dot{H}_0|n\rangle}{E_n - E_k} \quad (10)$$

Therefore

$$\dot{a}_k = -a_k\langle k|\dot{k}\rangle - \sum_{n \neq k} a_n e^{i(\theta_n - \theta_k)} \frac{\langle k|\dot{H}_0|n\rangle}{E_n - E_k} \quad (11)$$

Adiabatic processes and Berry compensation

Evolution under time-dependent Hamiltonian

Adiabatic approximation:

$$\left| \frac{\langle k | \dot{H}_0 | n \rangle}{E_n - E_k} \right| T \ll 1, \quad (12)$$

$$\dot{a}_k \approx -a_k \langle k | \dot{k} \rangle \quad (13)$$

$$a_k(t) \approx a_k(0) e^{i\gamma(t)} \quad (14)$$

Geometric phase γ :

$$\gamma(t) = i \int_0^t \langle k(t') | \dot{k}(t') \rangle dt' \quad (15)$$

Evolution under time-dependent Hamiltonian

Apply an additional Hamiltonian

[Demirplak and Rice, J. Phys. Chem. A **107**, 9937 (2003); Berry, J. Phys. A Math. Theor. **42**, 365303 (2009)] :

$$H_B = i\hbar \sum_n (|\dot{n}\rangle\langle n| - |n\rangle\langle n|\dot{n}\rangle\langle n|) \quad (16)$$

System starting in eigenstate $|n\rangle$ of H_0 , evolving under $H = H_0 + H_B$ stays **exactly** in eigenstate $|n\rangle$ of H_0 .

What if H_B is not available in the lab?

Partial compensation

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But we have couple of other controllable Hamiltonians instead:

L_1, L_2, \dots

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Task: Keep system as close as possible to eigenstate $|0(t)\rangle$ of $H_0(t)$.

Choose suitable $\alpha_1(t), \alpha_2(t) \dots$, the system evolves under Hamiltonian

$$H = H_0 + \alpha_1 L_1 + \alpha_2 L_2 \dots = H_0 + H_C \quad (17)$$

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Minimize norm of vector $(H_C - H_B)|0\rangle$, i.e., minimize

$$\langle 0 | \left(\sum_{k=1}^K \alpha_k L_k - H_B \right) \left(\sum_{k'=1}^K \alpha_{k'} L_{k'} - H_B \right) | 0 \rangle. \quad (18)$$

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Minimising the quadratic form by solving linear equations:

$$\sum_{k=1}^K A_{m,k} \alpha_k = C_m, \quad (19)$$

where

$$A_{m,k} = \langle L_m L_k + L_k L_m \rangle, \quad (20)$$

$$C_k = \langle L_k H_B + H_B L_k \rangle, \quad (21)$$

(mean values calculated in state $|0(t)\rangle\rangle$)

T. Opatrný and K. Mølmer, *New J. Phys.* **16** 015025 (2014).

Example: two spin- $\frac{1}{2}$ particles

$$H_0 = -B(\sigma_x^{(1)} + \sigma_x^{(2)}) + J\sigma_z^{(1)}\sigma_z^{(2)} \quad (22)$$

Eigenstates:

$|\rightarrow\rightarrow\rangle$ (for $|B/J| \gg 1$)

$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ (for $|B/J| \ll 1$).

$$H_0 = \begin{pmatrix} J & -B & -B & 0 \\ -B & -J & 0 & -B \\ -B & 0 & -J & -B \\ 0 & -B & -B & J \end{pmatrix}. \quad (23)$$

Example: two spin- $\frac{1}{2}$ particles

With parametrization

$$J = A \sin \varphi \quad (24)$$

$$B = \frac{A}{2} \cos \varphi \quad (25)$$

eigenvectors

$$|\phi_1\rangle = \frac{1}{2\sqrt{1+\sin\varphi}} \begin{pmatrix} \cos\varphi \\ 1+\sin\varphi \\ 1+\sin\varphi \\ \cos\varphi \end{pmatrix}, \quad |\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix},$$
$$|\phi_3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \quad |\phi_4\rangle = \frac{1}{2\sqrt{1+\sin\varphi}} \begin{pmatrix} 1+\sin\varphi \\ -\cos\varphi \\ -\cos\varphi \\ 1+\sin\varphi \end{pmatrix}. \quad (26)$$

Example: two spin- $\frac{1}{2}$ particles

Berry Hamiltonian

$$H_B = i\hbar|\dot{\phi}_1\rangle\langle\phi_1| + i\hbar|\dot{\phi}_4\rangle\langle\phi_4|, \quad (27)$$

$$H_B = \frac{\hbar\dot{\varphi}}{4} \begin{pmatrix} 0 & -i & -i & 0 \\ i & 0 & 0 & i \\ i & 0 & 0 & i \\ 0 & -i & -i & 0 \end{pmatrix} = \frac{\hbar\dot{\varphi}}{4} \left(\sigma_y^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_y^{(2)} \right). \quad (28)$$

Example: two spin- $\frac{1}{2}$ particles

Partial compensation

$$L = p\sigma_y^{(1)}\sigma_z^{(2)} + q\sigma_z^{(1)}\sigma_y^{(2)}. \quad (29)$$

$$\langle\phi_1|L^2|\phi_1\rangle = (q+p)^2, \quad (30)$$

$$\langle\phi_1|LH_B + H_B L|\phi_1\rangle = (q+p)\hbar\dot{\varphi}, \quad (31)$$

$$(32)$$

$$\alpha = \frac{\langle\phi_1|LH_B + H_B L|\phi_1\rangle}{2\langle\phi_1|L^2|\phi_1\rangle} = \frac{\hbar\dot{\varphi}}{2(q+p)}. \quad (33)$$

$$H_C = \alpha L = \hbar\dot{\varphi} \frac{p\sigma_y^{(1)}\sigma_z^{(2)} + q\sigma_z^{(1)}\sigma_y^{(2)}}{2(q+p)}. \quad (34)$$

Example: four spin- $\frac{1}{2}$ particles

Original Hamiltonian:

$$H_0 = -B(t) \sum_{j=1}^4 \sigma_x^{(j)} + J_0 \sum_{j=1}^3 \sigma_z^{(j)} \sigma_z^{(j+1)}. \quad (35)$$

Parameter change: $B(t) = B_0 \exp(-2.4t/t_0)$

State change from $|\rightarrow\rightarrow\rightarrow\rightarrow\rangle$ to $|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle$

For adiabatic transition: t_0 must be large.

Example: four spin- $\frac{1}{2}$ particles

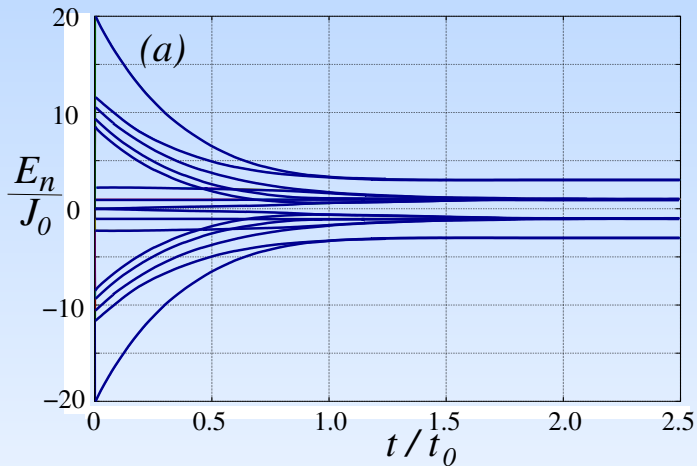


Figure: Eigenvalues of the Hamiltonian (35).

Example: four spin- $\frac{1}{2}$ particles

Possible choice of compensating operators:

$$L_1 = \sigma_y^{(1)} \sigma_z^{(2)} + \sigma_z^{(3)} \sigma_y^{(4)}, \quad (36)$$

$$L_2 = \sigma_z^{(1)} \sigma_y^{(2)} + \sigma_y^{(3)} \sigma_z^{(4)}, \quad (37)$$

$$L_3 = \sigma_y^{(2)} \sigma_z^{(3)} + \sigma_z^{(2)} \sigma_y^{(3)}, \quad (38)$$

$$L_4 = \sigma_z^{(1)} \sigma_y^{(4)} + \sigma_y^{(1)} \sigma_z^{(4)}. \quad (39)$$

Example: four spin- $\frac{1}{2}$ particles

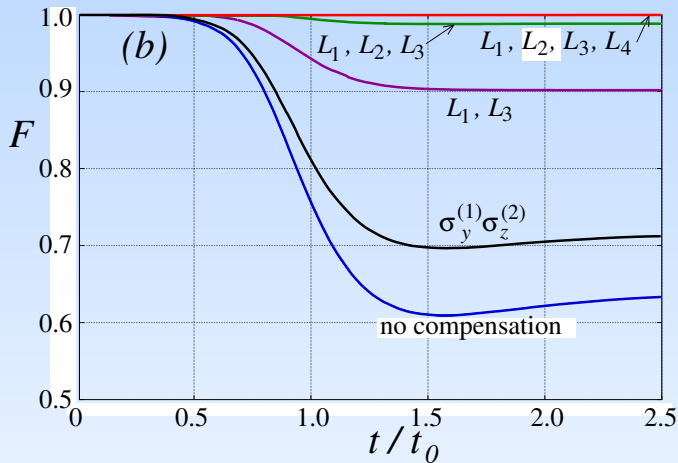


Figure: Fidelity of the evolved state

Example: four spin- $\frac{1}{2}$ particles

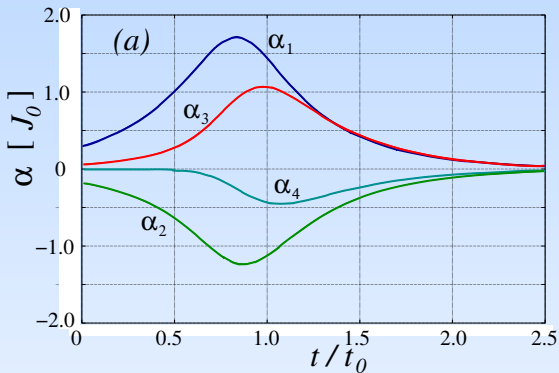


Figure: Compensation parameters α_1 – α_4 of the scheme with 4 operators.

Example: four spin- $\frac{1}{2}$ particles

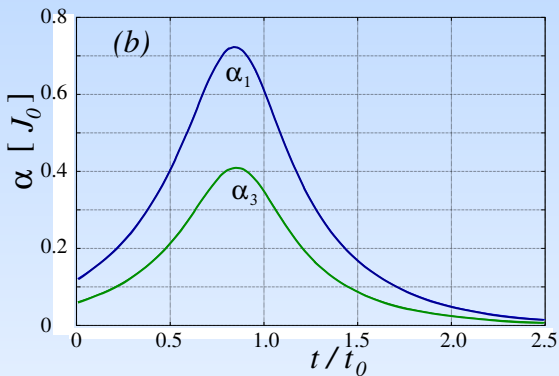


Figure: Compensation parameters α_1 and α_3 of the scheme with 2 operators.

Example: particle in an expanding box

Infinitely deep box, one wall moving:

$$H_0 = \frac{p^2}{2m} + U(x) \quad (40)$$

with

$$U(x) = \begin{cases} \infty & \text{for } x < 0, \\ 0 & \text{for } 0 < x < D(t), \\ \infty & \text{for } D(t) < x. \end{cases} \quad (41)$$

Berry compensation: solved by Jarzynski [arXiv:1305.4967 (2013)]

$$H_B = \frac{\dot{D}}{2D}(xp + px). \quad (42)$$

Example: particle in an expanding box

If H_B is not available, another option:

$$H_C = \frac{\dot{D}}{2} p. \quad (43)$$

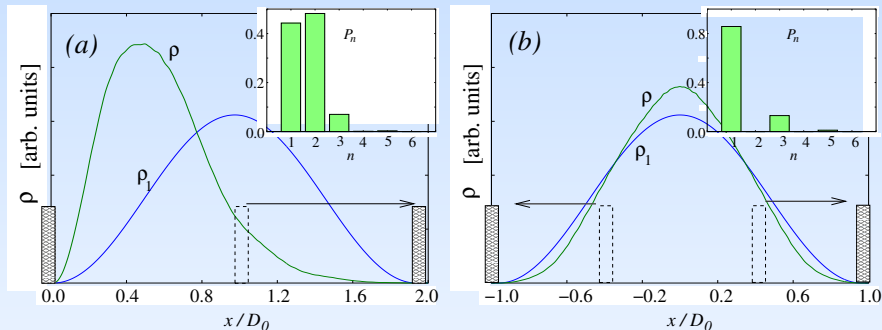
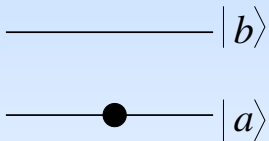


Figure: Particle in expanding box.

Example: squeezing with Rydberg blockade

Spin squeezing

Example: two-level atoms



Spin squeezing

Example: two-level atoms

Single-atom operators:

$$\begin{aligned}S_x &= \frac{1}{2}(|a\rangle\langle b| + |b\rangle\langle a|), \\S_y &= \frac{i}{2}(-|a\rangle\langle b| + |b\rangle\langle a|), \\S_z &= \frac{1}{2}(|a\rangle\langle a| - |b\rangle\langle b|).\end{aligned}$$

Example: squeezing with Rydberg blockade

Spin squeezing

Example: two-level atoms

Many atoms:

$$\vec{J} = \sum_k \vec{S}_k$$

$$J_x = \frac{1}{2}(a^\dagger b + ab^\dagger),$$

$$J_y = \frac{i}{2}(a^\dagger b - ab^\dagger),$$

$$J_z = \frac{1}{2}(a^\dagger a - b^\dagger b),$$

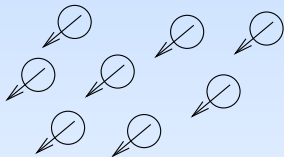
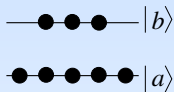
$$[a, a^\dagger] = [b, b^\dagger] = 1$$

$$[J_x, J_y] = -iJ_z$$

Example: squeezing with Rydberg blockade

Spin squeezing

Example: many two-level atoms

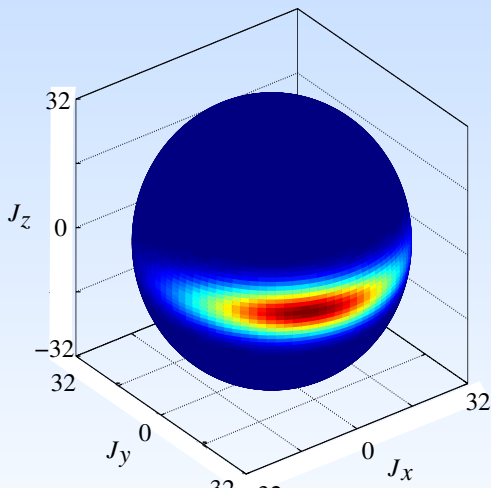


Example: squeezing with Rydberg blockade

Spin squeezing

Many two-level atoms

Poincare sphere



Spin squeezing

Many two-level atoms

PHYSICAL REVIEW A

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JULY 1994

Squeezed atomic states and projection noise in spectroscopy

D. J. Wineland, J. J. Bollinger, and W. M. Itano

Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303

D. J. Heinzen

Physics Department, University of Texas, Austin, Texas 78712

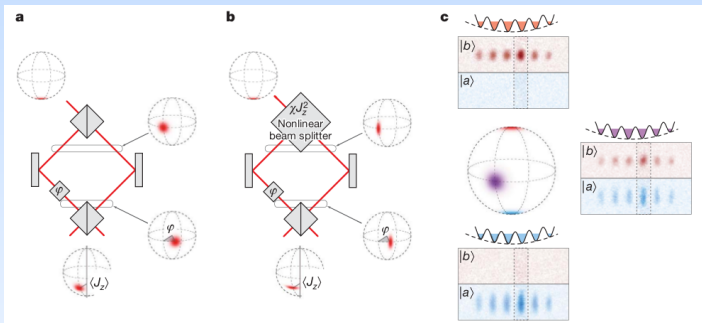
(Received 11 January 1994)

We investigate the properties of angular-momentum states which yield high sensitivity to rotation. We discuss the application of these "squeezed-spin" or correlated-particle states to spectroscopy. Transitions in an ensemble of N two-level (or, equivalently, spin- $\frac{1}{2}$) particles are assumed to be detected by observing changes in the state populations of the particles (population spectroscopy). When the particles' states are detected with 100% efficiency, the fundamental limiting noise is projection noise, the noise associated with the quantum fluctuations in the measured populations. If the particles are first prepared in particular quantum-mechanically correlated states, we find that the signal-to-noise ratio can be improved over the case of initially uncorrelated particles. We have investigated spectroscopy for a particular case of Ramsey's separated oscillatory method where the radiation pulse lengths are short compared to the time between pulses. We introduce a squeezing parameter ξ_R which is the ratio of the statistical uncertainty in the determination of the resonance frequency when using correlated states vs that when using uncorrelated states. More generally, this squeezing parameter quantifies the sensitivity of an angular-momentum state to rotation. Other squeezing parameters which are relevant for use in other contexts can be defined. We discuss certain states which exhibit squeezing parameters $\xi_R \approx N^{-1/2}$. We investigate possible experimental schemes for generation of squeezed-spin states which might be applied to the spectroscopy of trapped atomic ions. We find that applying a Jaynes-Cummings-type coupling between the ensemble of two-level systems and a suitably prepared harmonic oscillator results in correlated states with $\xi_R < 1$.

Example: squeezing with Rydberg blockade

Spin squeezing

Gross et al., Nature 464, 1165 (2010)

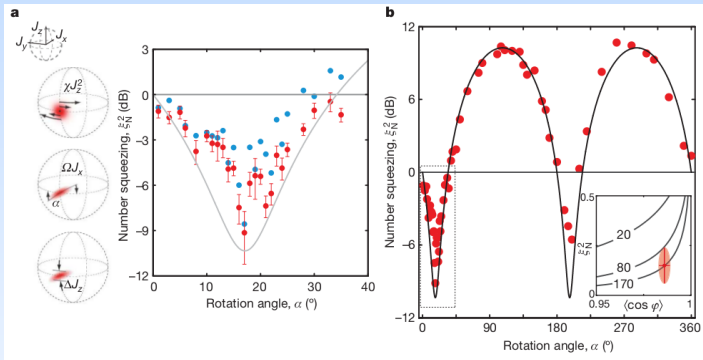


$\sim 10^3$ atoms squeezed by ~ 5 dB in ~ 10 ms

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Spin squeezing

Gross et al., Nature 464, 1165 (2010)

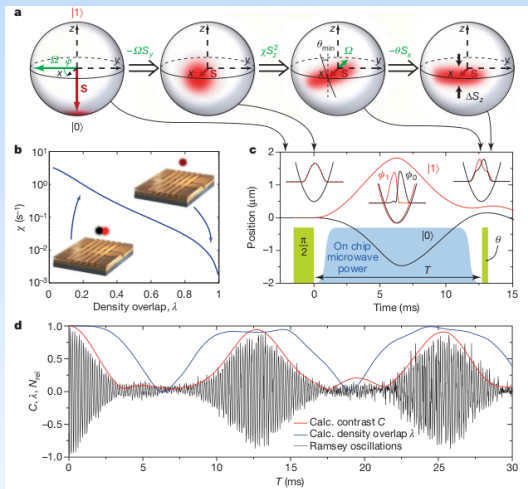


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Riedel et al., Nature 464, 1170 (2010)

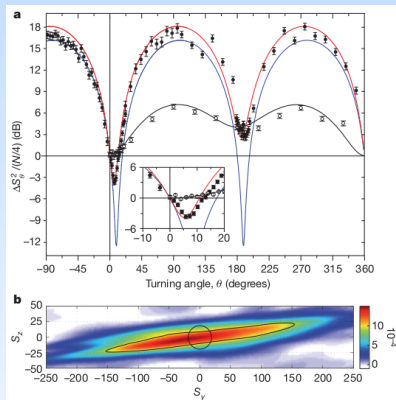


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Riedel et al., Nature 464, 1170 (2010)



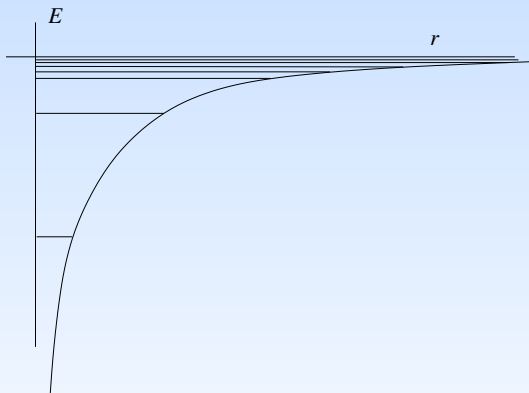
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Example: squeezing with Rydberg blockade

Rydberg atom

Excited atom with large principal number n

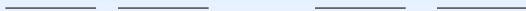
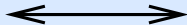
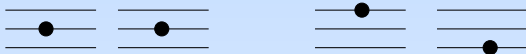
- size $\sim n^2$ ($\sim 0.3 \mu\text{m}$ for $n \approx 80$)
- lifetime $\sim n^3 - n^{4.5}$ ($\sim 600 \mu\text{s}$ for $n \approx 80$)



Example: squeezing with Rydberg blockade

Rydberg atom

Rydberg blockade: resonance transitions



Jaynes - Cummings model

A single two-level atom and a single-mode quantum field

$$H_{JC} = ga^+\sigma_- + g^*a\sigma_+$$

$$\sigma_+ = |b\rangle\langle a|$$

$$\sigma_- = |a\rangle\langle b|$$

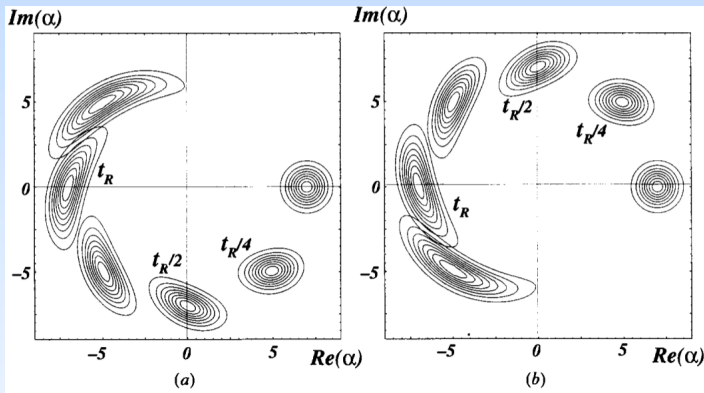
- Photon generation and atom deexcitation
- Photon absorption and atom excitation

Example: squeezing with Rydberg blockade

Jaynes - Cummings model

A single two-level atom and a single-mode quantum field

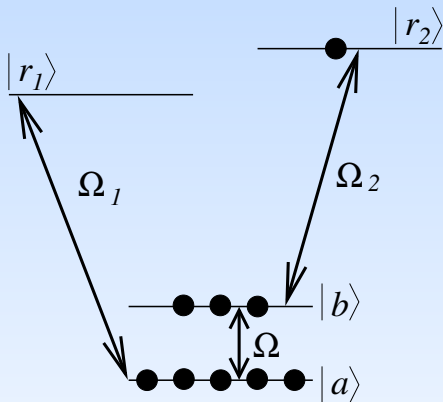
Squeezing of the field



G. Banacloche, PRL 65, 3385 (1990); picture from JMO 40, 2361 (1993).

Example: squeezing with Rydberg blockade

Spin squeezing and Schrödinger cat generation in atomic samples with Rydberg blockade



T. Opatrný and K. Mølmer, PRA 86, 023845 (2012)

Hamiltonian

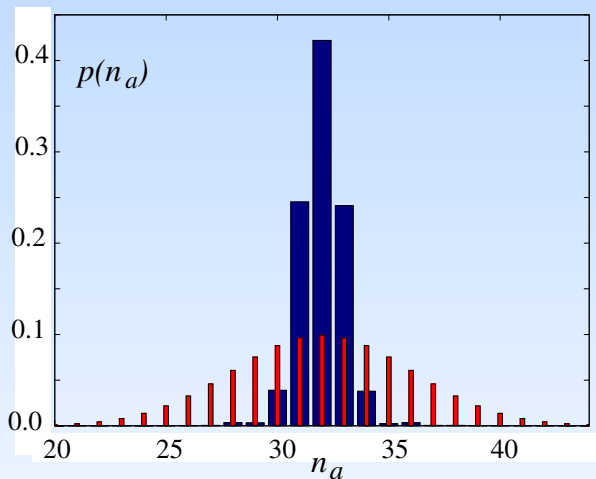
$$H_{JC1} = \Omega_1 a \sigma_+^{(1)} + \Omega_1^* a^\dagger \sigma_-^{(1)}$$

$$H_{JC2} = \Omega_2 b \sigma_+^{(2)} + \Omega_2^* b^\dagger \sigma_-^{(2)}$$

- Initialize the state
- Act with the Hamiltonian
- Rotate the state

Example: squeezing with Rydberg blockade

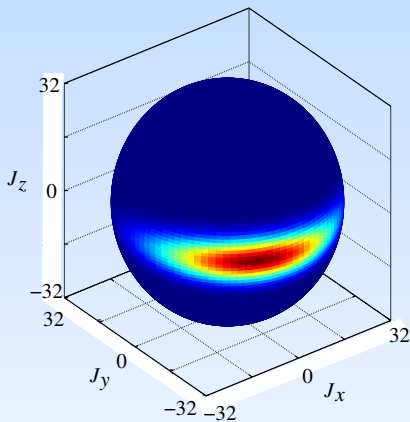
Results



Statistics of the atomic states $|a\rangle$ and $|b\rangle$ (64 atoms)

Example: squeezing with Rydberg blockade

Results



Q-function of the resulting state (64 atoms)

Example: squeezing with Rydberg blockade

Adiabatic squeezing: Hamiltonian eigenstates

$$\begin{aligned} |\psi_{+,+}^{(n_a, n_b)}\rangle &= \frac{1}{2} (|n_a, n_b, 0, 0\rangle + |n_a - 1, n_b, 1, 0\rangle \\ &\quad + |n_a, n_b - 1, 0, 1\rangle + |n_a - 1, n_b - 1, 1, 1\rangle), \\ |\psi_{+,-}^{(n_a, n_b)}\rangle &= \frac{1}{2} (|n_a, n_b, 0, 0\rangle + |n_a - 1, n_b, 1, 0\rangle \\ &\quad - |n_a, n_b - 1, 0, 1\rangle - |n_a - 1, n_b - 1, 1, 1\rangle), \\ |\psi_{-,+}^{(n_a, n_b)}\rangle &= \frac{1}{2} (|n_a, n_b, 0, 0\rangle - |n_a - 1, n_b, 1, 0\rangle \\ &\quad + |n_a, n_b - 1, 0, 1\rangle - |n_a - 1, n_b - 1, 1, 1\rangle), \\ |\psi_{-,-}^{(n_a, n_b)}\rangle &= \frac{1}{2} (|n_a, n_b, 0, 0\rangle - |n_a - 1, n_b, 1, 0\rangle \\ &\quad - |n_a, n_b - 1, 0, 1\rangle + |n_a - 1, n_b - 1, 1, 1\rangle), \end{aligned}$$

Adiabatic squeezing: Eigenenergies

$$E_{+,+}^{(n_a, n_b)} = \Omega_{JC} (\sqrt{n_a} + \sqrt{n_b}),$$

$$E_{+,-}^{(n_a, n_b)} = \Omega_{JC} (\sqrt{n_a} - \sqrt{n_b}),$$

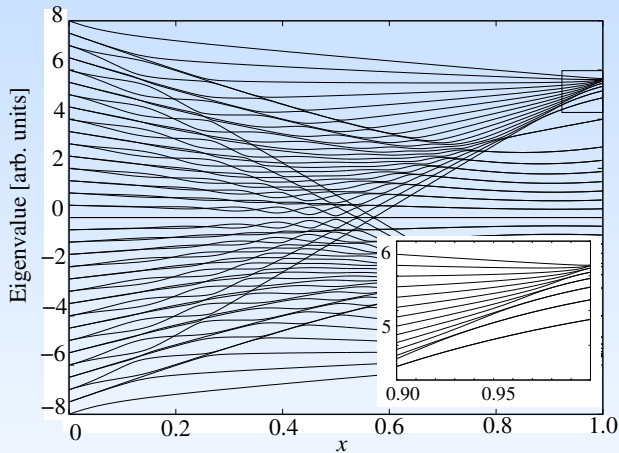
$$E_{-,+}^{(n_a, n_b)} = \Omega_{JC} (-\sqrt{n_a} + \sqrt{n_b}),$$

$$E_{-,-}^{(n_a, n_b)} = \Omega_{JC} (-\sqrt{n_a} - \sqrt{n_b}).$$

Example: squeezing with Rydberg blockade

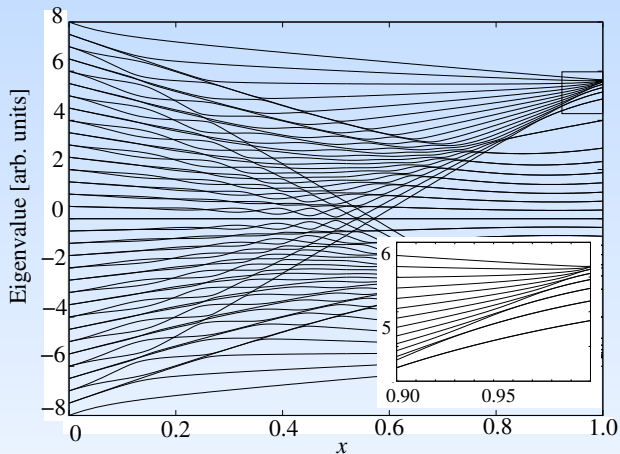
Adiabatic squeezing: Combine Hamiltonian

$$H = uH_{JC} + (1 - u)J_x$$



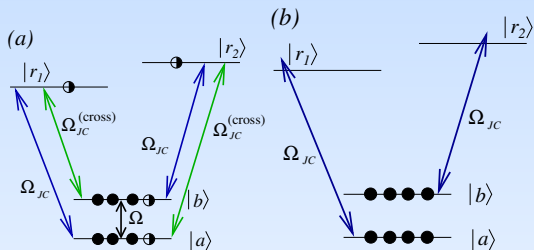
Example: squeezing with Rydberg blockade

BUT PROBLEM: LINES TOO CLOSE!



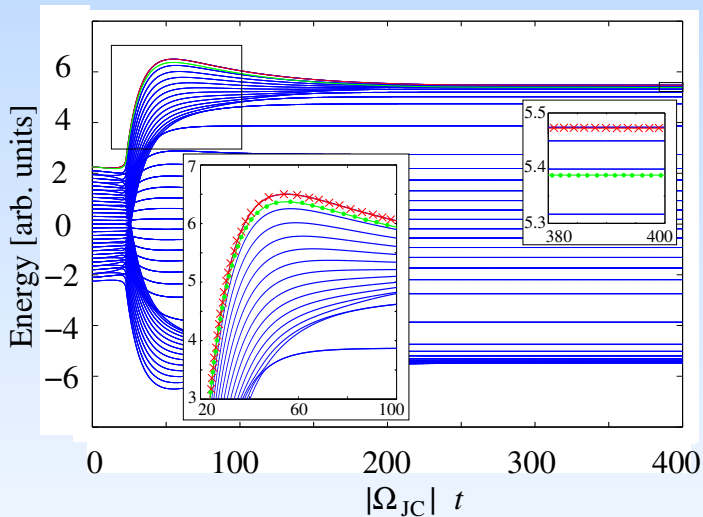
Example: squeezing with Rydberg blockade

Adiabatic squeezing



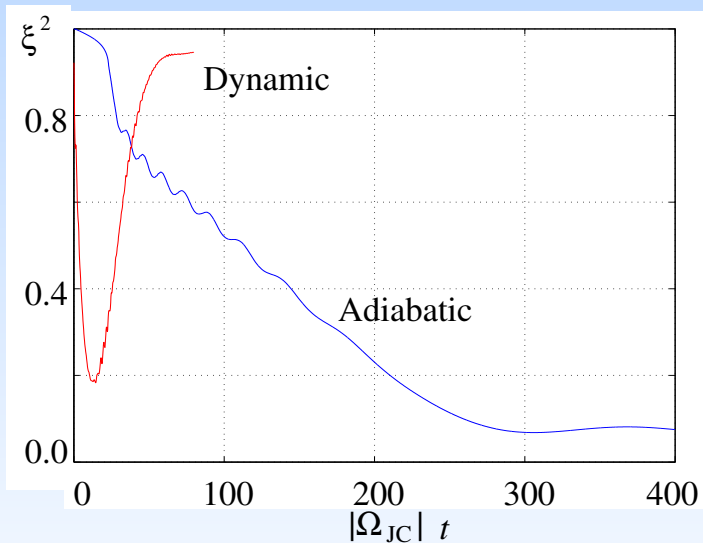
Example: squeezing with Rydberg blockade

Adiabatic squeezing



Example: squeezing with Rydberg blockade

Adiabatic squeezing



Discussion, open questions

- Any general rule for the spin systems?
- Applicability in trapped ion systems?
- Is there any possibility for compensation of nonadiabatic processes in superfluid — Mott insulator transitions?
- Any suitable approximative methods for large systems (Hilbert space expands, impossible to solve exactly)?
- So far optimization for a single state $|0\rangle$. Any possibility for optimization of more states? What about qubit?

Summary

- Adiabatic processes - robust, but slow. Speeding up means transitions to unwanted states.
- Additional Hamiltonian H_B can fully compensate nonadiabatic transitions. Easy to compute, but often impossible to produce in a lab.
- Partial compensation using available operators L_k : need to have nonzero averages $\langle L_k H_B + H_B L_k \rangle$ in the wanted state $|0\rangle$.
- Several examples: paramagnetic vs. antiferromagnetic spin interactions, expanding box, atoms with Rydberg blockade.
- Many open questions remain.

Squeezing with Rydberg blockade

Thanks for your attention!

