Visit to Le LIMOS (France)

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european social fund in the czech republic
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OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

Basic Information

- institution: Le LIMOS, Universit Blaise Pascal
- location: Clermont-Ferrand, France
- date: November 17th - December 2nd, 2013
- guarantee: prof. Lhouari Nourine

Le LIMOS (Laboratoire d'Informatique, de Modlisation et d'Optimisation des Systemes)

Research

- Models and Algorithms of Decision Support (Algorithms, Graphs, Complexity)
- Information Systems and Communication
- Production Systems
- Operations Research Industrial Engineering

Members

- 8 professors (Lhouari Nourine)
- 24 research and teaching assistants
- 32 PhD students

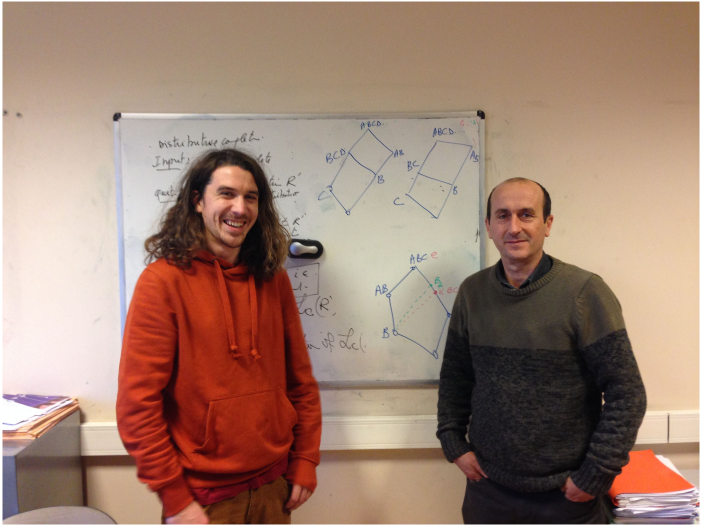
Seminar

New Cluster Mining algorithm via Data Cleaning by Takeaki Uno

PhD defence

Enumeration of minimal dominating sets of graphs by Amaud Mary

Research



Jan Laštovička and Lhouari Nourine

Distributive concept lattices

Let $\langle X, Y, I\rangle$ is a formal context, $x \in X$, and $y \in Y$.

Definition (Arrow relations)

$$
\begin{array}{ll}
x \swarrow_I y & \text{if } \begin{cases} \langle x,y\rangle \notin I \text{ and} \\ \{x\}^{\uparrow} \subset \{x_1\}^{\uparrow} \text{ imply } \langle x_1, y\rangle \in I \end{cases} \\
x \nearrow_I y & \text{if } \begin{cases} \langle x,y\rangle \notin I \text{ and} \\ \{y\}^{\downarrow} \subset \{y_1\}^{\downarrow} \text{ imply } \langle x, y_1\rangle \in I \end{cases} \\
x \swarrow\!\!\!\nearrow_I y & \text{if } x \swarrow_I y \text{ and } x \nearrow_I y
\end{array}
$$

Proposition

For a finite concept lattice $\mathscr{B}(X,Y,I)$, the following conditions are equivalent:

- $\mathscr{B}(X,Y,I)$ *is distributive.*
- *From* $x \swarrow\!\!\!\nearrow y_1$ *and* $x \nearrow y_2$ *it follows that* $\{y_1\}^{\downarrow} = \{y_2\}^{\downarrow}$.
- *From* $x_1 \swarrow\!\!\!\nearrow y$ *and* $x_2 \swarrow y$ *it follows that* $\{x_1\}^{\uparrow} = \{x_2\}^{\uparrow}$.

$\mathbf{L}$-relations of $\mathbf{L}$-sets

Let $\mathbf{L}$ be a Boolean algebra with variables and $A, B \in L^X$. Then *the degree* $S_{\subset}(A,B)$ *to which* A *is strictly contained in* B is defined by

$$S_{\subset}(A,B) = S(A,B) \wedge \big(S(B,A)\big)'.$$

Lemma

For each admissible assignment v *we have*

1. $\bar{v}(S(A,B)) = 1$ *iff* $\bar{v} \circ A \subseteq \bar{v} \circ B$,
2. $\bar{v}(A \approx B) = 1$ *iff* $\bar{v} \circ A = \bar{v} \circ B$,
3. $\bar{v}(S_{\subset}(A,B)) = 1$ *iff* $\bar{v} \circ A \subset \bar{v} \circ B$.

Incomplete arrow relations

Let $\langle X, Y, I\rangle$ be an incomplete $\mathbf{L}$-context.

$$x \nearrow_I y = I(x,y)' \wedge \bigwedge_{y_1 \in Y} S_{\subset}(\{y\}^{\downarrow}, \{y_1\}^{\downarrow}) \to I(x, y_1)$$

$$x \swarrow_I y = I(x,y)' \wedge \bigwedge_{x_1 \in X} S_{\subset}(\{x\}^{\uparrow}, \{x_1\}^{\uparrow}) \to I(x_1, y)$$

$$x \swarrow\!\!\!\nearrow_I y = (x \swarrow_I y) \wedge (x \nearrow_I y)$$

Lemma

For an admissible assignment v *we have* $x \nearrow_{\bar{v}\circ I} y = \bar{v}(x \nearrow_I y)$, $x \swarrow_{\bar{v}\circ I} y = \bar{v}(x \swarrow_I y)$, *and* $x \swarrow\!\!\!\nearrow_{\bar{v}\circ I} y = \bar{v}(x \swarrow\!\!\!\nearrow_I y)$.

Completations with distributive concept lattice

Let $\langle X, Y, I \rangle$ be an incomplete $\mathbf{L}$-context.

Theorem

Let v an admissible assignment. Then the following conditions are equivalent.

1. $\mathscr{B}(X, Y, \bar{v} \circ I)$ *is distributive.*
2. $\bar{v}\big(\bigwedge_{x \in X, y_1 \in Y, y_2 \in Y} (x \mathrel{\swarrow\!\!\!\!\!\nearrow}_I y_1) \wedge (x \nearrow_I y_2) \rightarrow (\{y_1\}^{\downarrow_I} \approx \{y_2\}^{\downarrow_I})\big) = 1.$
3. $\bar{v}\big(\bigwedge_{x_1 \in X, x_2 \in X, y \in Y} (x_1 \mathrel{\swarrow\!\!\!\!\!\nearrow}_I y) \wedge (x_2 \swarrow_I y) \rightarrow (\{x_1\}^{\uparrow_I} \approx \{x_2\}^{\uparrow_I})\big) = 1.$

Theorem

A problem whether an incomplete $\mathbf{L}$-context where each assignment is admissible has a completion with distributive concept lattice is NP-complete.

Example

Let $\mathbf{L}$ he Boolean algebra with variables u_1, u_2, u_3 such that each admissible assignment v satisfy $v(u_1)' \leq v(u_2)' \wedge v(u_3)'$, $v(u_1) \wedge v(u_2) \leq v(u_3)$.

Admissible assignments: $v_1 = \emptyset, v_2 = \{u_1, u_3\}, v_3 = \{u_1, u_2\}, v_4 = \{u_1, u_2, u_3\}$.

Incomplete $\mathbf{L}$-context $\langle X, Y, I \rangle$:

	y_1	y_2	y_3
x_1	u_1		
x_2		u_2	u_3
x_3			$\times$

$\bigwedge_{x \in X, y_1 \in Y, y_2 \in Y} (x \swarrow\!\!\nearrow_I y_1) \wedge (x \nearrow_I y_2) \to (\{y_1\}^{\downarrow_I} \approx \{y_2\}^{\downarrow_I}) = u_2'$

Completions with distributive concept lattices: $\langle X, Y, \bar{v}_1 \circ I \rangle$ and $\langle X, Y, \bar{v}_2 \circ I \rangle$.