Quantum Information:

Noiseless Amplification of light and other stories

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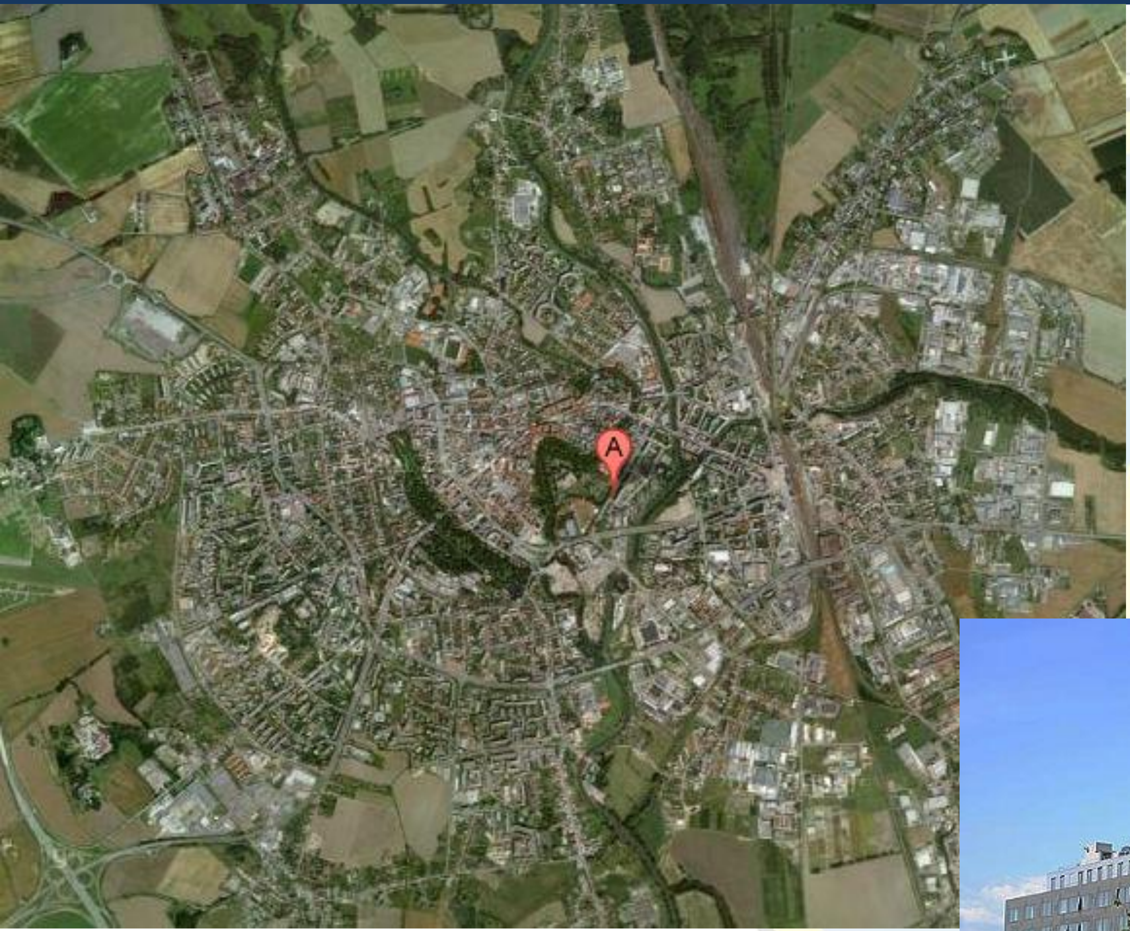
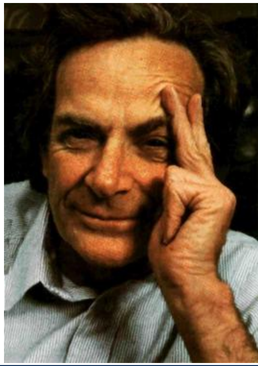


What is Quantum Information?

- Quantum systems exhibit some very strange features
  - Uncertainty relations
  - Entanglement

- Instead of working around them, let’s use them.



Some examples?

- Quantum computation
  - Exponential speedup over classical protocols
- Quantum cryptography
  - Unconditionally secure key distribution

To approaches to Quantum Information:

- Top to bottom:
  - A distant goal
  - Universal can-do-all operations
  - Principal possibility is more important than experimental feasibility
  - *What can be done.*
- Bottom to top:
  - A specific task
  - A specific device
  - Strong focus on experimental feasibility
  - *What can be done with resources at hand.*

Noiseless amplification of light

- Light is a very good carrier of information
  - Classically encoded into intensity
- In classical communication, amplification is important tool for compensation of losses and noise
- Quantum light useful for quantum communication (quantum cryptography)
  - Single photons
  - Coherent states (collective states of many photons)

Brief introduction to quantum optics

- Light = harmonic oscillator

$$\hat{H} = \hbar\omega(\hat{x}^2 + \hat{p}^2) = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

  – Annihilation and creation operators

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

  – Quantum states can be expressed a vector…

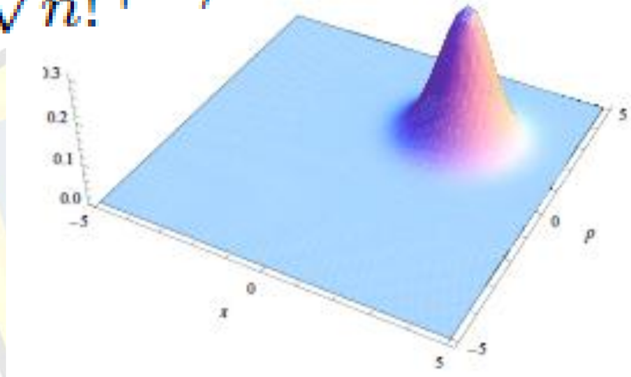
$$|\psi\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$$

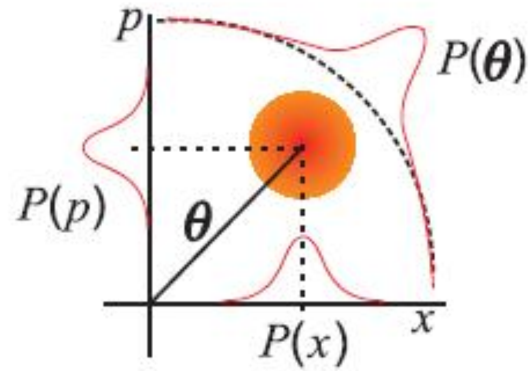
  – Or by a two-dimensional quasi-probability distribution

$$W_\psi(x,p)$$

Bottom to top – amplification of coherent states

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$



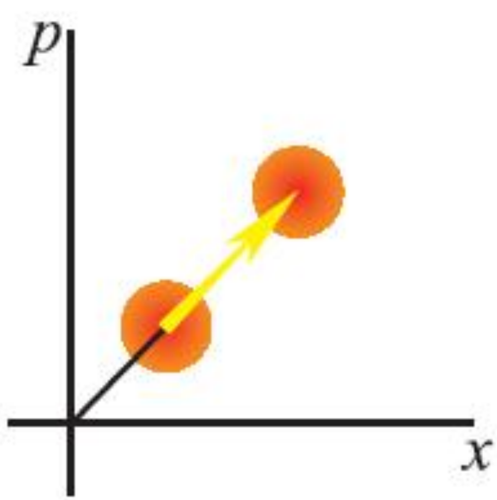


- Approximation of single mode laser light
- Almost classical state
- Composed of independent photons
- No entanglement
- Quantum nature manifests as irreducible noise

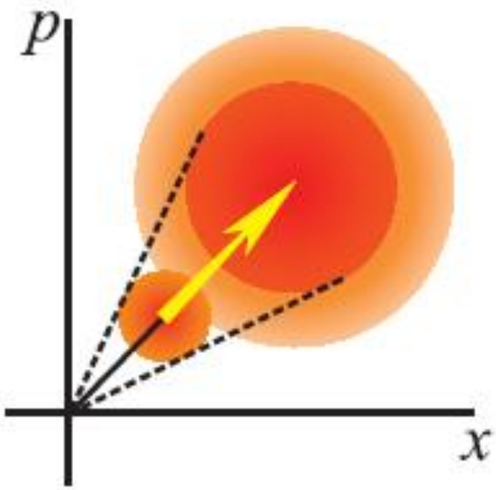
Amplification of coherent states

- Idealistic amplification
  - Does not exist

$$|\alpha\rangle \rightarrow |g\alpha\rangle$$



- Gaussian amplification
  - From population inversion in nonlinear media
  - Adds extra noise
    - This actually makes things worse

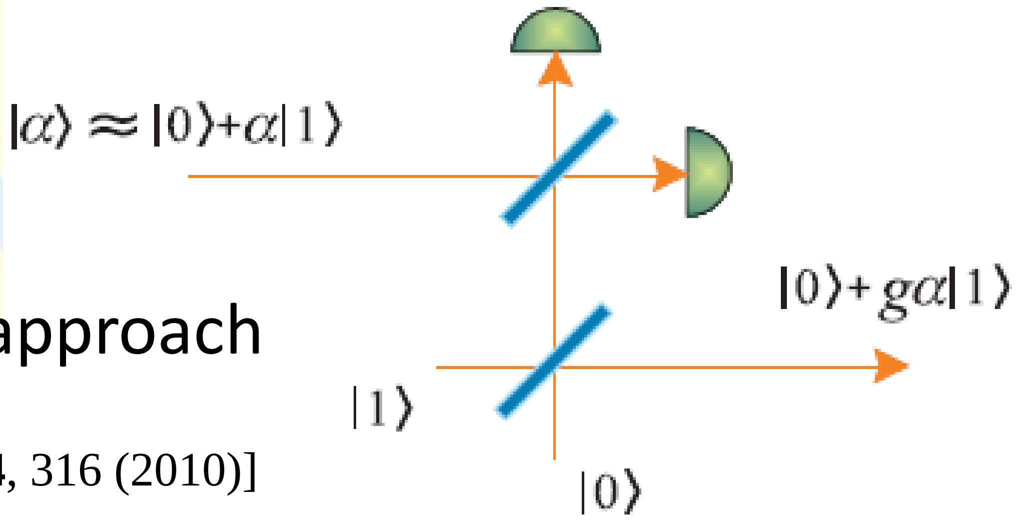


Weak coherent state amplification

$$|\alpha\rangle \propto |0\rangle + \alpha|1\rangle + \frac{\alpha^2}{\sqrt{2}}|2\rangle + \frac{\alpha^3}{\sqrt{6}}|3\rangle + \cdots$$

$$|\alpha\rangle \approx |0\rangle + \alpha|1\rangle$$

- Strong coherent states can be divided into weak coherent states, amplified, and combined again
  - For example:
  - The "universal" approach

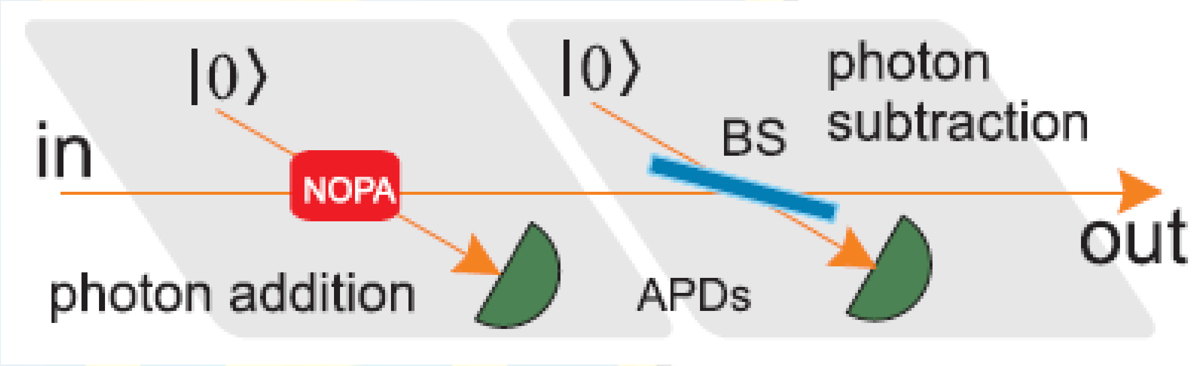


[Xiang et al. Nature Photonics **4**, 316 (2010)]

[Ferreyol et al., Phys. Rev. Lett **104**, 123603 (2010)]

Weak coherent state amplification, take 2

- What about something less resource intensive?



$$\hat{a}\hat{a}^{\dagger}(|0\rangle + \alpha|1\rangle) = \hat{a}(|1\rangle + \sqrt{2}\alpha|2\rangle) = |0\rangle + 2\alpha|1\rangle$$

- Amplification by photon addition and subtraction
- Effective gain g=2
- But… we still need single photons.

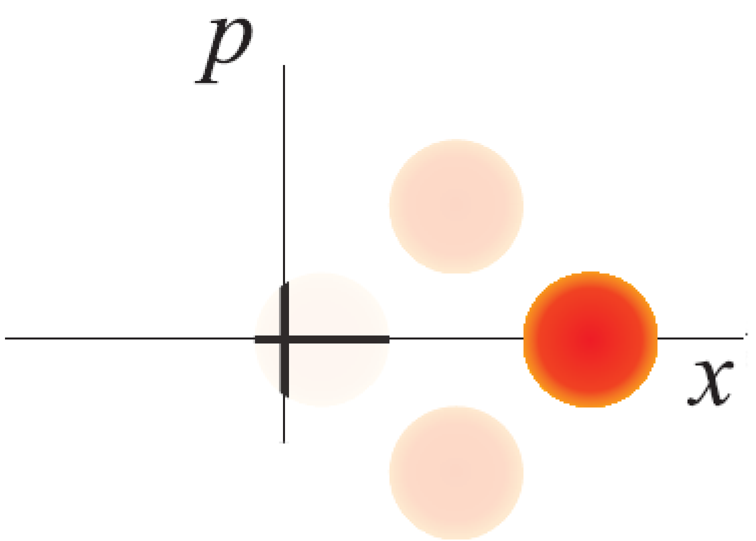
A detour: What's so bad about single photons?

- Single photons are extremely potent resource
  - In principle, any quantum state and operation can be constructed if we have good enough manipulation at the single photon level
  - They are "universal" tool - it is not surprising they can help with a specific task
- Single photons are expensive resource
  - They can be used, but it's not exactly easy…

[Zavatta et al., Nature Photonics **5**, 52 (2011)]

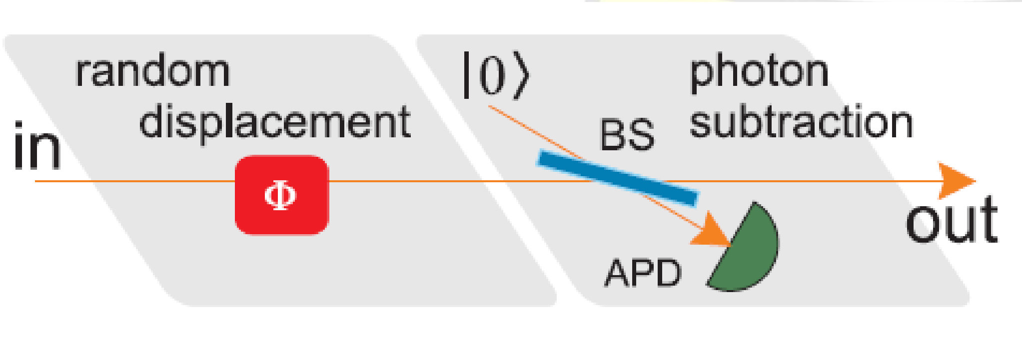
Amplification without single photons

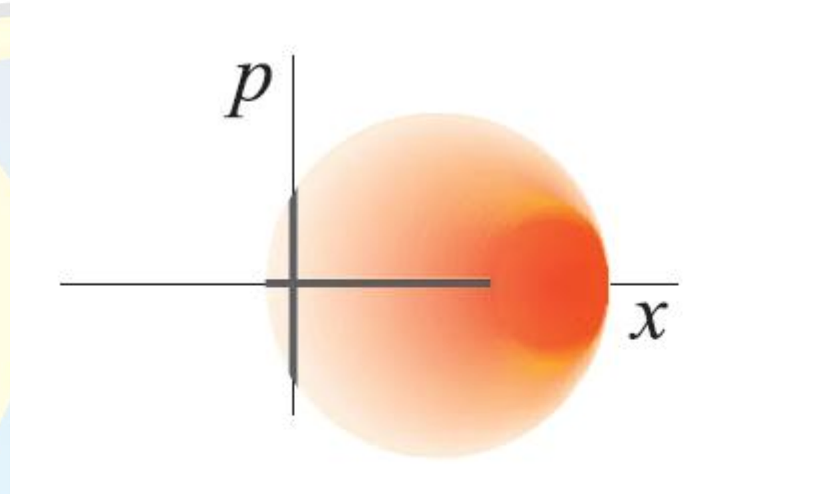
- Let’s say the state is not completely unknown
  - It has one of the four phases
- Instead of single photon addition, we displace the state randomly
- Photon subtraction serves as intensity filter



- Subtracting more photons improves the result

Full amplification



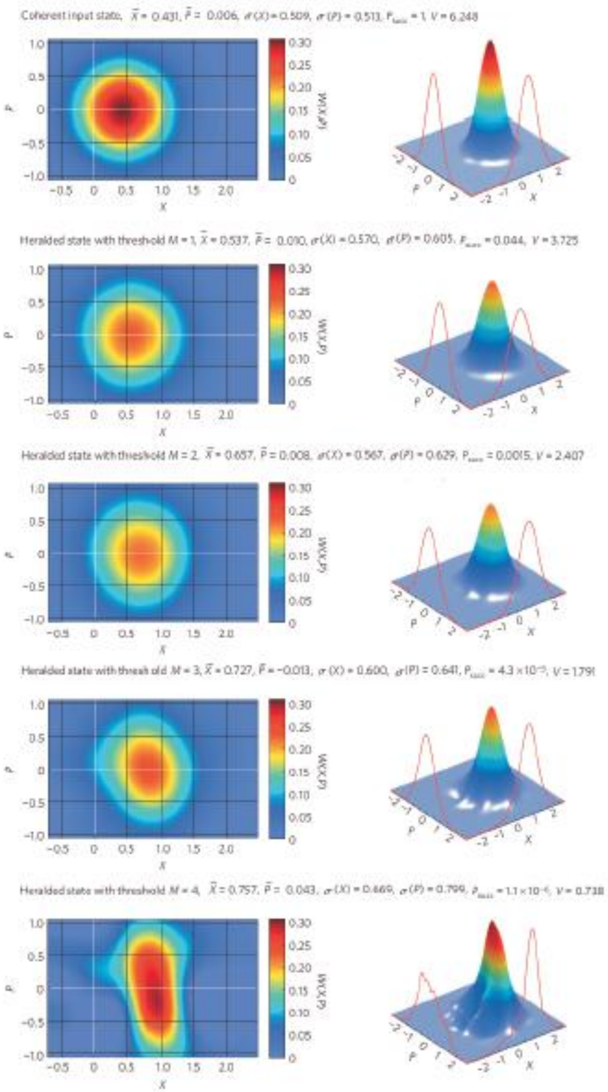


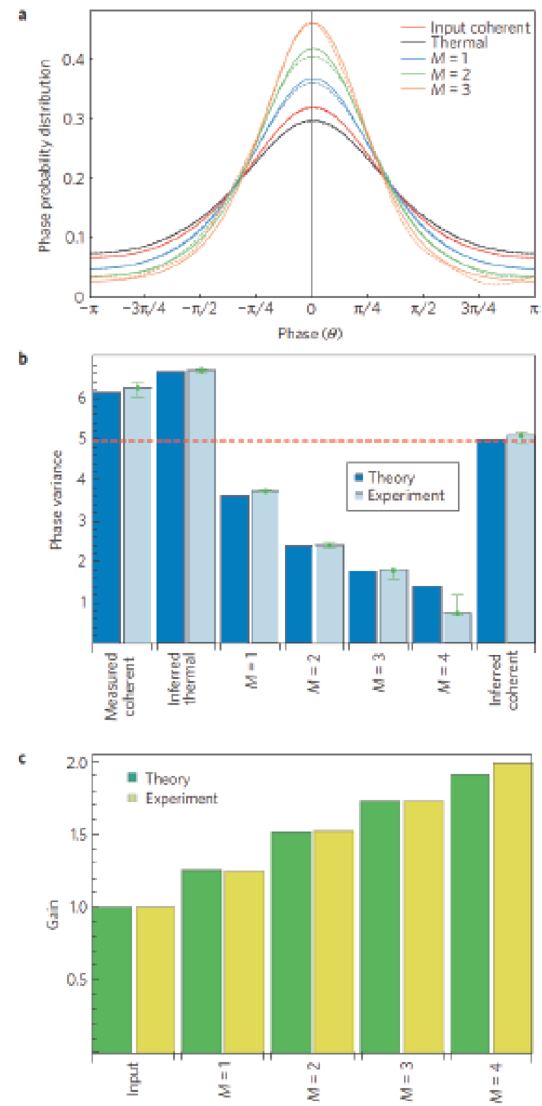
- The idea is the same as in the previous example
- The displacement is completely random this time

- The final state is distorted, but amplified
- Subtracting more photons again helps a lot

Does it work?

[Usuga et al., Nature Phys. 6, 767 (2010)]





Top to bottom approach to QIP

- What do we need for universal quantum computation?
- We need absolute control of quantum systems.
- We need the ability to perform an arbitrary quantum unitary operation

$$\hat{U} = e^{i\hat{H}t}$$

$$\hat{H} = \sum \omega_{jk} \hat{x}^j \hat{p}^k$$

Top to bottom approach to QIP

- Can it be simplified?
  - Is there a way of decomposing an arbitrary Hamiltonian to some elementary building blocks?

$$e^{iAt}e^{iBt}e^{-iAt}e^{-iBt} = e^{-[A,B]t^2} + O(t^3)$$

[Lloyd and Braunstein, Phys. Rev. Lett. **82**, 1784]

- The building blocks:

[Sefi and van Loock, arXiv:1010.0326]

$$\hat{H}_0 = \omega_0(\hat{x}^2 + \hat{p}^2)$$

$$\hat{H}_1 = \omega_1\hat{x}$$

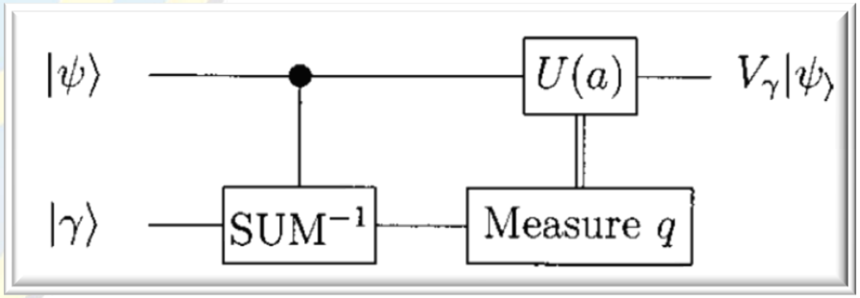
$$\hat{H}_2 = \omega_2\hat{x}^2$$

$$\hat{H}_3 = \omega_3\hat{x}^3$$

How to perform cubic operation?

- Not easily
- Third order nonlinearities are difficult to come by
- Simplification:
  - Use ancilla-and-measurement-and-feedforward driven operation

$$|\gamma\rangle = \int e^{i\chi x^3} |x\rangle dx$$



[Gottesman *et al.* , PRA 64 012310 (2001)]

A bit of math...

$$|\psi\rangle|\gamma\rangle = \int \psi(x)|x\rangle dx \int e^{i\chi y^3}|y\rangle dy$$

$$\Rightarrow \quad \int \psi(x)e^{i\chi y^3}|x, y-x\rangle dx dy$$

- The two mode state...
- ...is QND coupled...

- ...and one of the modes is measured, providing value q, and leaving the state as:

$$\int \psi(x)e^{i\chi(x+q)^3}|x\rangle dx = \int \psi(x)e^{i\chi(x^3+\underline{3qx^2+3q^2x+q^3})}|x\rangle dx$$

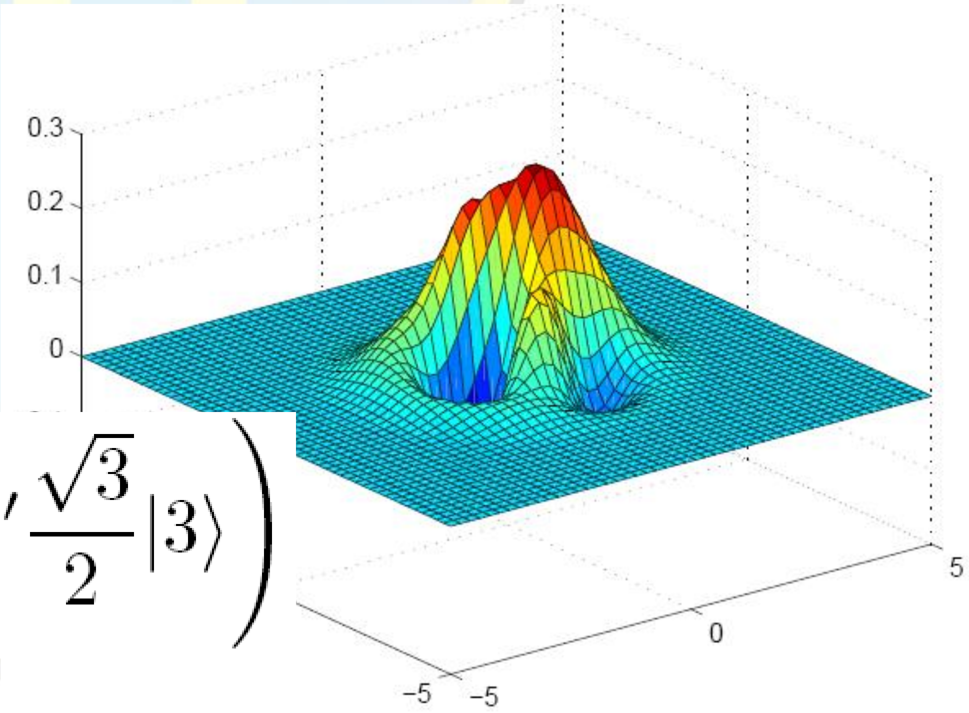
How do we get a cubic state?

- Not so easily – it has infinite energy, after all
- What about some approximation?

$$e^{i\chi\hat{x}^3}\hat{S}|0\rangle$$

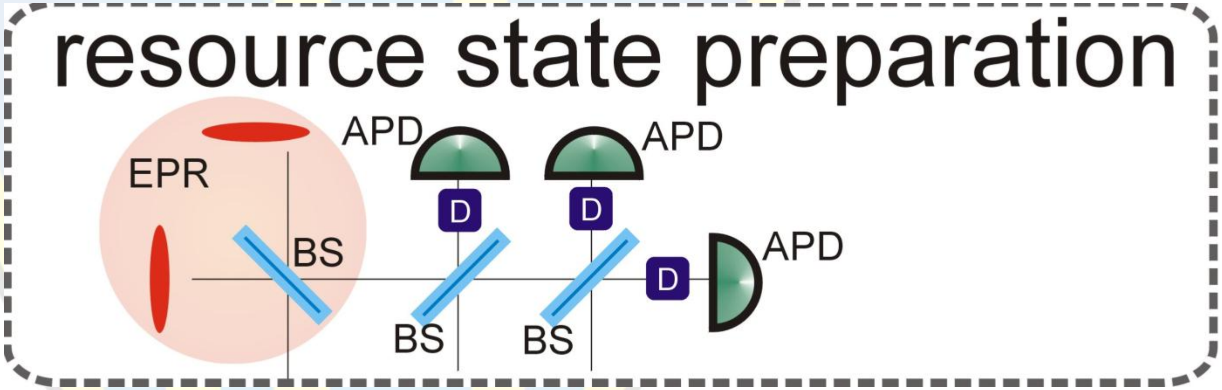
$$(1+i\chi\hat{x}^3)\hat{S}|0\rangle$$

$$\hat{S}\left(|0\rangle+\chi'\frac{3}{2\sqrt{2}}|1\rangle+\chi'\frac{\sqrt{3}}{2}|3\rangle\right)$$



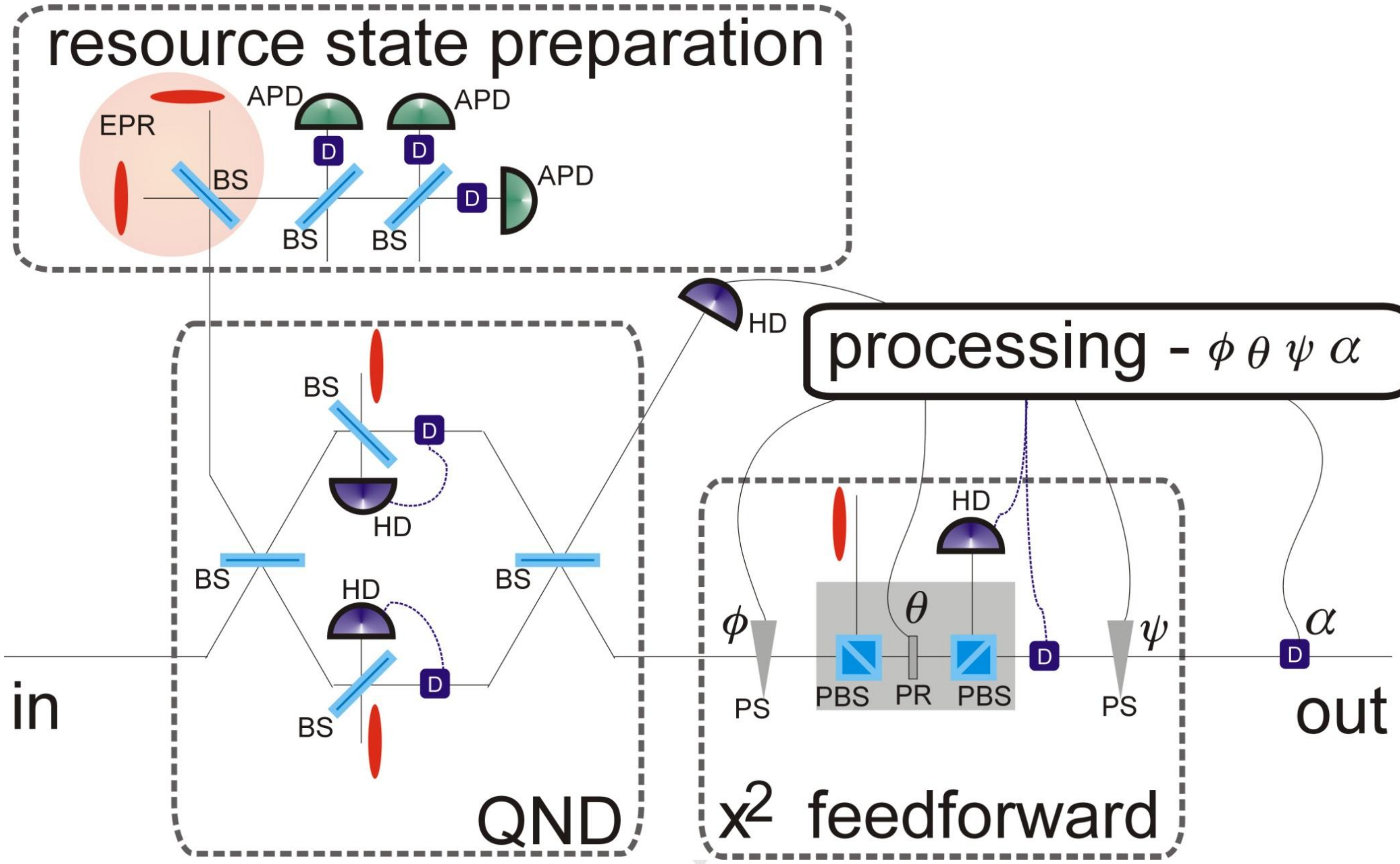
Single photons strike back!

- The state can be constructed at the single photon level!



$$\langle 0|(\hat{a}-\alpha)(\hat{a}-\beta)(\hat{a}-\gamma)\sum_{n=0}^{\infty}\lambda^n|n,n\rangle$$

Full Scheme



resource state preparation
EPR
APD
APD
APD
D
D
D
BS
BS
BS
HD
processing - $\phi\ \theta\ \psi\ \alpha$
BS
D
HD
BS
BS
HD
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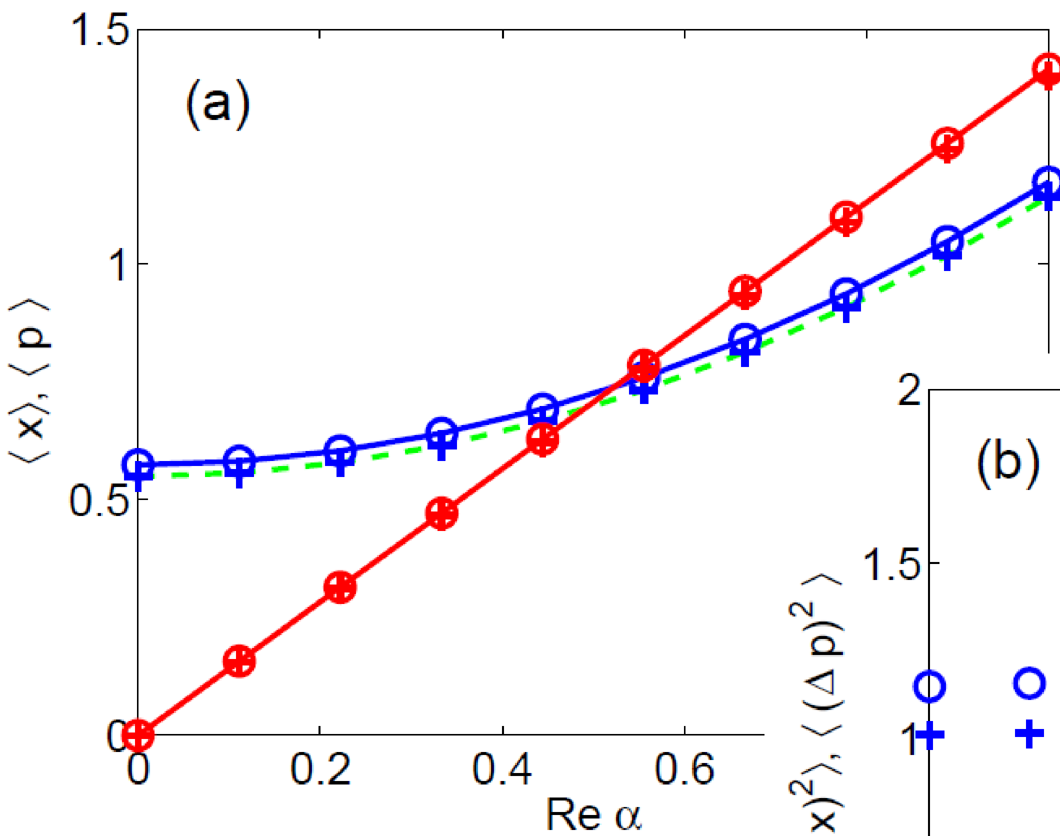
D
D
PS
PBS
PR
PBS
PS
out
x^2 feedforward

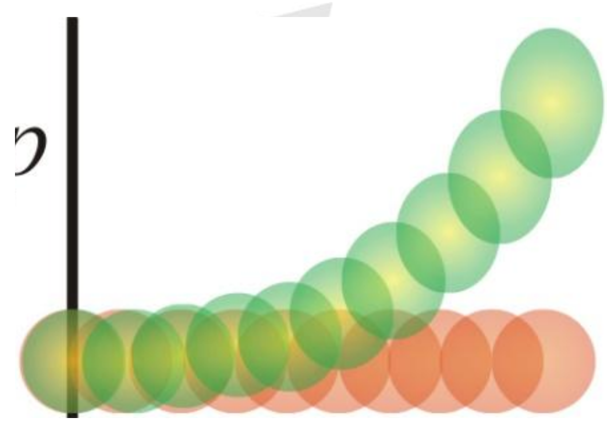
How good is the operation?

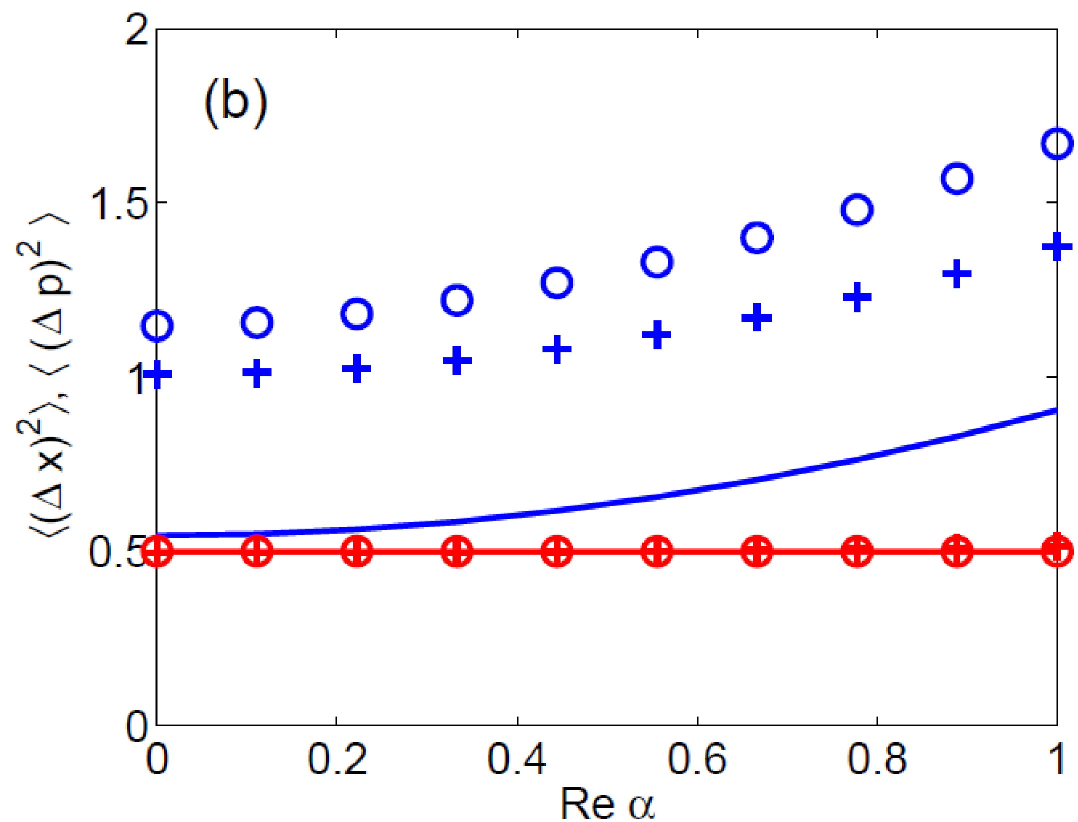
- High squeezing = many photons
- High nonlinearity = many photons
- We want both, but have only three photons
- We need to find middle ground
  - That means, some noise is unavoidable
  - How can we tell something nontrivial is going on?

$$\hat{H}_3 = \omega_3 \hat{x}^3 \quad \Rightarrow \quad \begin{aligned} \hat{x} &\rightarrow \hat{x} \\ \hat{p} &\rightarrow \hat{p} + \chi \hat{x}^2 \end{aligned}$$

Transforming coherent states







What have we learned:

- Amplification of coherent states:
  - Noise = power
    - It is random, incoherent, but there are ways of harnessing it
  - Interesting results can be obtained even with “discounted” resources
  - Don’t ask what you can do about the noise, ask what can the noise do for you

What have we learned:

- Deterministic cubic nonlinearity
  - Single photons can substitute macroscopic nonlinearities
  - We're not there yet, but even today's resources can show something non-trivial going on
  - It's not perfect.
  - But first steps never are.

Thank you for the attention!