

Nový pohled na slabé světlo

Radim Filip, Petr Marek, Vlad Usenko, Miroslav Gavenda,
Ladislav Mišta Jr., Lukáš Lachman



Department of Optics
Palacký University



MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



OP Vzdělávání
pro konkurenceschopnost

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Nový pohled na slabé světlo

nové aplikace žádají hlubší poznání

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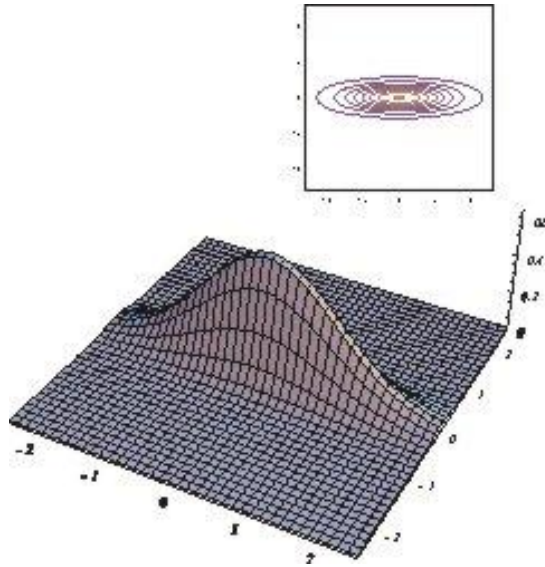
MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



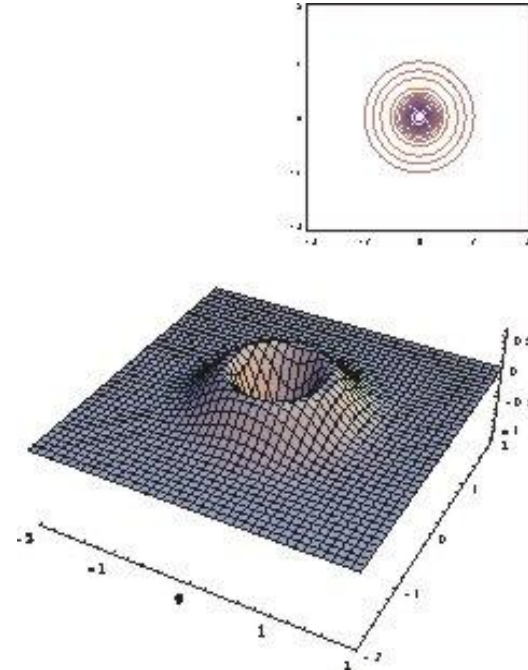
OP Vzdělávání
pro konkurenceschopnost

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

NONCLASSICAL QUANTUM STATES OF LIGHT:



<http://qis.ucalgary.ca/quantech/wiggallery.php>



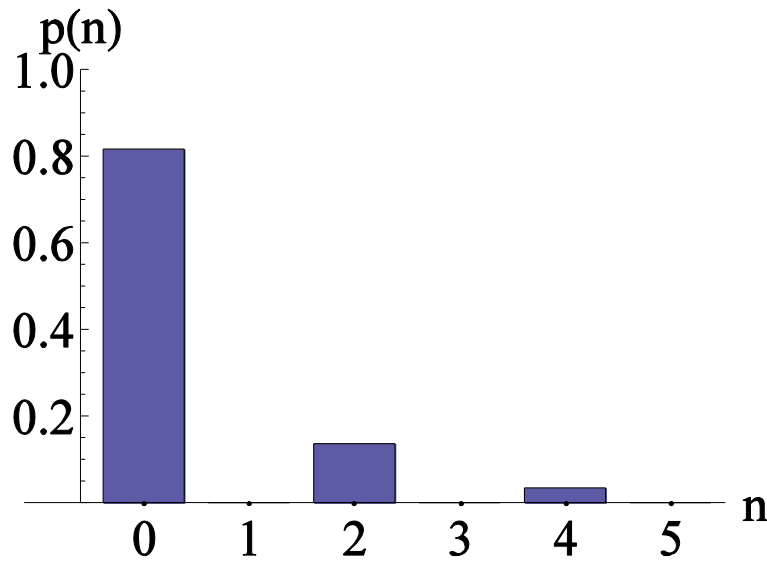
Gaussian squeezed state:

- positive Wigner function
- single quadrature variance below vacuum level

non-Gaussian Fock state:

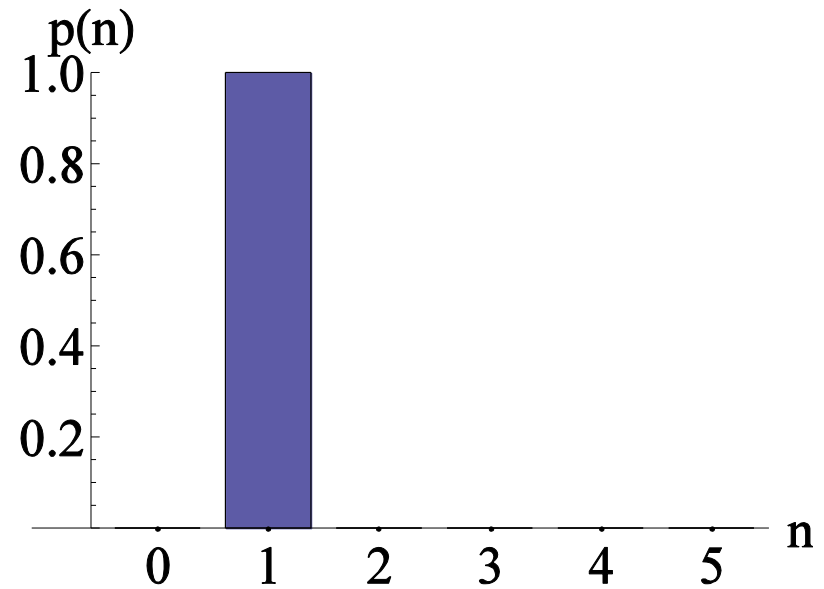
- negative Wigner function
- all quadrature variance above vacuum level

NONCLASSICAL QUANTUM STATES OF LIGHT:



Gaussian squeezed state:

- only even number of photons
- large uncertainty in photon number



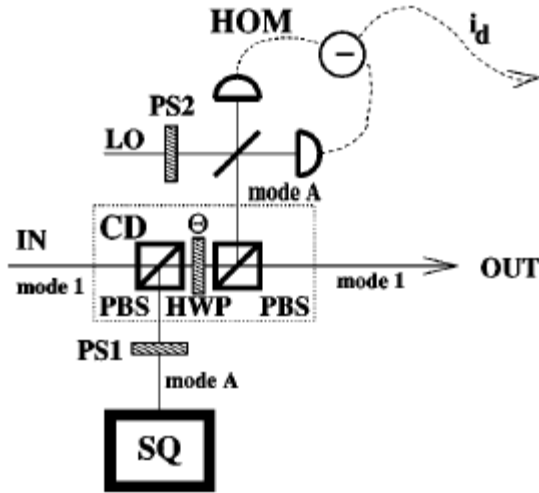
non-Gaussian Fock state:

- exact number of photons
- minimal uncertainty in photon number

SQUEEZING FOR WHAT ? APPLICATIONS ?



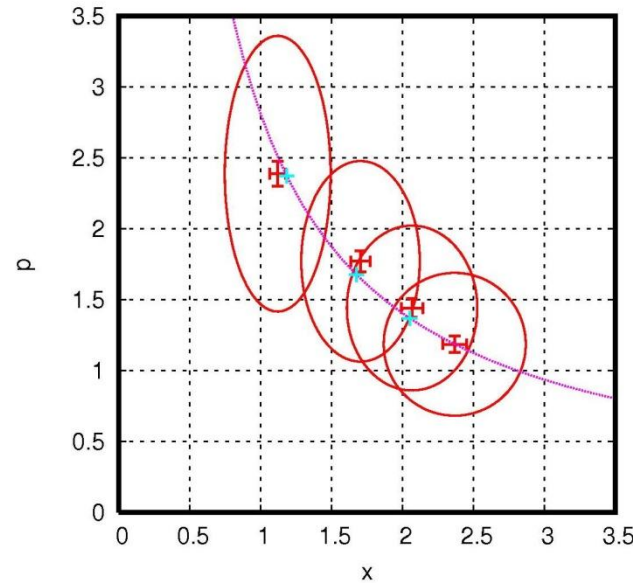
SQUEEZING = IRREDUCIBLE RESOURCE FOR GAUSSIAN SQUEEZER



Off-line squeezed state ->
On-line squeezer

$$X'_1 = \sqrt{T}X_1 + \sqrt{1-T}X_A,$$

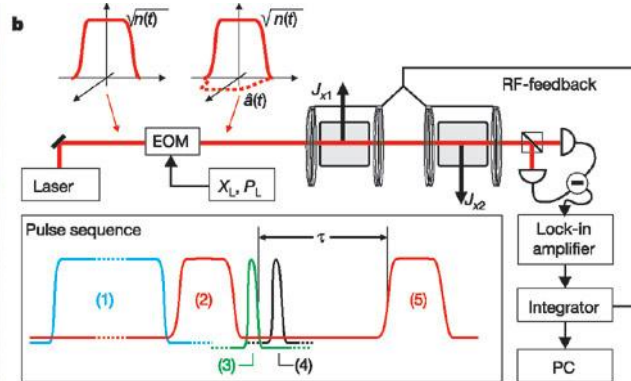
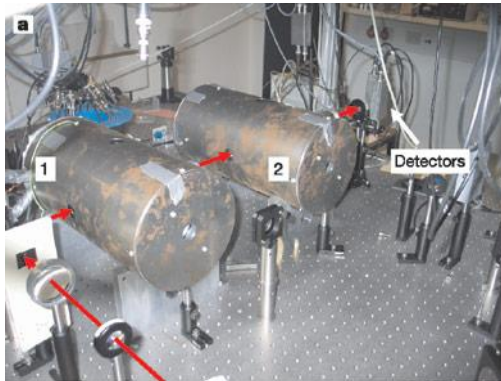
$$P'_1 = \frac{1}{\sqrt{T}}P_1 - \frac{\sqrt{(1-T)(1-\eta)}}{\sqrt{T\eta}}P_0,$$



R. Filip, P. Marek and U.L. Andersen,
Phys. Rev. A 71, 042308 (2005).

J. Yoshikawa et al., Phys. Rev. A 76,
060301(R) (2007)

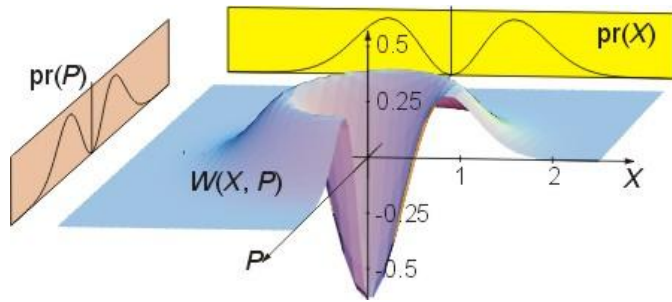
PRE-SQUEEZING FOR QUANTUM MEMORY



Copenhagen's memory
(2004)

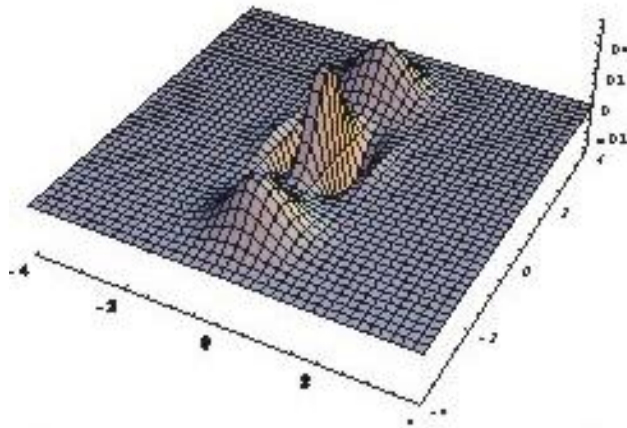
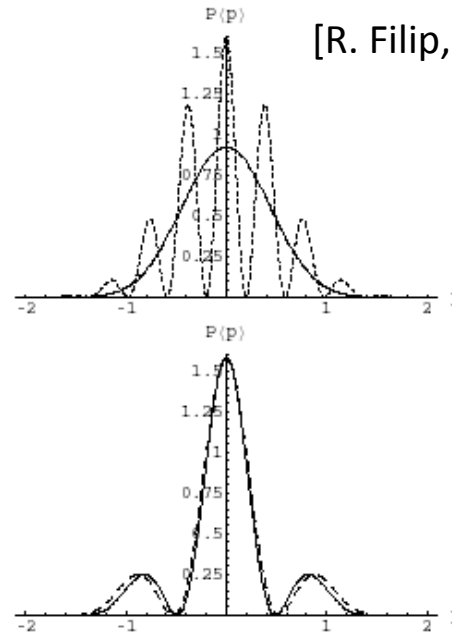
- single-mode pre-squeezing effectively **enhances** QND **interaction**, Radim Filip, Phys. Rev. A 78, 012329 (2008)
- upload can be **limited only by loss** (up to fixed squeezing).

UPLOAD OF $|1\rangle$ OR “CAT” STATE



$$\langle x = 0 |_L \left(1 + \frac{\kappa}{4} \tau (a_A + a_A^\dagger)(a_L^\dagger - a_L) \right) |1\rangle_L |0\rangle_A = -\frac{\kappa}{4} |1\rangle_A$$

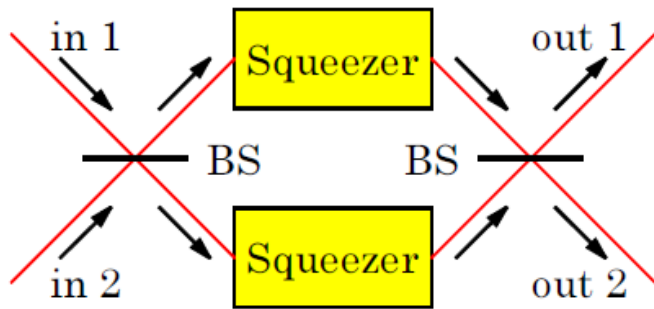
[R. Filip, PRA 2008]



- post-selection transforms **loss to reduction of amplitude**, but uploaded state remains pure!
- pre-squeezing **increases interference**.

QUANTUM GAUSSIAN OPERATIONS

Quantum operations
based on online squeezer:



S.L. Braunstein, Phys.Rev. A71,
055801 (2005).

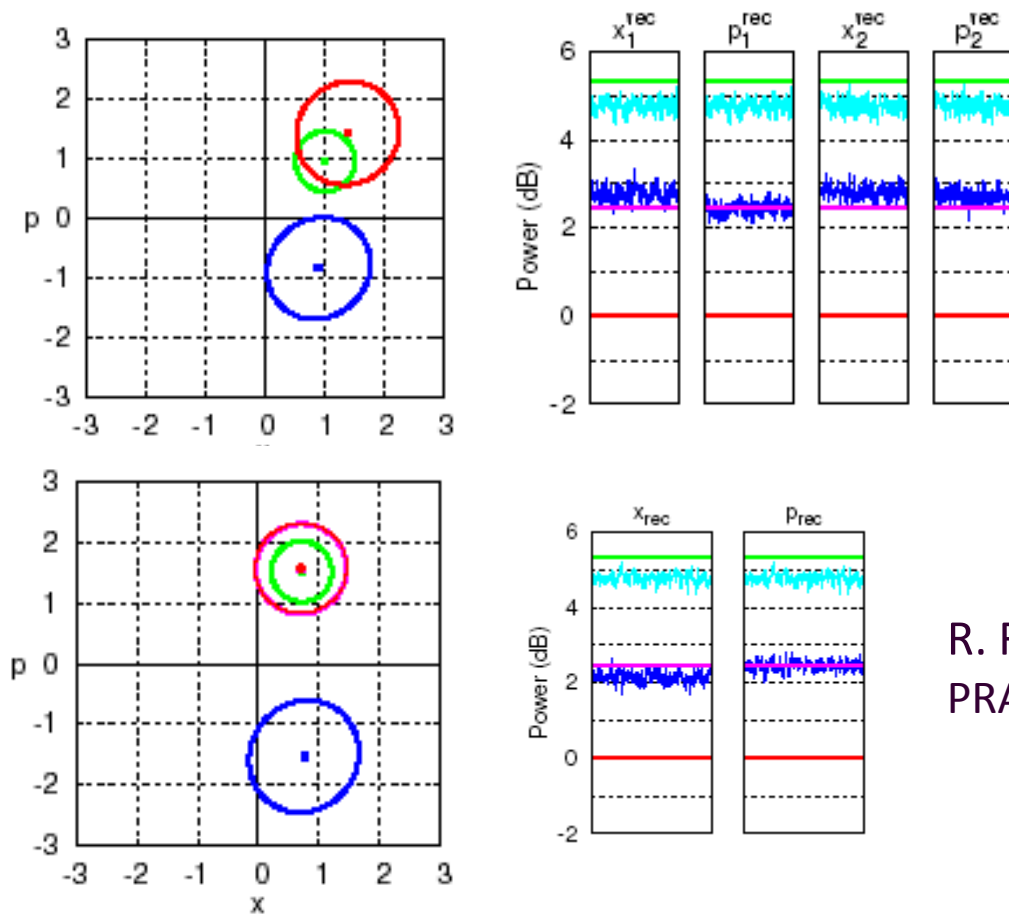


Scheme used for **QND interaction** and
Gaussian amplifier in Furusawa's lab.

J. Yoshikawa et al., Phys. Rev. Lett. 101, 250501
(2008).

J. Yoshikawa, Y. Miwa, R. Filip, A. Furusawa,
Phys. Rev. A 83, 052307 (2011).

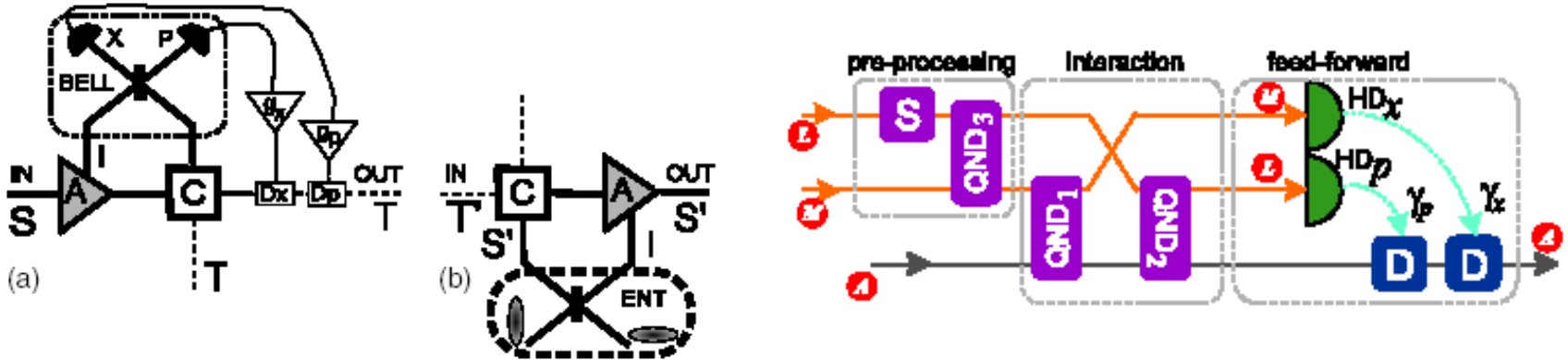
REVERSIBLE QUANTUM AMPLIFIER AND CLONER



R. Filip, J. Fiurasek , P. Marek,
PRA 69 , 012314 (2004).

J. Yoshikawa, Y. Miwa, R. Filip, A. Furusawa, *Demonstration of reversible phase-insensitive optical amplifier*, Phys. Rev. A 83, 052307 (2011)

APPLICATION: QUANTUM INTERFACES BETWEEN LIGHT AND MATTER



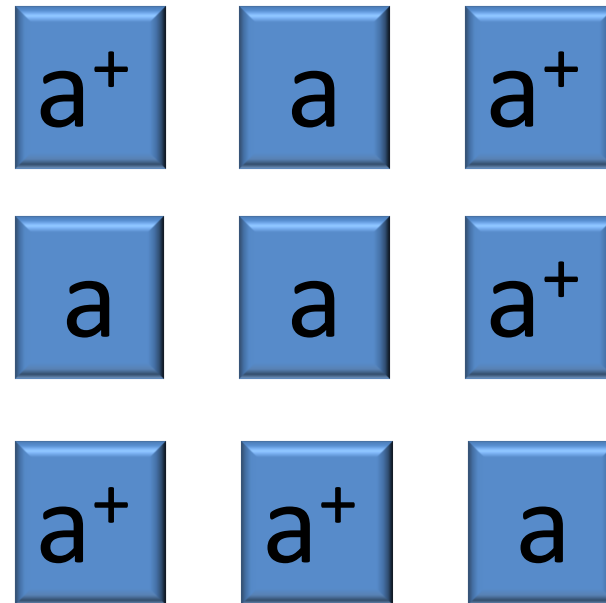
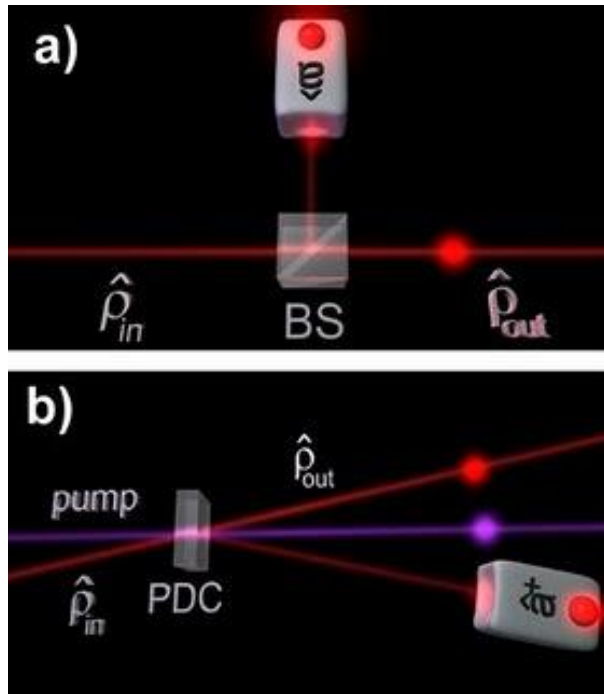
R. Filip, Phys. Rev. A 80, 022304 (2009); P. Marek and R. Filip, Phys. Rev. A 81, 042325 (2010).

- Quantum pre-processing and feed-forward control **perfectly transfer** any quantum state to noisy system through arbitrarily weak coupling.
- **Full quantum linear amplifier and QND interaction are useful tool for quantum pre-processing!**

|1> FOR WHAT ? APPLICATIONS ?



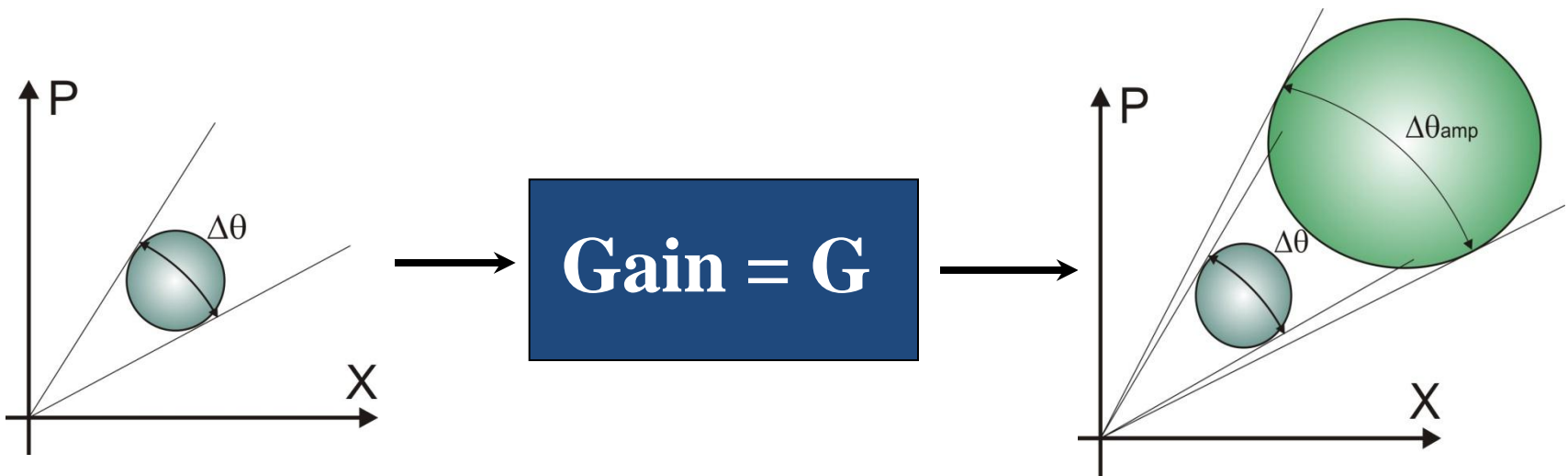
PHOTON SUBTRACTION & ADDITION = ANY PROBABILISTIC OPERATION



- U.L. Andersen, M. Bellini, A. Furusawa, P. Grangier, G. Leuchs, A. Lvovsky, E. Polzik, M. Sasaki, C. Silberhorn, NIST etc.

QUANTUM NOISE LIMITED AMPLIFIER

Linear (Gaussian) quantum amplifier

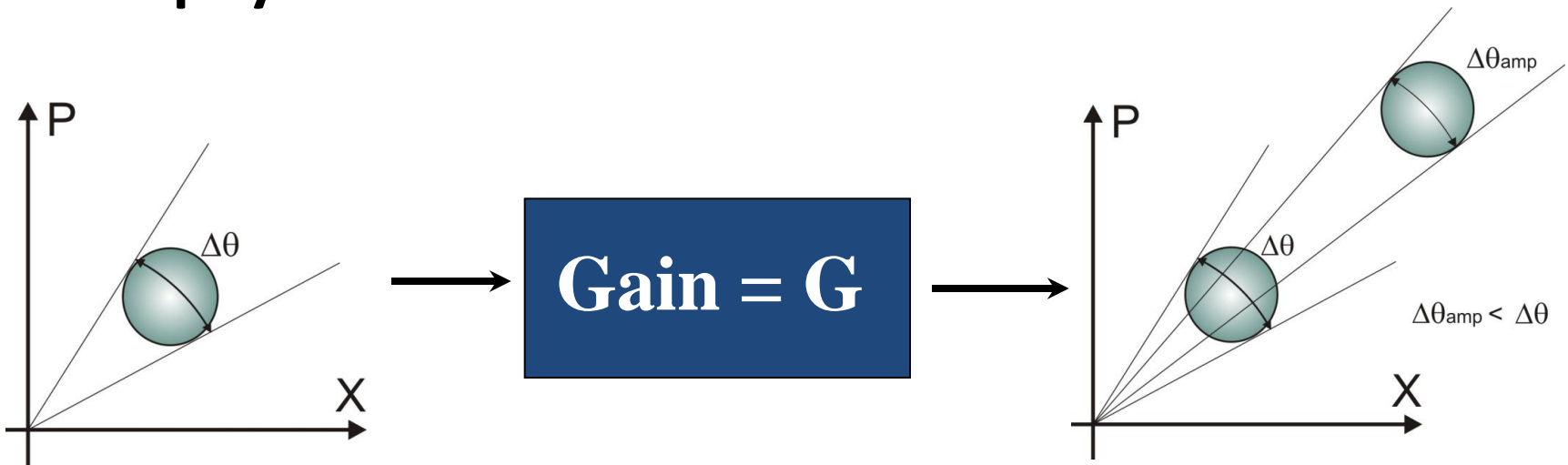


Input-output relation:

$$a_{out} = \sqrt{G}a_{in} + \sqrt{G-1}v^+$$

QUANTUM NOISELESS AMPLIFIER

Is it physical? Is it feasible?



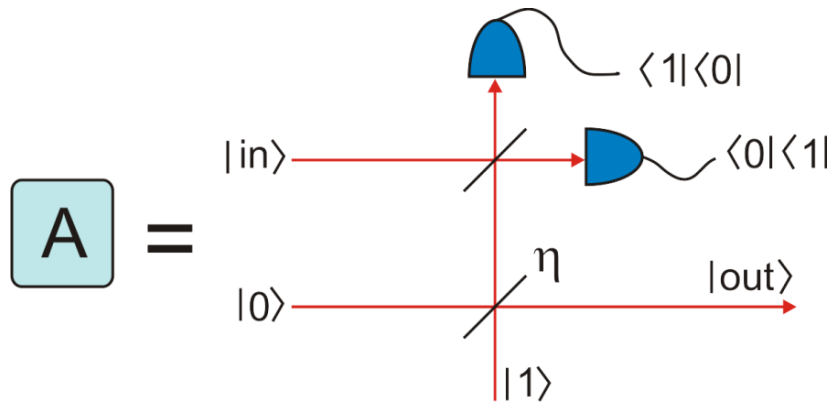
Input-output relation:

$$a_{out} = \sqrt{G}a_{in} + \sqrt{G-1}v^+$$



Who first asked this question?

NOISELESS AMPLIFIER BY QUANTUM SCISSORS

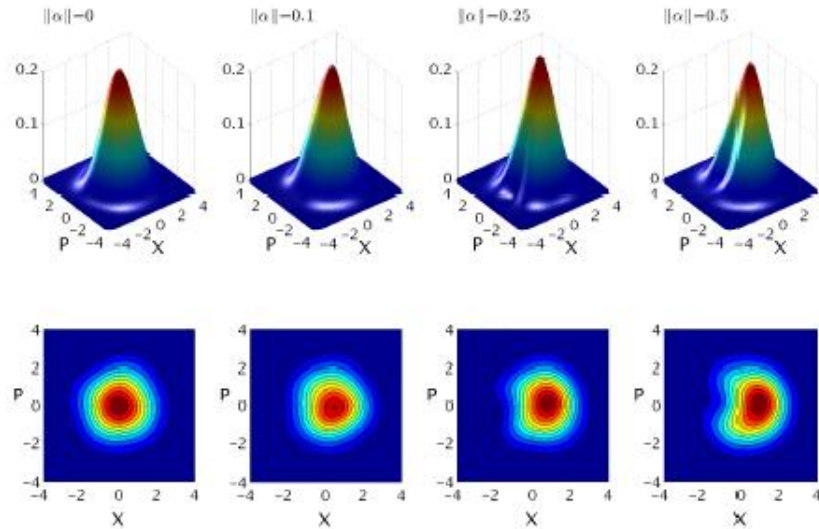


$$|\alpha'\rangle_{\alpha'} \rightarrow e^{-\frac{|\alpha'|^2}{2}} \sqrt{\frac{\eta}{2}} \left(1 \pm \sqrt{\frac{1-\eta}{\eta}} \hat{a}^\dagger \alpha'\right) |0\rangle$$

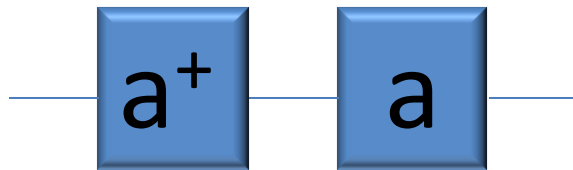
T.C. Ralph and A.P. Lund,
QCMC Proc. of 9th Int. Conf. 155
(2008)

F. Ferreyrol, M. Barbieri, R. Blandino, S. Fossier, R. Tualle-Brouri, and P. Grangier, *Phys. Rev. Lett.* 104, 123603 (2010).

G. Y. Xiang, T. C. Ralph, A. P. Lund, N. Walk and G. J. Pryde, *Nature Photonics* 4, 316 - 319 (2010);



NOISELESS AMPLIFIER BY aa^\dagger



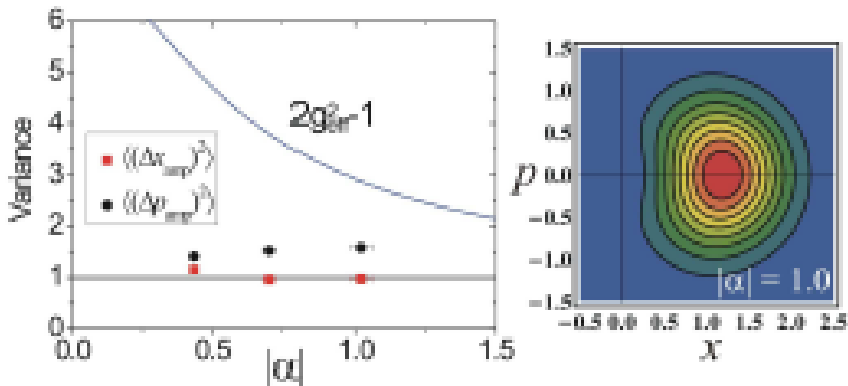
$$|\alpha\rangle = |0\rangle + \alpha |1\rangle + \dots$$

$$a^\dagger |\alpha\rangle = |1\rangle + 2^{1/2} \alpha |2\rangle + \dots$$

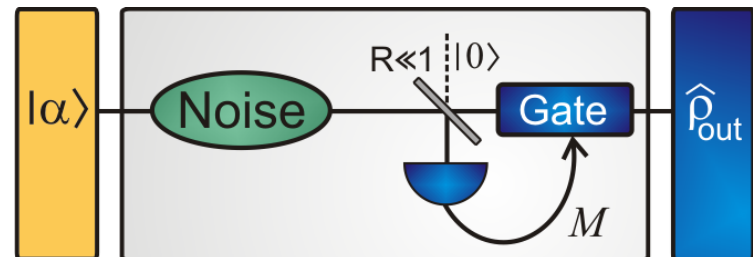
$$aa^\dagger |\alpha\rangle = |0\rangle + 2\alpha |1\rangle + \dots$$

P. Marek and R. Filip, Phys. Rev. A 81, 022302 (2010).

A. Zavatta, J. Fiurášek, M. Bellini,
Nature Phot. 5, 52 (2011)



M.A. Usuga, Ch. R. Müller, Ch. Wittmann, P.
Marek, R. Filip, Ch. Marquardt, G. Leuchs,
U.L. Andersen, Nature Phys. 6, 767–771
(2010)



? NOISE ADDITION ?



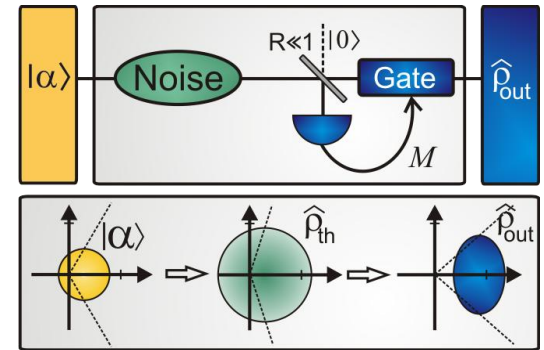
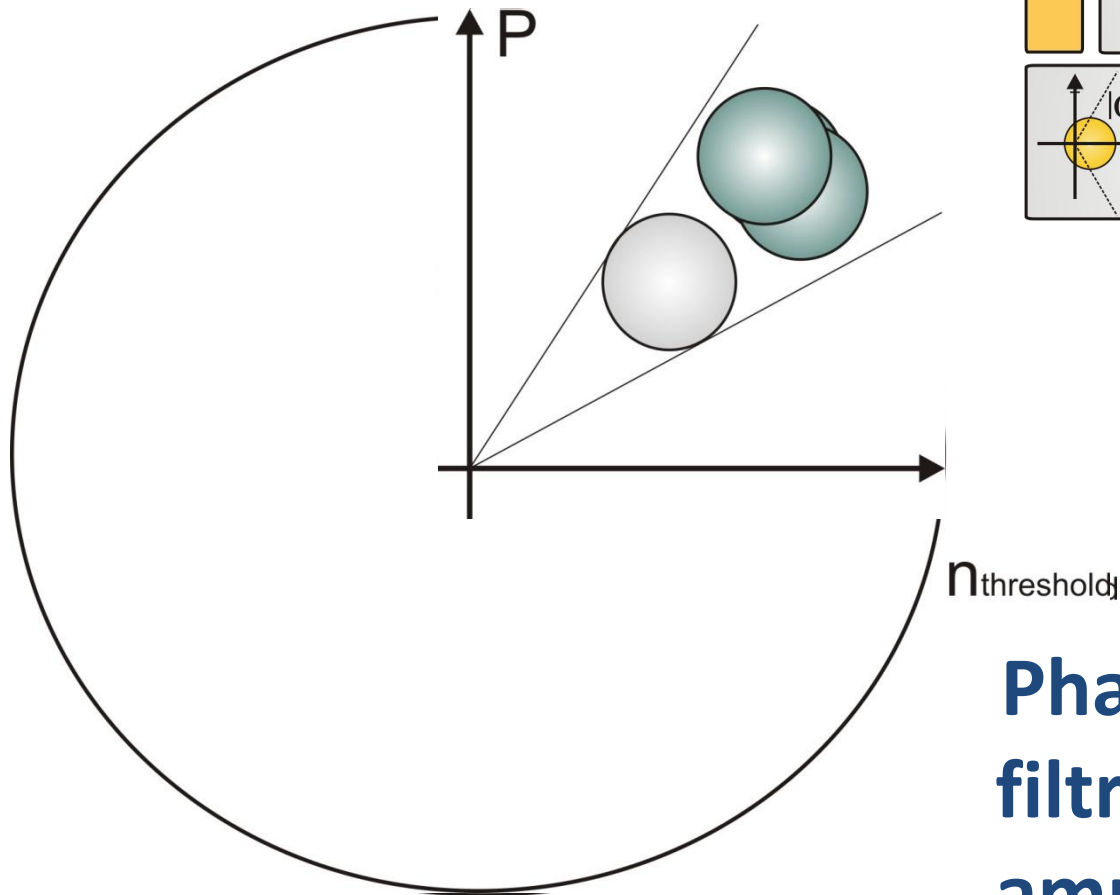
- $|\alpha\rangle = |0\rangle + \alpha|1\rangle + \dots$, $\rho \rightarrow a\rho a^\dagger + n_{\text{TH}} a a^\dagger \rho a a^\dagger \rightarrow$
 $[|\alpha|^2 |0\rangle\langle 0| + n_{\text{TH}} (|0\rangle + 2\alpha|1\rangle)(\langle 0| + 2\alpha^*\langle 1|)]/N$
 $N = |\alpha|^2 + n_{\text{TH}} + 4|\alpha|^2 n_{\text{TH}}$

- For low $|\alpha|^2 < 0.1$ and small n_{TH} :

$$V_C \rightarrow \mathbf{1/(4|\alpha|^2)} < 1/|\alpha|^2 \text{ (coherent state)}$$

Noise addition can contribute to reduction of phase variance instead single photon addition!

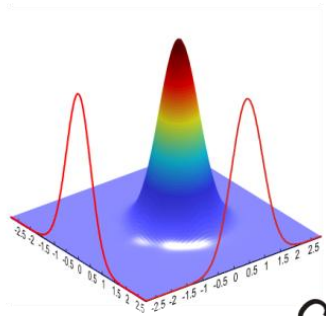
PRINCIPLE ?



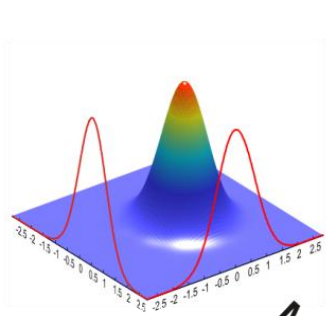
**Phase insensitive
filtration of high
amplitudes**

WIGNER FUNCTION PICTURES

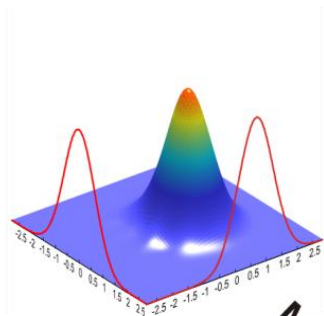
$$|\alpha_{\text{coh}}|^2 = 0.186$$



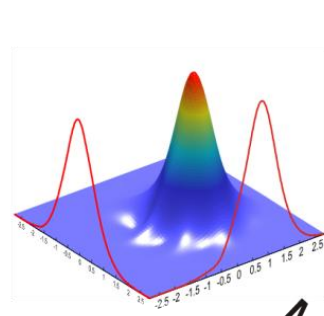
coherent
input



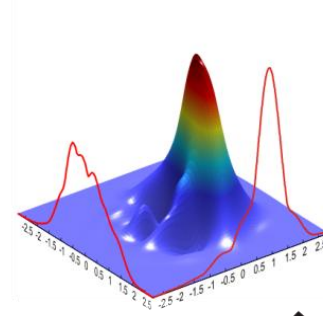
M=1



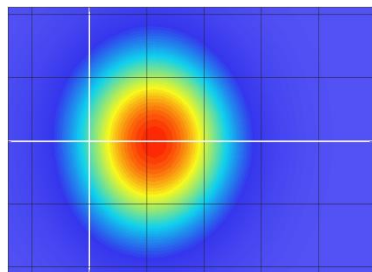
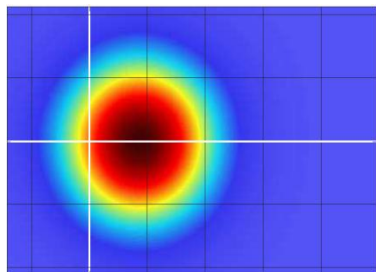
M=2



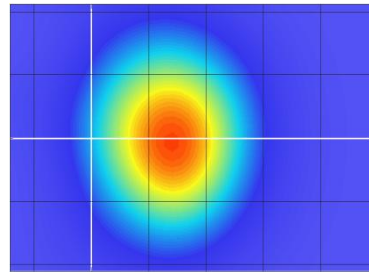
M=3



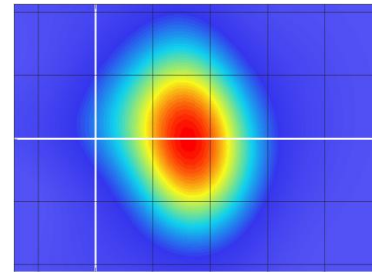
M=4



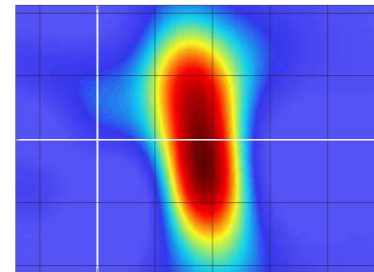
PS=0.044



PS=0.0015

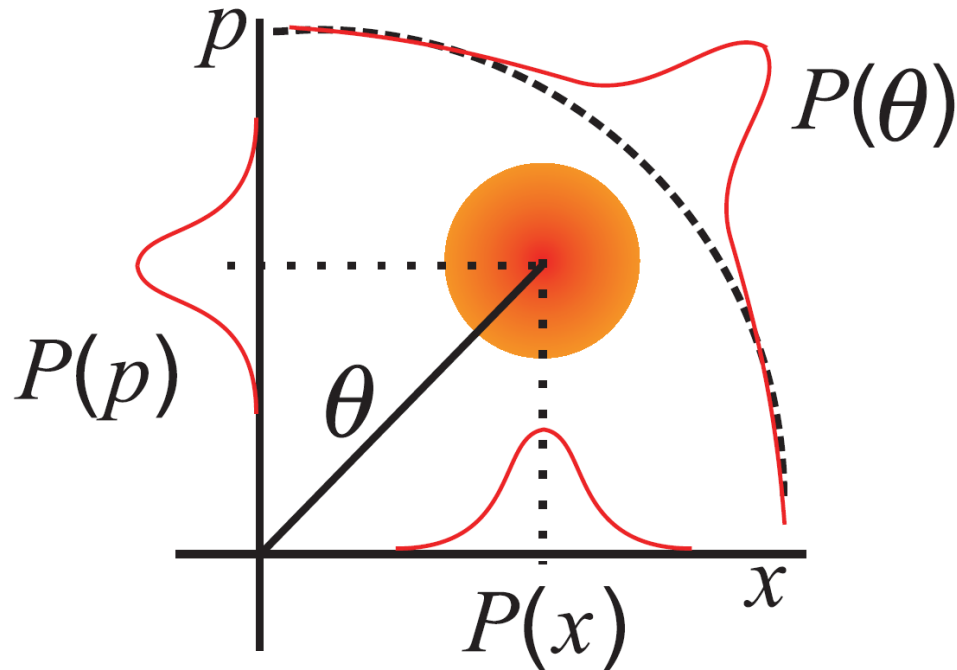


PS=4.3x10⁻⁵



PS=1.5x10⁻⁶

PHASE UNCERTAINTY MEASURE



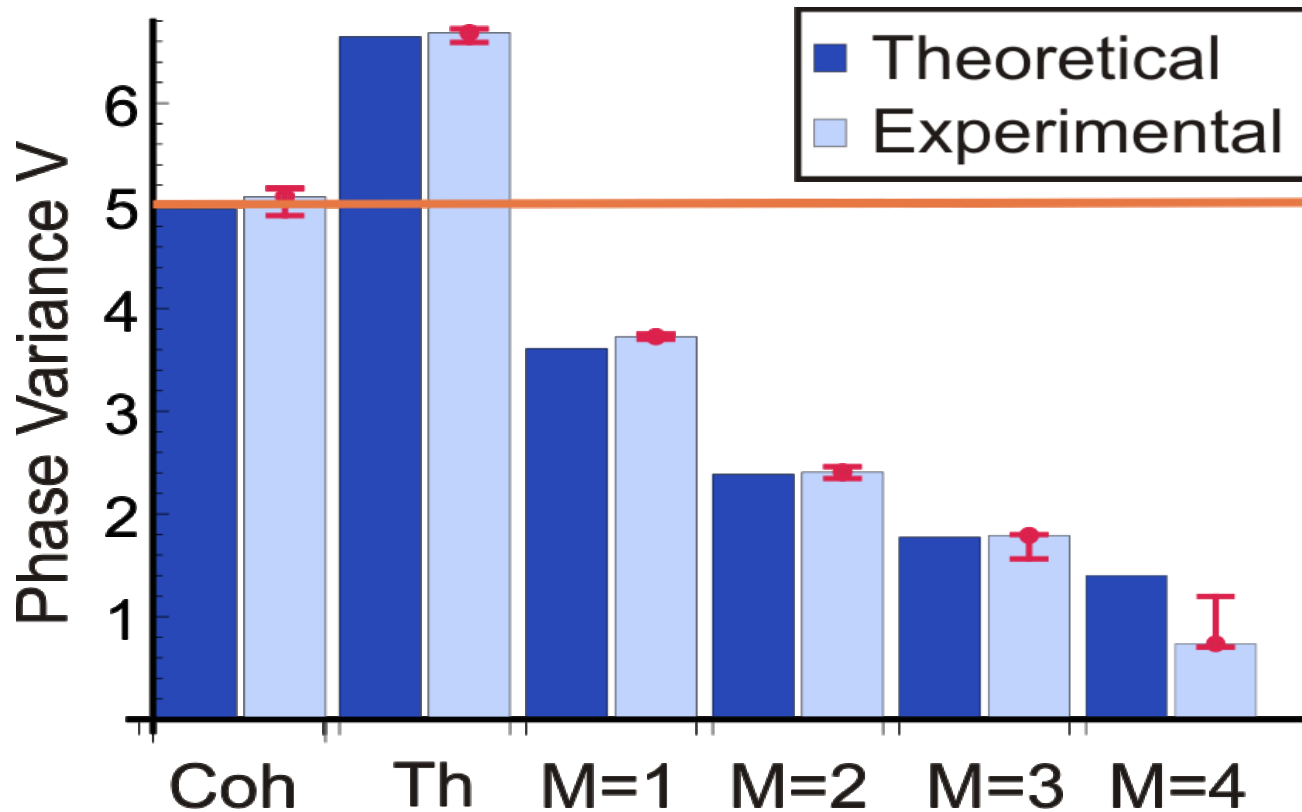
Phase uncertainty increases under standard Gaussian amplification.

- Phase distributions (optimal canonical measurement):

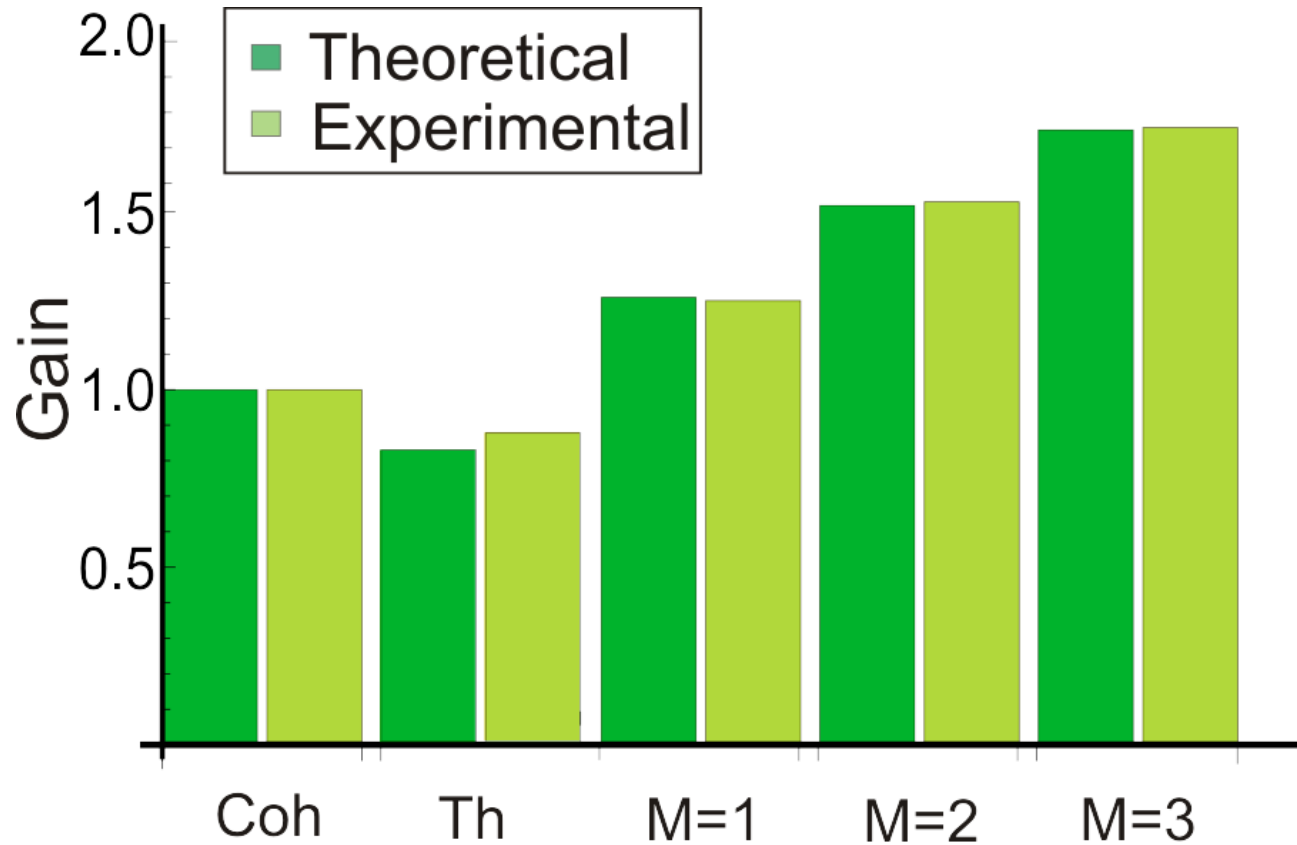
$$P(\theta) = \text{Tr}[F(\theta)\rho],$$

$$F(\theta) = \frac{1}{2\pi} \sum \exp(i\theta(m-n)) |m\rangle\langle n|$$
- Phase variance $V = |\mu|^{-2} - 1$, $\mu = \langle \exp(i\theta) \rangle$

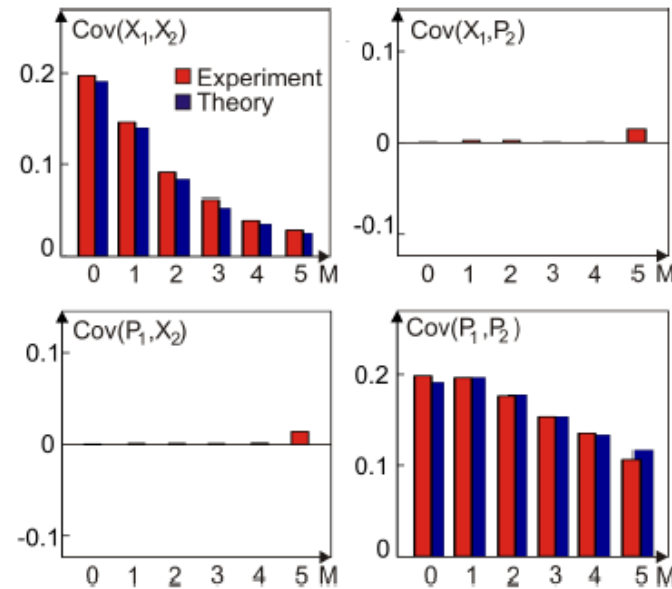
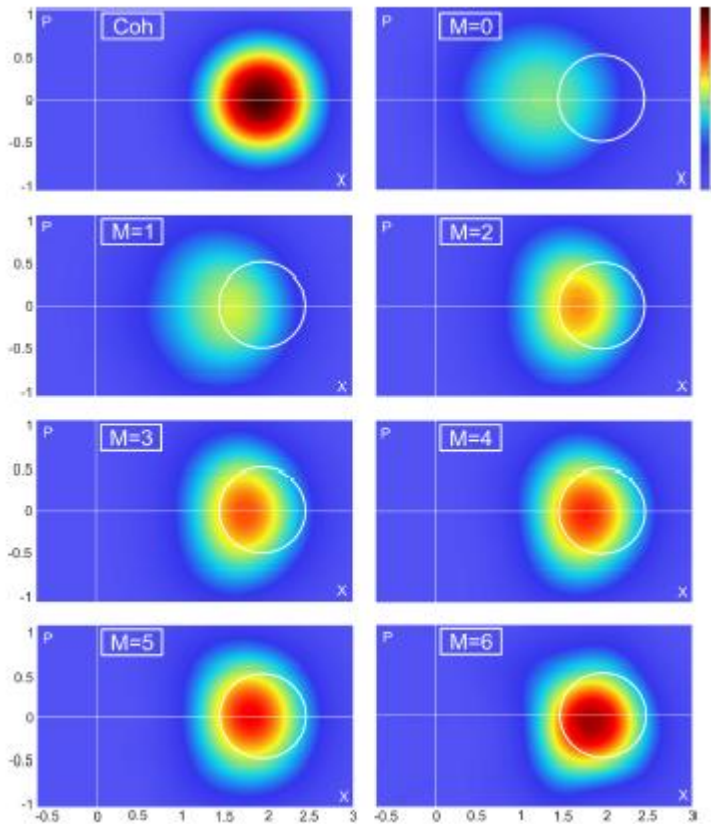
PHASE VARIANCE AFTER AMPLIFICATION



AMPLIFICATION GAIN



OVERAL PERFORMANCE IN CLONING OF COHERENT STATES

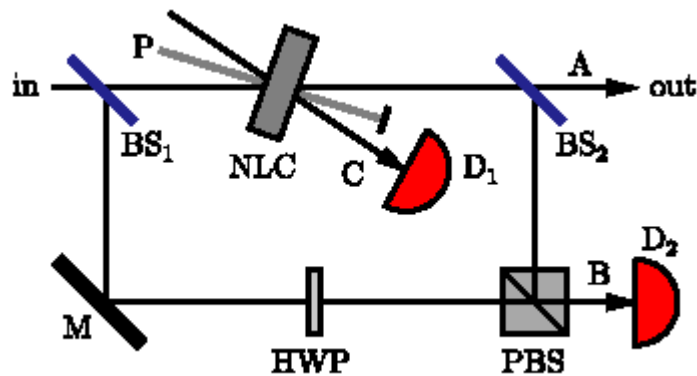


Ideal loss compensation by noiseless amplification T.C. Ralph, arXiv:1105.4309

Christian R. Müller, Christoffer Wittmann, Petr Marek, Radim Filip, Christoph Marquardt, Gerd Leuchs, Ulrik L. Andersen, quant-ph arXiv:1108.4241

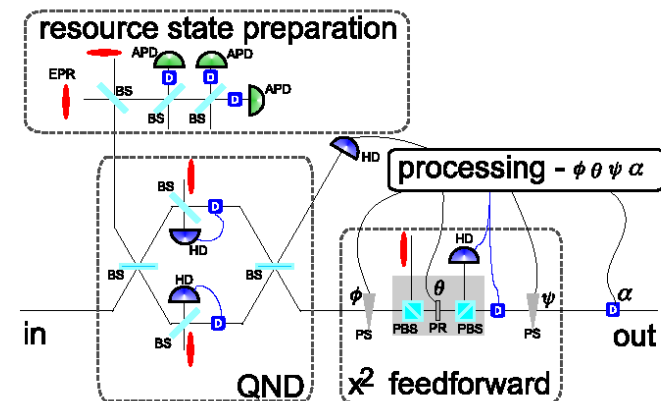
PROBABILISTIC AND DETERMINISTIC OPERATIONS BASED ON $|1\rangle$ etc.

Probabilistic
Kerr effect



J. Fiurášek, Phys. Rev. A 80, 053822 (2009).

Deterministic
weak X^3 nonlinearity



P. Marek, R. Filip, A. Furusawa, Phys. Rev. A **84**, 053802 (2011)

See Petr's Poster No.1

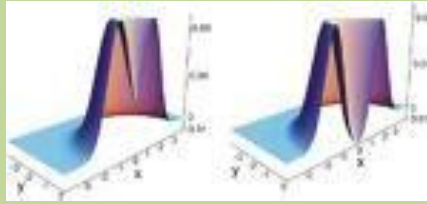
HOW GOOD “SINGLE PHOTON” IS REQUIRED FOR GIVEN TASK?



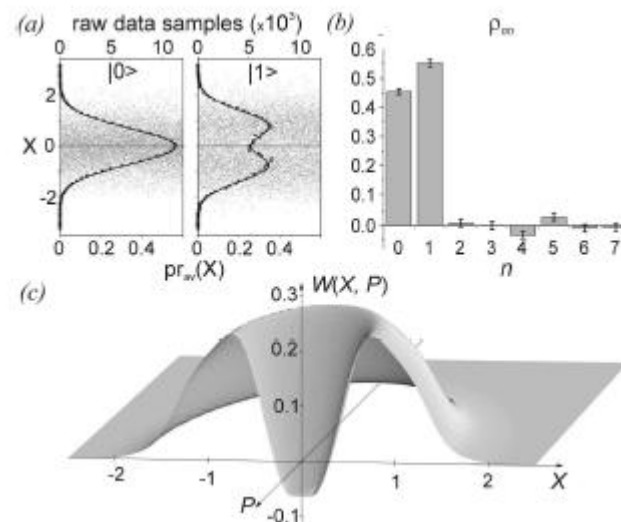
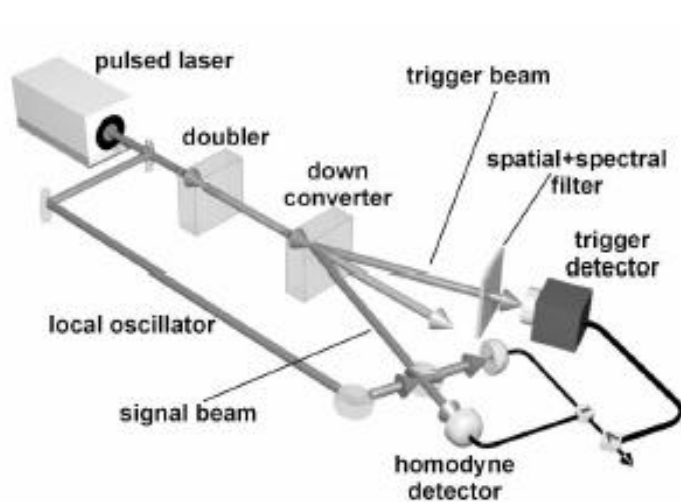
HOW ARE “SINGLE PHOTONS” CLASSIFIED ?



REALISTIC QUANTUM STATE OF SINGLE PHOTONS ?



ATTENUATED SINGLE PHOTON



A. Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)

$$\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$$

- Wigner function negative only for $\eta > 0.5$
- Non-classical state for any $\eta > 0$

PHOTON NONCLASSICALITY

- “Classical states” = **mixtures of coherent states**

Why?

They can be prepared in an oscillator driven by classical force (laser and modulators).

- **Criterion for $\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$:** charact. fce

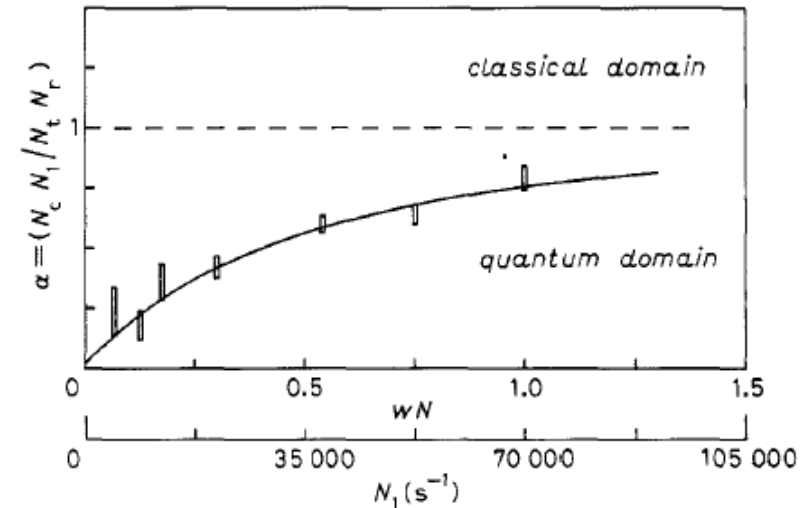
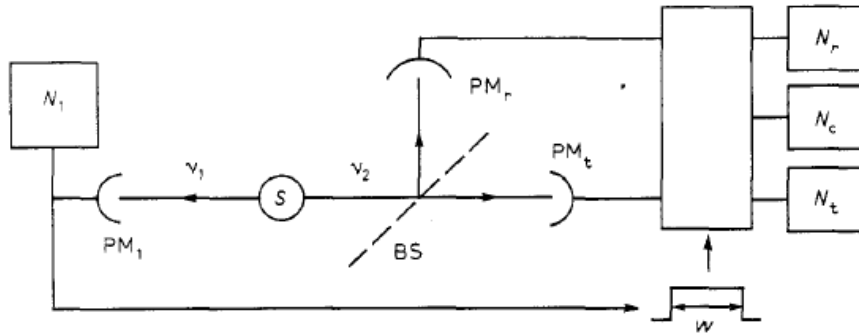
$$|\eta C_1(u) + (1-\eta) C_0(u)| > C_0(u)$$

for some u , therefore is non-classical for $\eta > 0$.

Th. Richter and W. Vogel, Phys. Rev. Lett. **89**, 283601 (2002)

- $C(u)$ can be estimated from homodyne detection

PHOTON ANTIBUNCHING



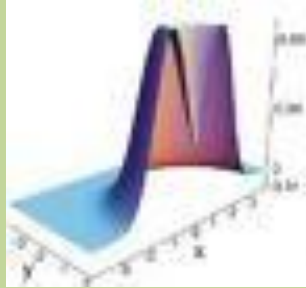
P. Grangier, G. Roger and A. Aspect,
Europhysics Lett. 1, 173 (1986)

photon becomes a particle: $\alpha = p_c / (p_r p_t)$, $\alpha < 1$

For $\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$: anti-correlation always!

- measurement insensitive to vacuum
- detectable using photon threshold detectors

**WHICH STATES WITH POSITIVE
WIGNER FUNCTION ARE NOT
MIXTURES OF GAUSSIAN STATES ?**



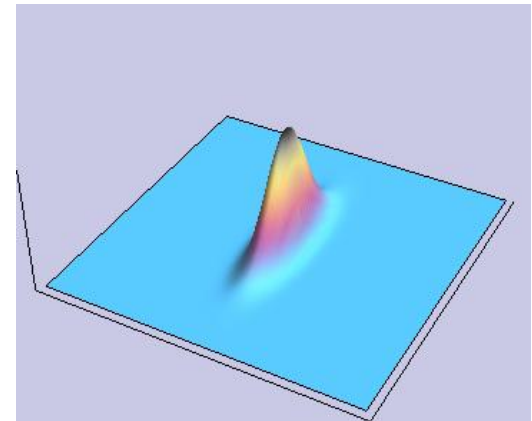
MIXTURES OF GAUSSIANS

$$\rho_c = \int \mathcal{P}(\lambda) |\lambda\rangle \langle \lambda| d\lambda, \quad |\lambda\rangle = S(r, \psi) D(\beta) |0\rangle$$

- **positive Wigner function**
- prepared by quantum

Hamiltonians up to second

order of a, a^\dagger (**quadratic Hamiltonians**) with arbitrary fluctuating coupling constants



HOW TO EXCLUDE THEM ???

PHOTON NUMBER FLUCTUATIONS

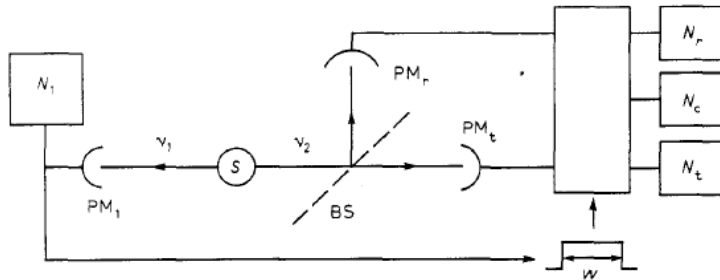
$$\langle \Delta^2 N \rangle = \langle N^2 \rangle - \langle N \rangle^2$$

Any value of $\langle \Delta^2 N \rangle$ can be reached by pure Gaussian states (displaced squeezed state) \rightarrow mixtures of Gaussians **cannot** be excluded (without additional energy constraint)

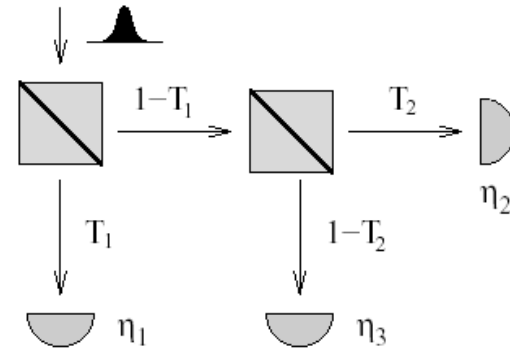
Similar for other measure based on $\langle \Delta^2 N \rangle$.

WHAT TO USE ???

INSPIRATION



P. Grangier, G. Roger and A. Aspect,
Europhysics Lett. 1, 173 (1986)



J. Řeháček, Z. Hradil, O. Haderka, J. Peřina, Jr.,
and M. Hamar, Phys. Rev. A **67**, 061801 (2003)

- Anti-correlation measurement is actually close to photon number “resolving” measurement.
- Instead of $\langle \Delta^2 N \rangle = \langle N^2 \rangle - \langle N \rangle^2$ we estimate $\mathbf{p(n)}$.
- Full information about $\mathbf{p(n)}$ is hardly accessible, but few first $\mathbf{p(0)}$, $\mathbf{p(1)}$, ... can be estimated.

P(n) FOR MIXTURE OF GAUSSIANS

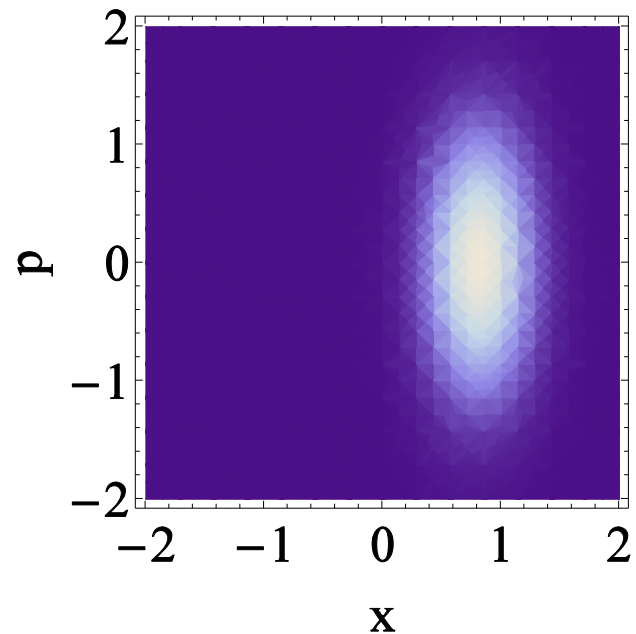
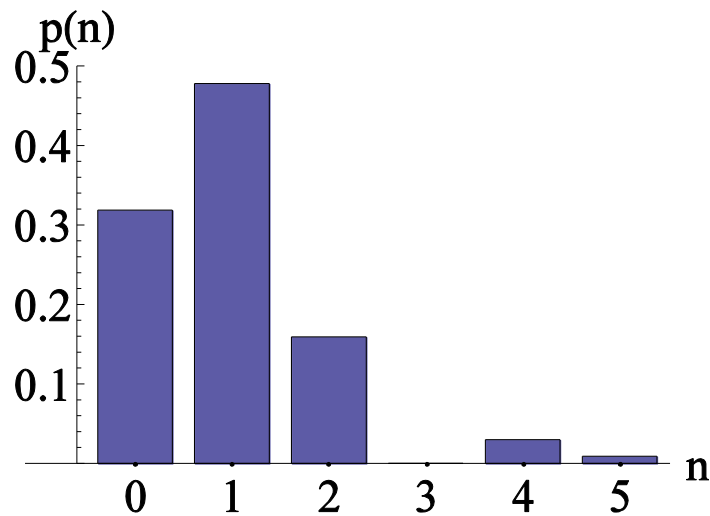
- linear: $p_n(\int \mathcal{P}(\lambda) |\lambda\rangle\langle\lambda| d\lambda) = \int \mathcal{P}(\lambda) p_n(|\lambda\rangle\langle\lambda|) d\lambda$
optimization over pure Gaussians

$$p_n(|\lambda\rangle\langle\lambda|) = \left| \frac{1}{\sqrt{n!\mu}} \left(\frac{\nu}{2\mu} \right)^{\frac{n}{2}} H_n \left(\frac{\beta}{\sqrt{2\mu\nu}} \right) \times \right. \\ \left. \exp \left(-\frac{|\beta|^2}{2} + \frac{\beta^2 \nu^*}{2\mu} \right) \right|^2,$$

$$\beta = |\beta| \exp(i\phi), \quad \mu = \cosh r, \quad \nu = \sinh r \exp(i\psi)$$

“GAUSSIAN” SINGLE PHOTON STATE

$$p(1) \leq 3\sqrt{3}/(4e) \approx 0.477889$$



$$V_x = 1/12, V_p = 3/4 \text{ and } \langle X \rangle = \sqrt{2}/\sqrt{3}$$

“GAUSSIAN” HIGHER PHOTON STATES

analytically for $n=2$, numerically for $n>2$

In general:

$$\theta = 0$$
$$|\beta|^2 = e^{2r} [n - \sinh^2(r)]$$

TABLE I: Maximal probabilities p_n^{max} .

n	p_n^{max}	$ \beta_{max} ^2$	r_{max}	n	p_n^{max}	$ \beta_{max} ^2$	r_{max}
1	0.4779	2	0.5493	5	0.2792	21.1232	0.8088
2	0.3813	5.5981	0.6584	6	0.2623	27.3948	0.8391
3	0.3326	10.1188	0.7243	7	0.2488	34.0822	0.8647
4	0.3014	15.3351	0.7717	8	0.2376	41.1522	0.8868

PHOTON STATES AND LOSS

- $|1\rangle$ passes a transmission η :
Wigner function positive for $p > 0.5$
not mixture of Gaussians for $p > 0.477889$
- $|2\rangle$ passes a transmission η :
Wigner function positive for $p > 0.5$
not mixture of Gaussians for $p > 0.394855$

Higher $|n\rangle$ are more preserving higher order non-classicality incompatible with the mixture of Gaussians.

HOW TO MAKE THE CRITERION FOR SINGLE PHOTON STRONGER ?

$$\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$$



WITNESS WITH NOISE SUBTRACTION

$$\Pi_1 = |1\rangle\langle 1| - \sum_{n=2} |n\rangle\langle n|$$

If $P \equiv \text{Tr}(\Pi\rho) > P^{max}$ then ρ is not the mixture of Gaussian states.

$\Pi_1 : P_1^{max} = 0.349409$, stronger than the previous criterion!

MORE ???

FINAL FORM OF WITNESS

$$\Pi_a = |1\rangle\langle 1| - a \sum_{n=2} |n\rangle\langle n| \quad a > -1$$

If $P \equiv \text{Tr}(\Pi\rho) > P^{max}$ then ρ **is not the mixture of Gaussian states.**

$$P_a^{max} = \frac{\sqrt{3 + 4a(3 + 2a)} + \sqrt{9 + 8a} \exp\left(\frac{2}{1 - \sqrt{9 + 8a}}\right)}{8\sqrt{2}(1 + a)} \\ \times (3 + 4a + \sqrt{9 + 8a}) - a$$

TEST STATE $\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$

- **Any state with $\eta > 0$ is not the mixture of Gaussians.** Follows from

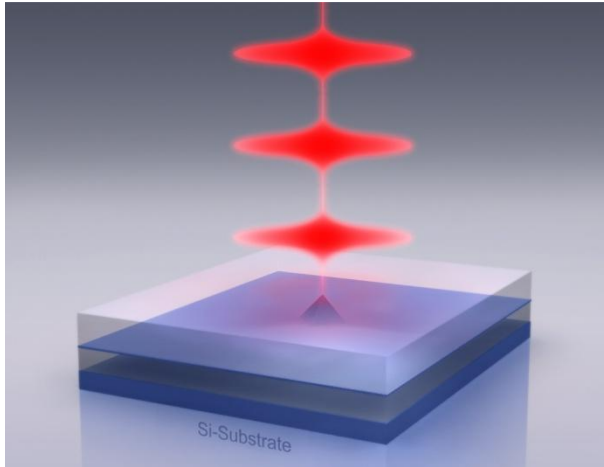
$$P_a = \eta$$

$$\lim_{a \rightarrow \infty} P_a^{max} = 0.$$

- **We detect states with a positive Wigner function which are not mixtures of the Gaussian states.**



MORE REALISTIC STATE



**First single photon source on silicon substrate
(indistinguishable photons generation)**

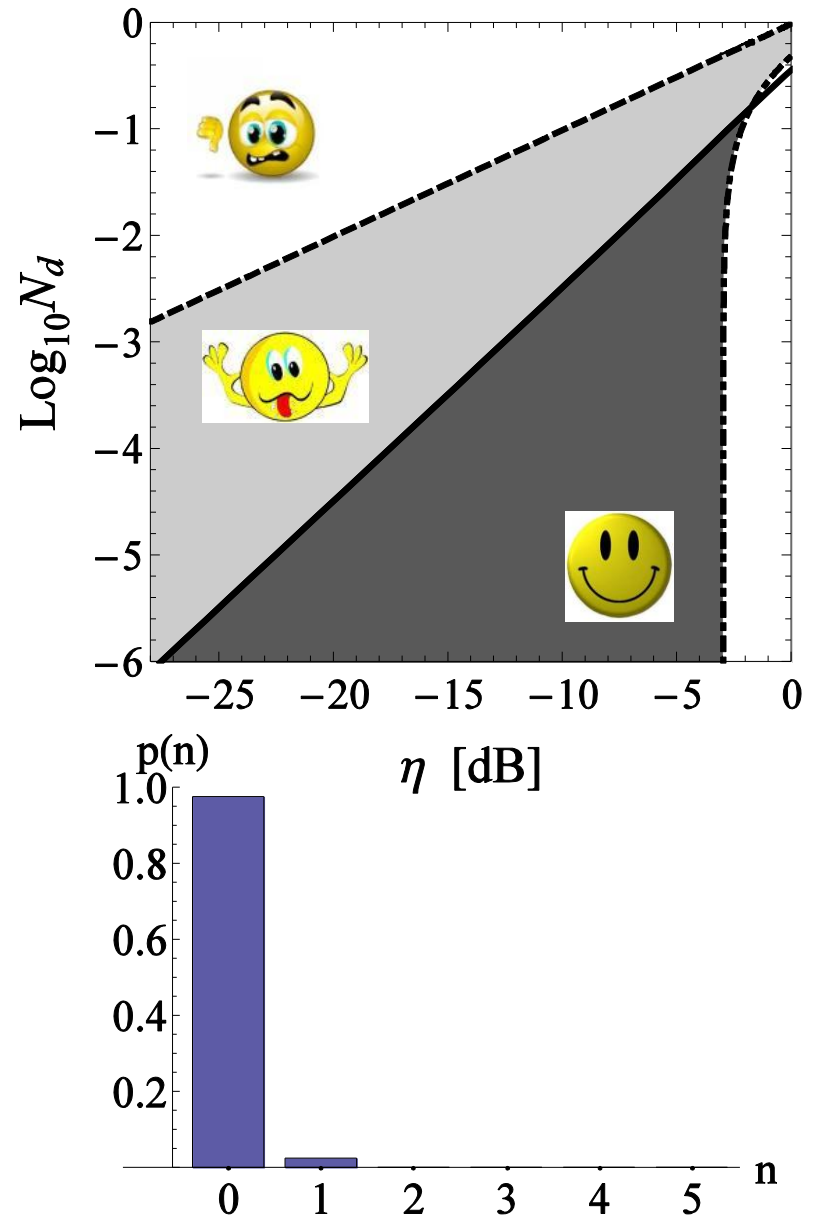
M. Benyoucef et al., Nano Letters 9, 304 (2009)

Model: single photon from our source propagates with probability of η to the detector, which is also receiving a background noise radiation.

$$p_0 = \frac{1 - \eta + N_d}{(1 + N_d)^2}$$

$$p_1 = \frac{\eta + N_d(1 - \eta + N_d)}{(1 + N_d)^3}$$

- witness tolerates quite high loss of photon
- We detect quite region of realistic single photon states.



HOW MUCH IS REQUIRED?

$$\Pi_a = |1\rangle\langle 1| - a \sum_{n=2} |n\rangle\langle n|$$

- Rewriting $\sum_{n=2} |n\rangle\langle n|$ as $1 - |0\rangle\langle 0| - |1\rangle\langle 1|$, **only $p(0)$, $p(1)$ are required**. They are achievable from the anti-correlation measurement interpreted as the photon number "resolving" detector.
- **equivalently, $p(1) + ap(0)$** can be used to construct the witness (optimal linear witness).

MULTIMODE SINGLE PHOTON?

- Thanks to linearity, we have to find maximum only **over multimode pure Gaussian states**.
- Using Bloch-Messiah decomposition, we can decompose it to a set of factorizable pure Gaussian states.
- Two modes A and B: $P_a = \text{Tr} \Pi_a \rho_{AB}$

$$\Pi_a = \Pi(1) - a(1 - \Pi(0) - \Pi(1))$$

$$\Pi(0) = |00\rangle\langle 00| \quad \Pi(1) = |10\rangle\langle 01| + |01\rangle\langle 10|$$

- We have to find a maximum of

$$P_a = (1 + a)(p_1^A p_0^B + p_1^B p_0^A) + a p_0^A p_0^B - a.$$

$$p_0^i = \exp(-\bar{n}_i(\cos 2\psi_i \tanh r_i - 1)) \operatorname{sech} r_i$$

$$p_1^i = \bar{n}_i \exp(-\bar{n}_i(\cos 2\psi_i \tanh r_i - 1)) \operatorname{sech}^3 r_i$$

- Optimization over ψ_1, ψ_2 & r_1, r_2 & \bar{n}_1, \bar{n}_2
- For $a > -1$, maximum is for $\bar{n}_2 = 0$ and $r_2 = 0$, therefore the single mode case is optimal.
- Extension to multimode case.

CRITERIUM FOR MORE REALISTIC MEASUREMENT

Instead estimation of $p(0)$ and $p(1)$ we consider the detector set-up.

Detector: 50:50 beam splitter

1- $|0\rangle\langle 0|$ threshold detectors ($\eta=1$)

$$\tilde{\Pi}_a = \tilde{p}_1 - a(1 - \tilde{p}_{00} - \tilde{p}_1)$$

$$\tilde{p}_{00} = \text{Tr} \rho U_{BS} |0\rangle_A \langle 0| \otimes |0\rangle_B \langle 0| U_{BS}^\dagger$$

$$\tilde{p}_1 =$$

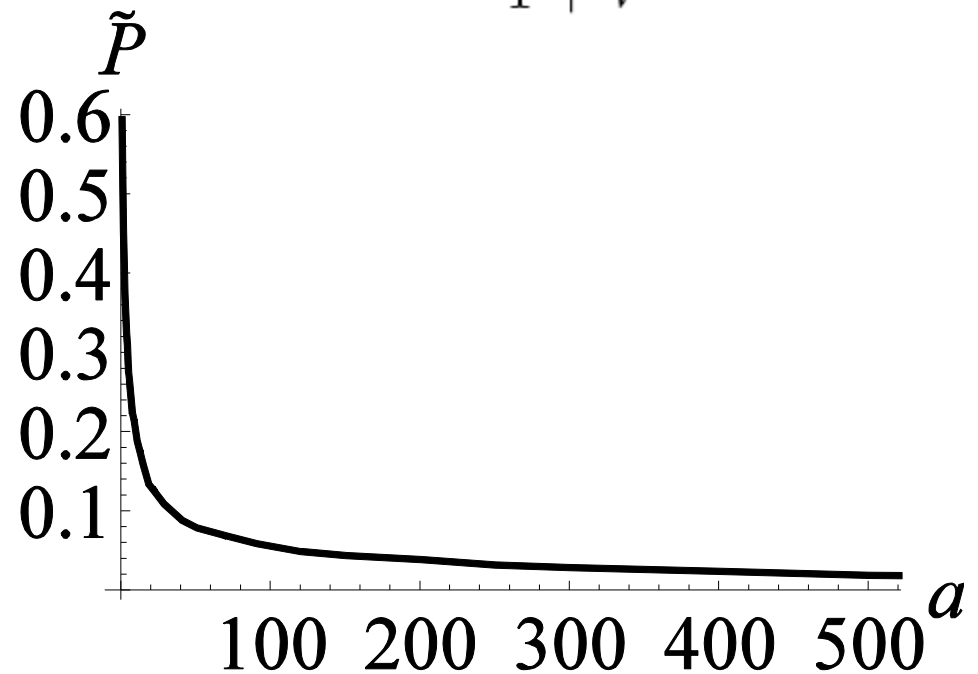
$$\text{Tr} \rho U_{BS} ((1 - |0\rangle_A \langle 0|) \otimes |0\rangle_B \langle 0| + (1 - |0\rangle_B \langle 0|) \otimes |0\rangle_A \langle 0|) U_{BS}^\dagger$$

$$\tilde{P}(a) = \text{Tr} \rho \tilde{\Pi}_a = 2(\tilde{p}_0 - \tilde{p}_{00}) - a(1 - \tilde{p}_{00} - 2(\tilde{p}_0 - \tilde{p}_{00}))$$

Pure Gaussian: $\tilde{p}_0 = 4\sqrt{V} \frac{\exp\left(-\frac{1+V(6+V)+(1-V^2)\cos 2\psi}{4(3+V)(1+3V)}|\alpha|^2\right)}{\sqrt{(3+V)(1+3V)}}$,

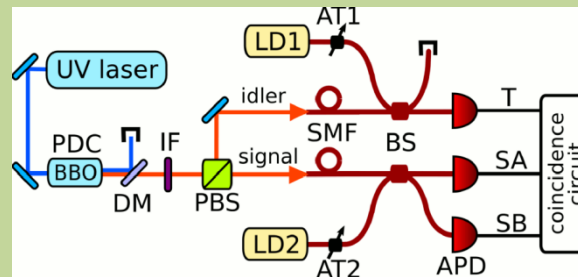
$$\tilde{p}_{00} = 2\sqrt{V} \frac{\exp\left(-\frac{(1+V)+(1-V)\cos 2\psi}{4(1+V)}|\alpha|^2\right)}{1+V}$$

Num. optimization:



- monotonously decreasing function \rightarrow
 $\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$ is always detected as incompatible with a mixture of Gaussians.
- similarly strong as the original witness for strongly attenuated states.
- true photon number resolving detector is not required.
- can be extended: known properties of detector can be taken into account.
- alternative to estimation of $p(0)$ and $p(1)$ for restricted density matrix.

EXPERIMENTS WITH SINGLE PHOTON SOURCES ?



OLOMOUC'S EXPERIMENT:

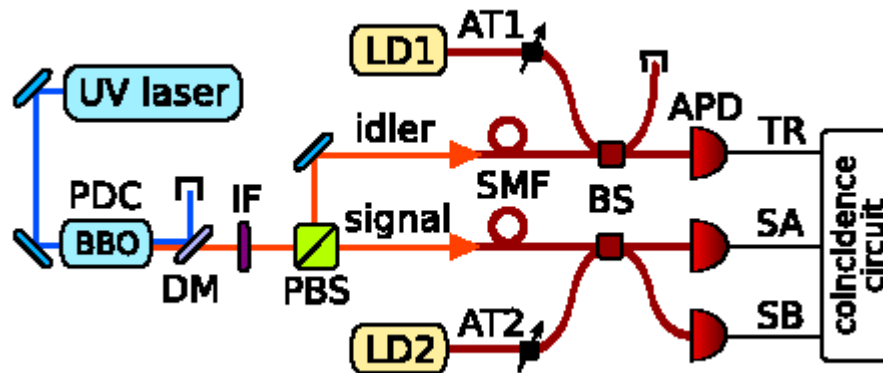
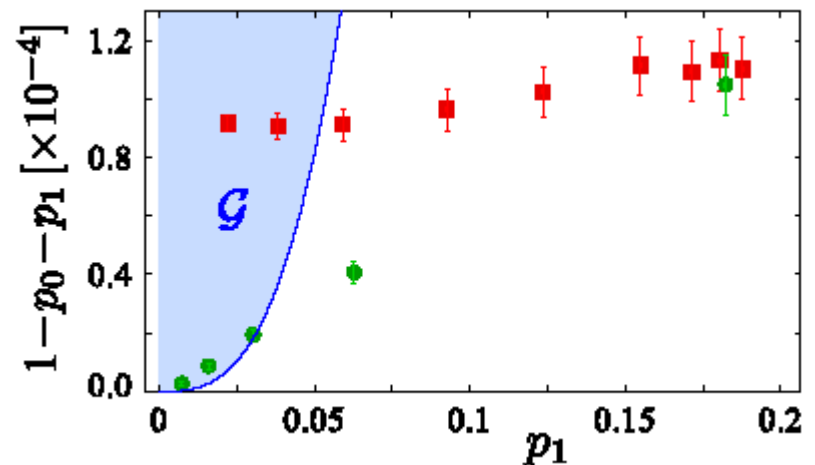


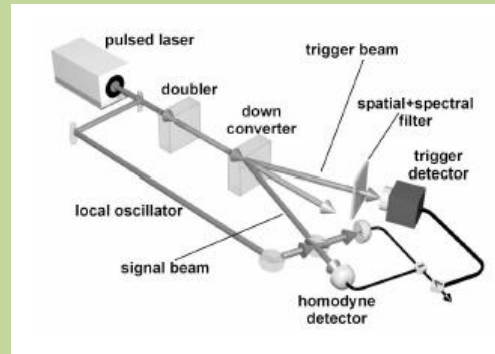
TABLE II: The same as Table I, n_{rel} indicates the amount of noise injected from LD1 and LD2 into trigger and signal detectors, $P = 50$ mW, and IF width $w = 10$ nm.

n_{rel}	p_0	p_1	a_{opt}	$\Delta W [\times 10^{-6}]$
0.0	0.8195	0.1804	0.94018	3479 ± 7
0.1	0.9073	0.0926	0.98389	406 ± 3
0.2	0.9408	0.0591	0.99332	42 ± 2
1.0	0.9777	0.0222	0.99903	-84 ± 1

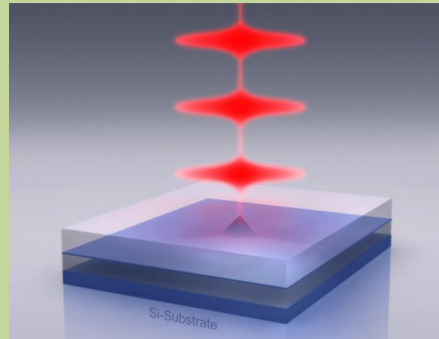


Miroslav Jezek, Ivo Straka, Michal Micuda, Miloslav Dusek, Jaromir Fiurasek, Radim Filip, Experimental test of quantum non-Gaussian character of heralded single photon state, Phys. Rev. Lett. 107, 213602 (2011)

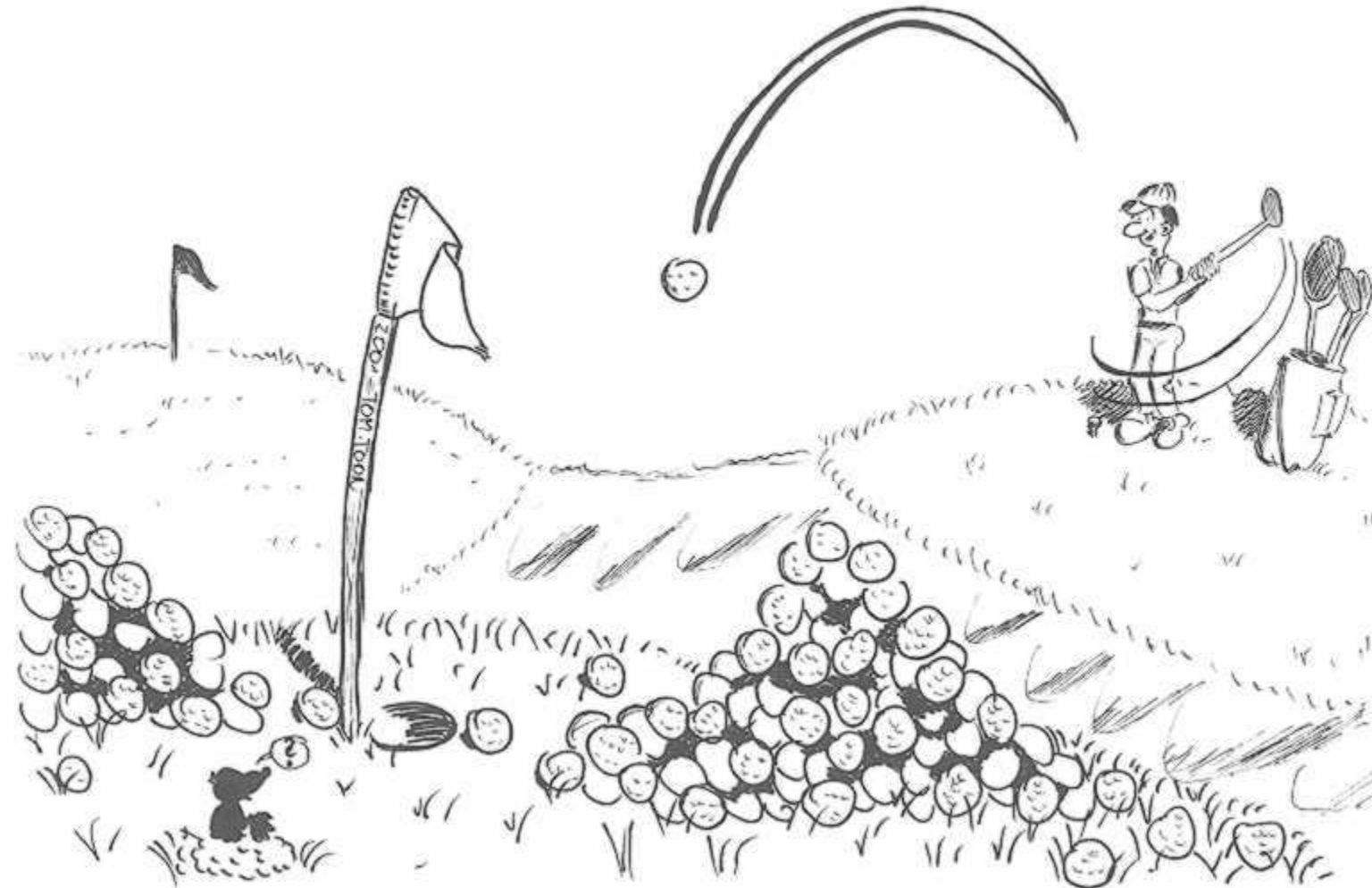
EXPERIMENT WITH SINGLE PHOTON SOURCE & HOMODYNE?



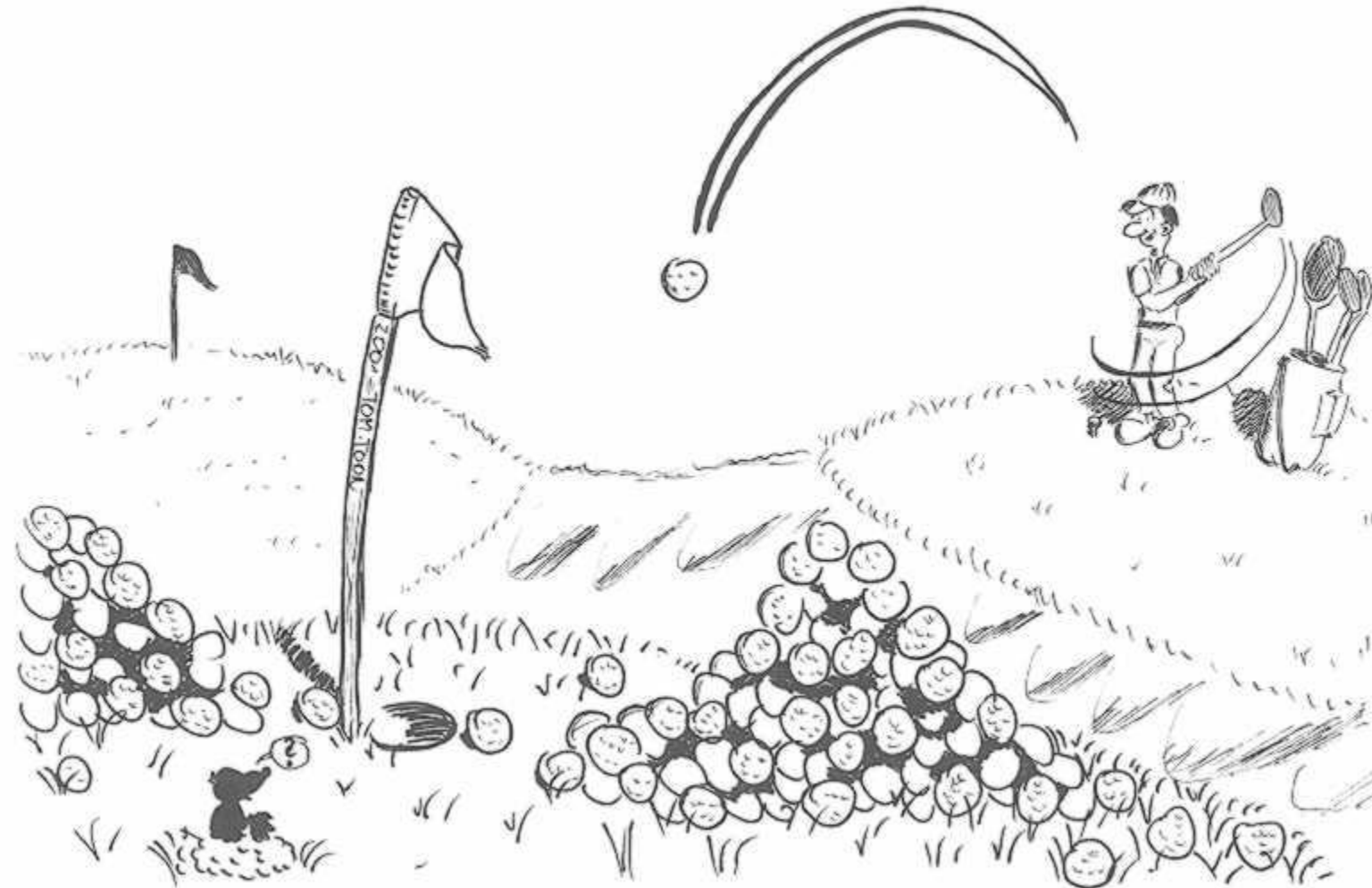
EXPERIMENTS WITH SOLID-STATE SINGLE PHOTON SOURCES ?



WHAT IS SINGLE PHOTON?



WHEN SINGLE PHOTON IS ..?



$$\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$$

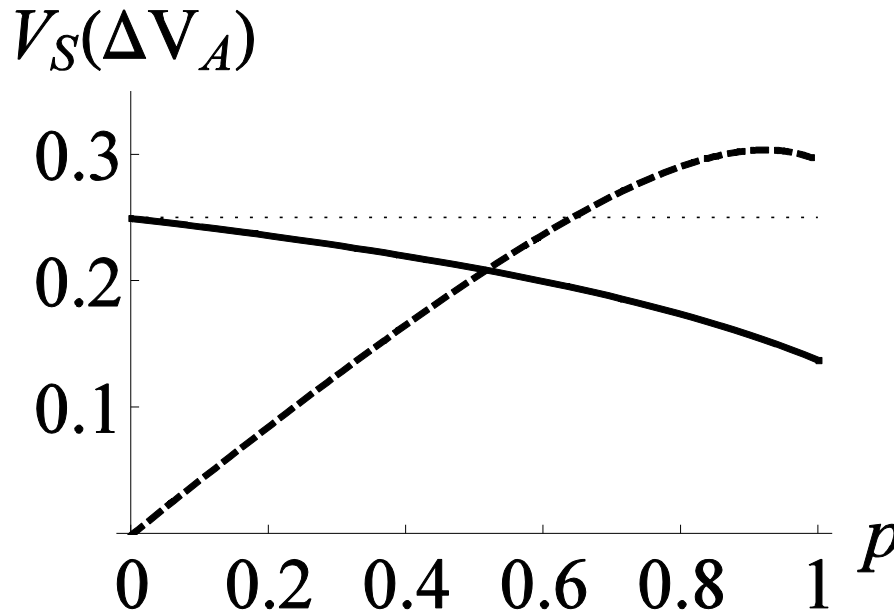
FOR WHAT?



SQUEEZING FROM

$$\eta |1\rangle\langle 1| + (1-\eta) |0\rangle\langle 0|$$

- Linear-optical processing **cannot** increase photon efficiency. D. W. Berry, A. I. Lvovsky, Phys. Rev. Lett. 105, 203601 (2010)
- Squeezing **can be** extracted already from two-copies by probabilistic Gaussian procedure.



more details on another
conference

R. Filip, submitted.