Optimal probabilistic measurement of phase

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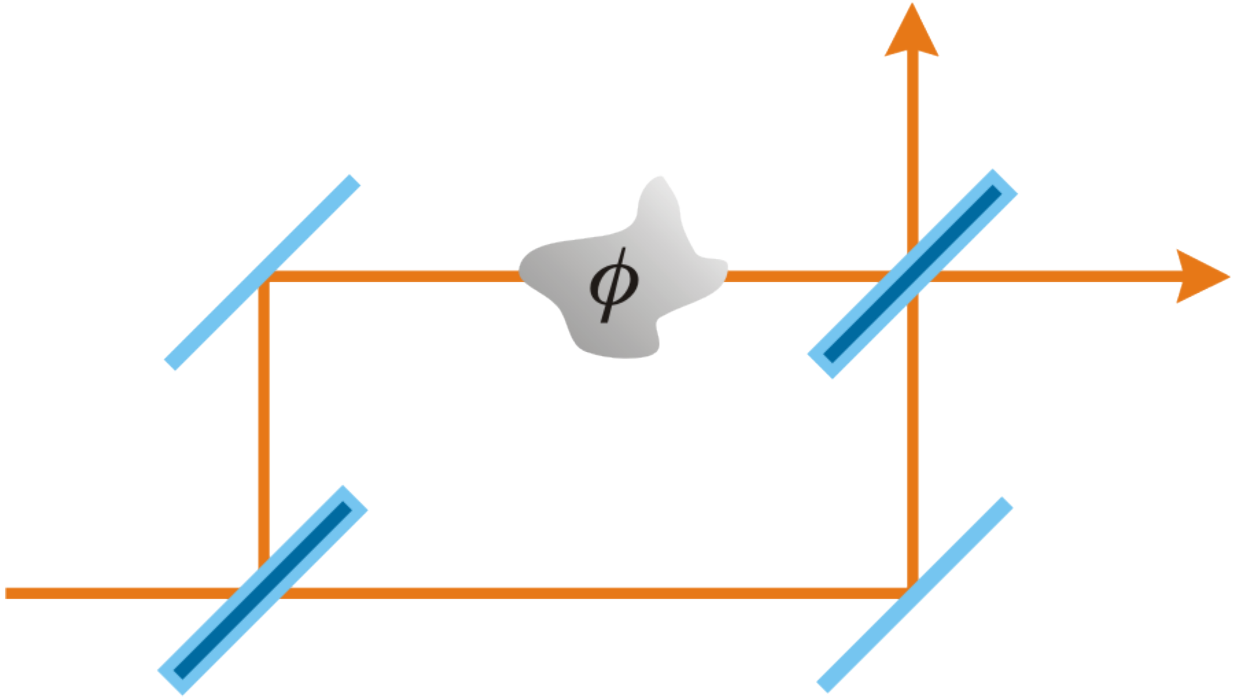
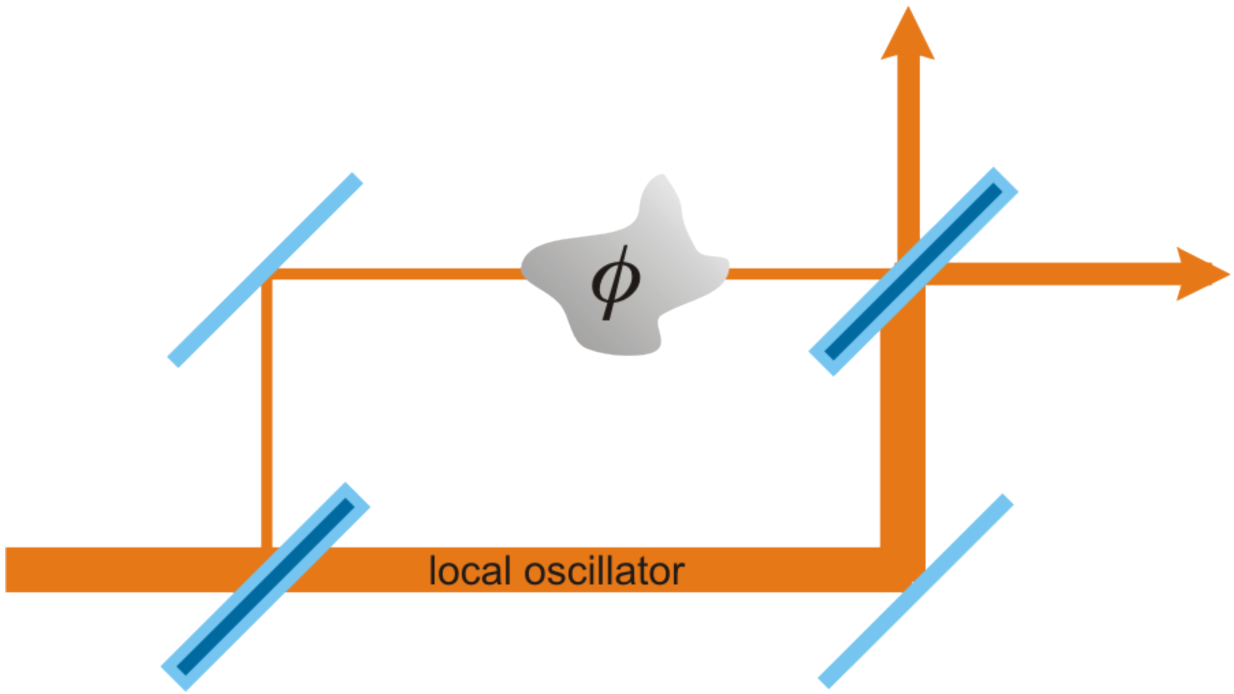
What is phase?



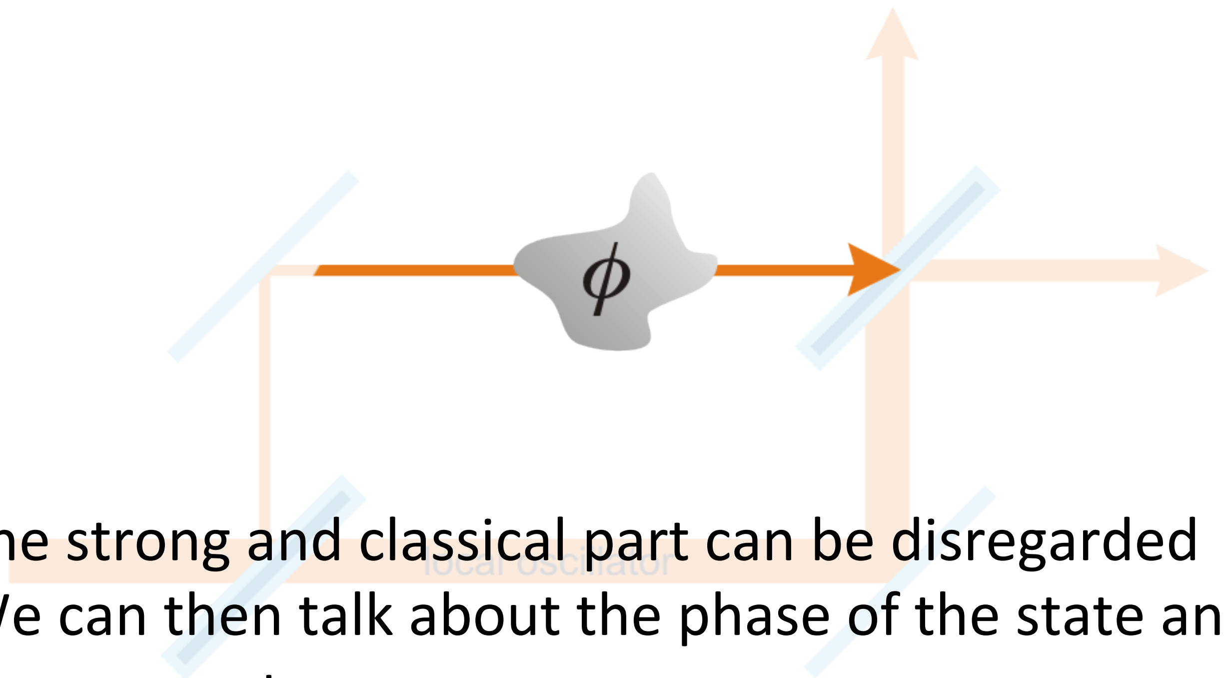
Relative difference of an optical path length in an interferometer

What is phase for Continuous Variables?



Basically the same, only the interferometer is strongly unbalanced

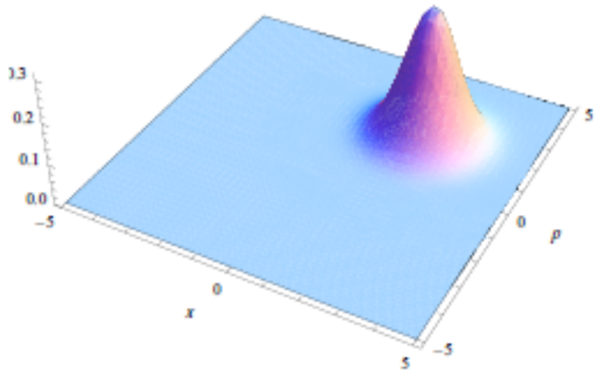
What is phase for Continuous Variables?

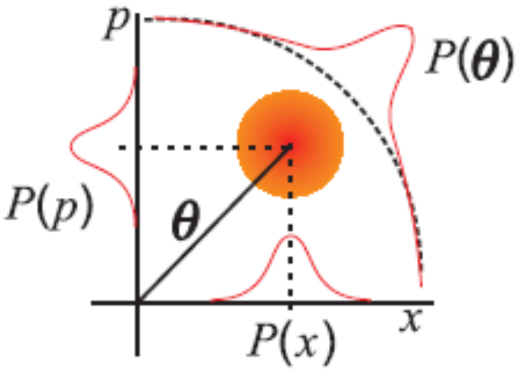


- The strong and classical part can be disregarded
- We can then talk about the phase of the state and its measurement

Example – phase of coherent states

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$



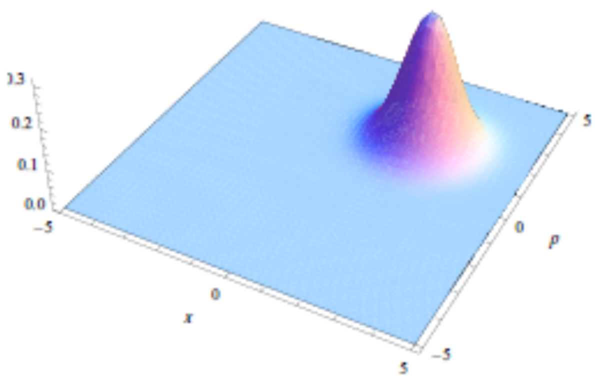


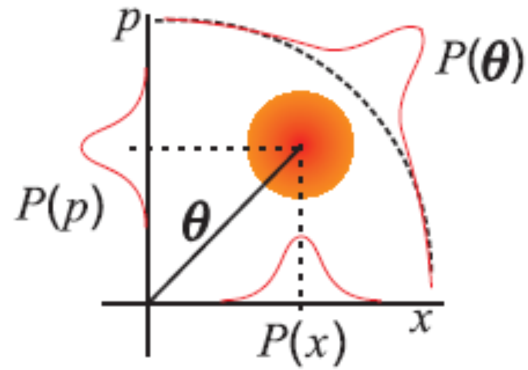
- Approximation of single mode laser light
- Almost classical state
- Can be used for quantum communication

Measurement versus estimation

- Phase measurement
  - Extracts information about the state from a single copy of the state
  - Gives immediate result, but it can be wrong
- Phase estimation
  - Extracts information about the state from many copies of the state
  - The quality of the outcome depends on the number of copies

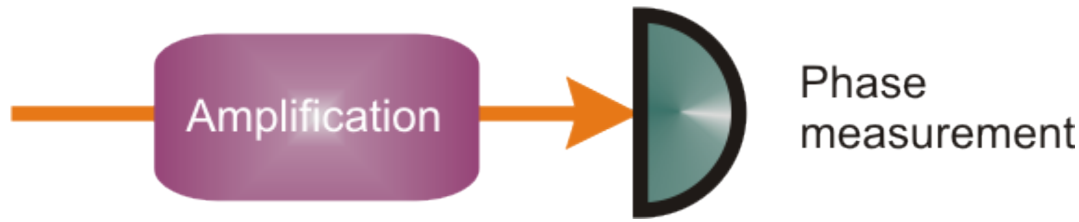
Back to coherent states



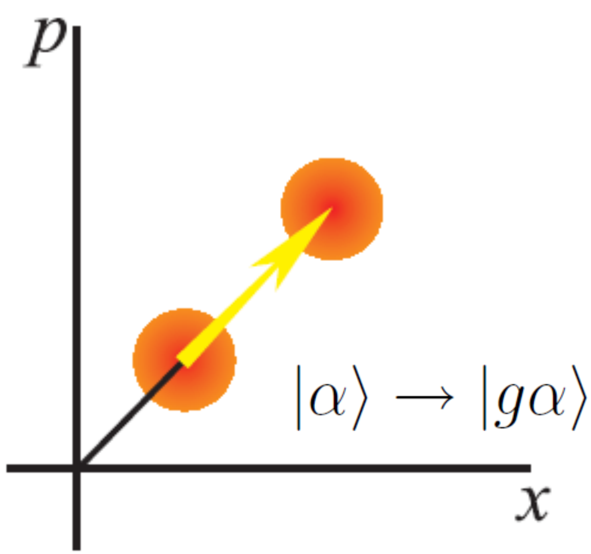


- Estimation reveals the phase distribution
- Measurement yields single value
- Quality of phase encoding given by the amplitude

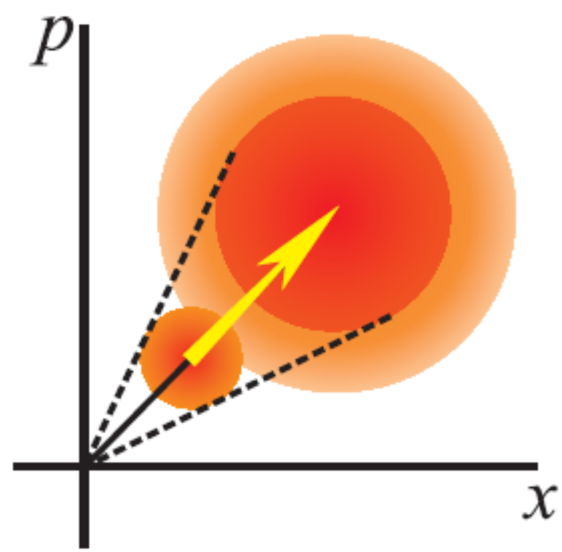
Improving the measurement outcome by amplification



Ideal amplification



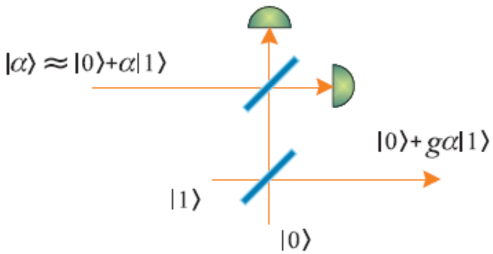
Gaussian amplification
(NOPA)



Ideal amplification does not exist…

…but it can be implemented approximately

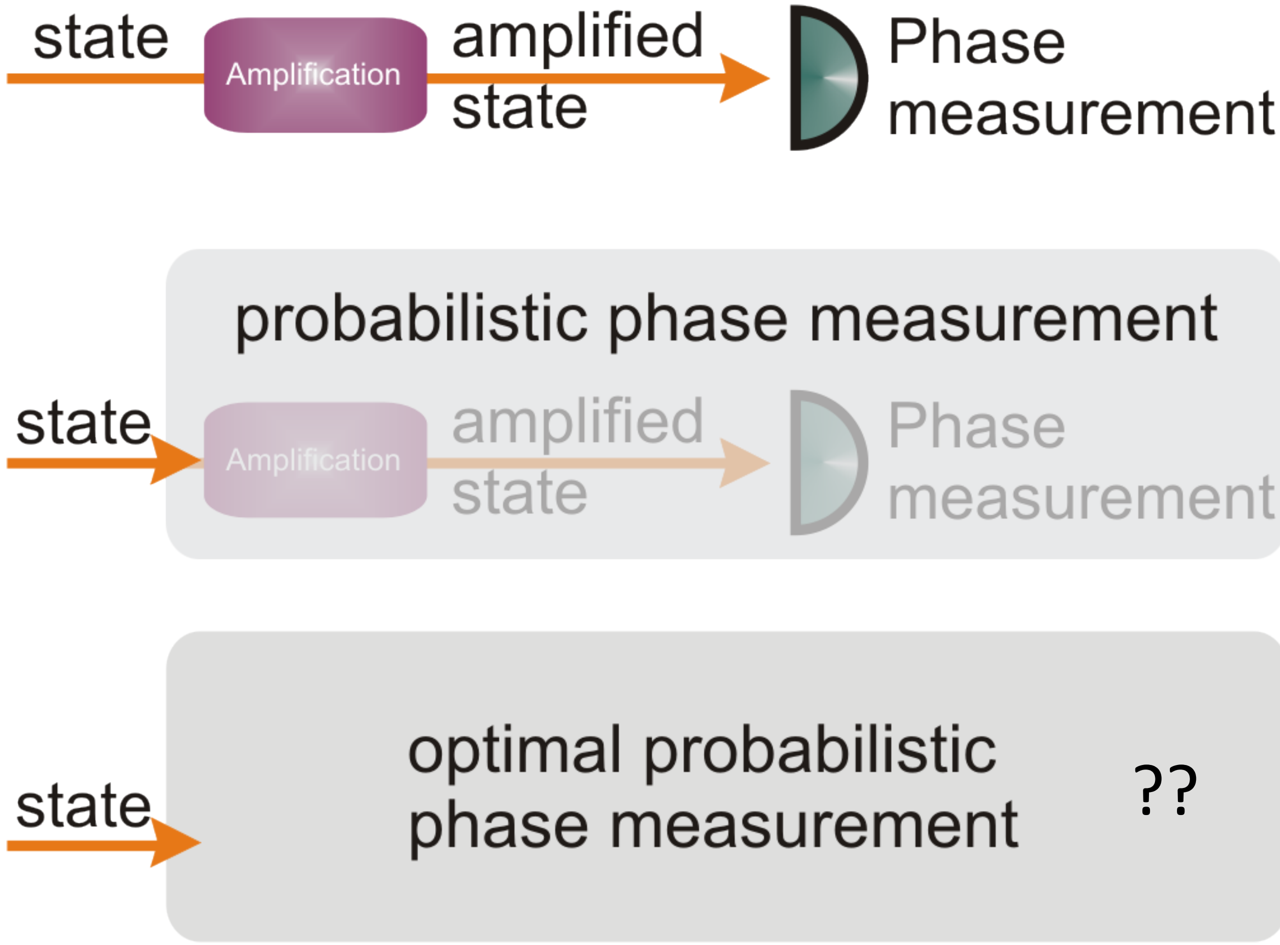
- Quantum scissors approach
  - [Xiang et al. Nature Photonics **4**, 316 (2010)]
  - [Ferreyol et al., Phys. Rev. Lett **104**, 123603 (2010)]
- Photon addition and subtraction
  - [Marek and Filip, Phys Rev A **81**, 022302 (2010)]
  - [Zavatta et al., Nature Photonics **5**, 52 (2011)]
- Noise powered amplification
  - [Marek and Filip, Phys Rev A **81**, 022302 (2010)]
  - [Usuga et al., Nature Phys. 6, 767 (2010)]



$$\hat{a}\hat{a}^\dagger(|0\rangle + \alpha|1\rangle)$$
$$= \hat{a}(|1\rangle + \sqrt{2}\alpha|2\rangle)$$
$$= |0\rangle + 2\alpha|1\rangle$$

- It can be used for improving phase measurements…

… but how good can it be?



Optimal phase measurement

- Deterministic
  - Projects on idealized phase states

$$|\theta\rangle = \sum_{n=0}^{\infty} e^{i\theta n}|n\rangle$$

- Probabilistic
  - Projects on generalized phase states
  - Nonzero probability of an inconclusive result
  - The probability of success is state dependant

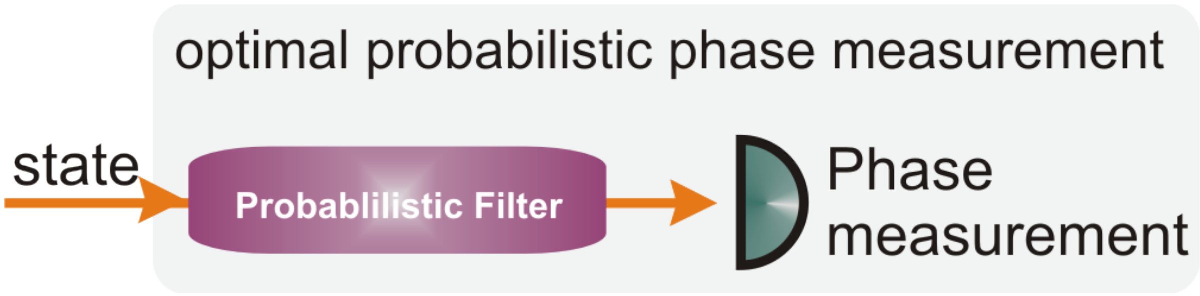
$$|\theta_f\rangle = \sum_{n=0}^{\infty} f_n e^{i\theta n}|n\rangle$$

$$0 \leq f_n \leq 1$$

Optimal phase measurement

- Can be represented by the optimal deterministic phase measurement and a trace decreasing filtering operation

$$F = \sum_{n=0}^{\infty} f_n |n\rangle\langle n|$$



- To find the optimal measurement one needs only to find the optimal filter

Finding the optimal filter

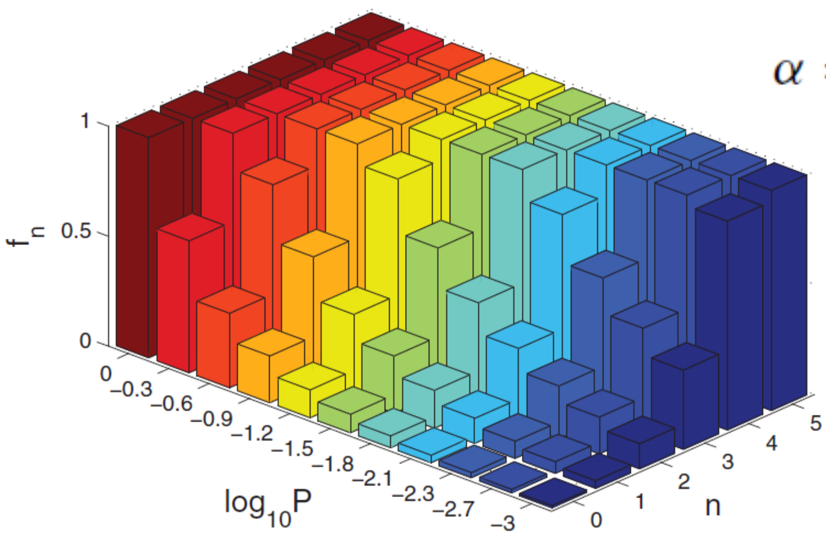
- What do we mean by optimal?
  - For any given state $|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$ and any given probability of success, we look for minimal value of phase variance

$$V = \left|\langle e^{i\phi}\rangle\right|^{-2} - 1$$

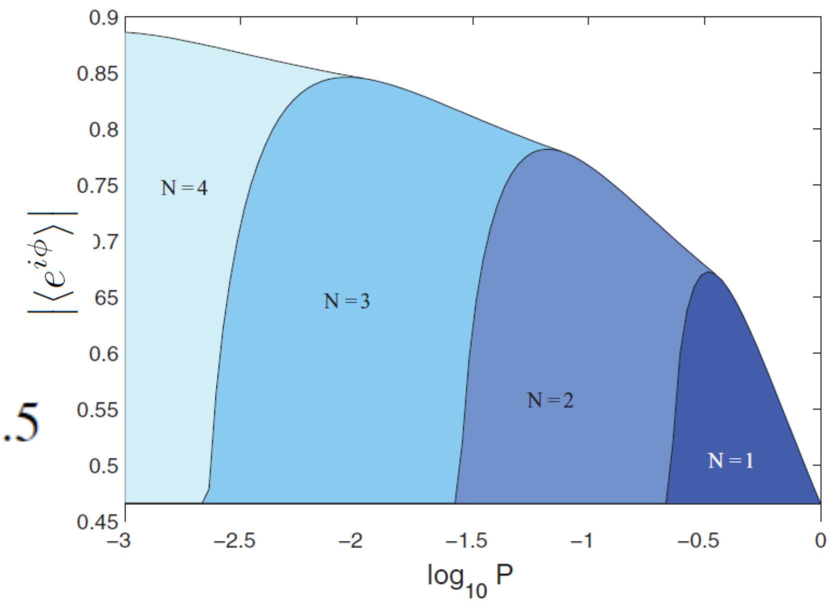
- That means:
  - Maximization of $$\left|\langle e^{i\phi}\rangle\right| = \left|\sum_{n=0}^{\infty} f_n f_{n+1}^* c_n c_{n+1}^*\right|$$
  - Under the condition $$\sum_{n=0}^{\infty} |f_n|^2 |c_n|^2 = P$$

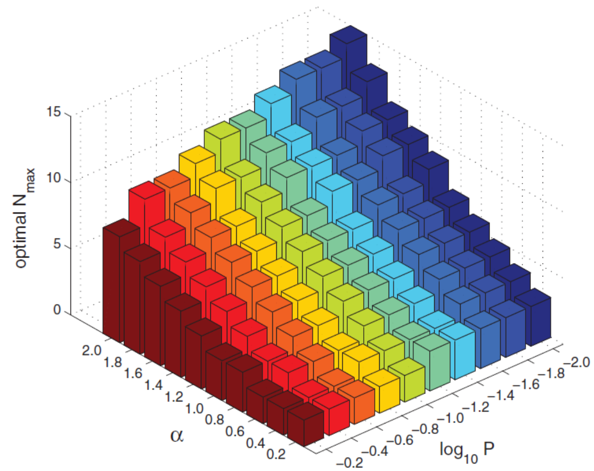
Optimal filters for coherent states

- Can be found semi-analytically



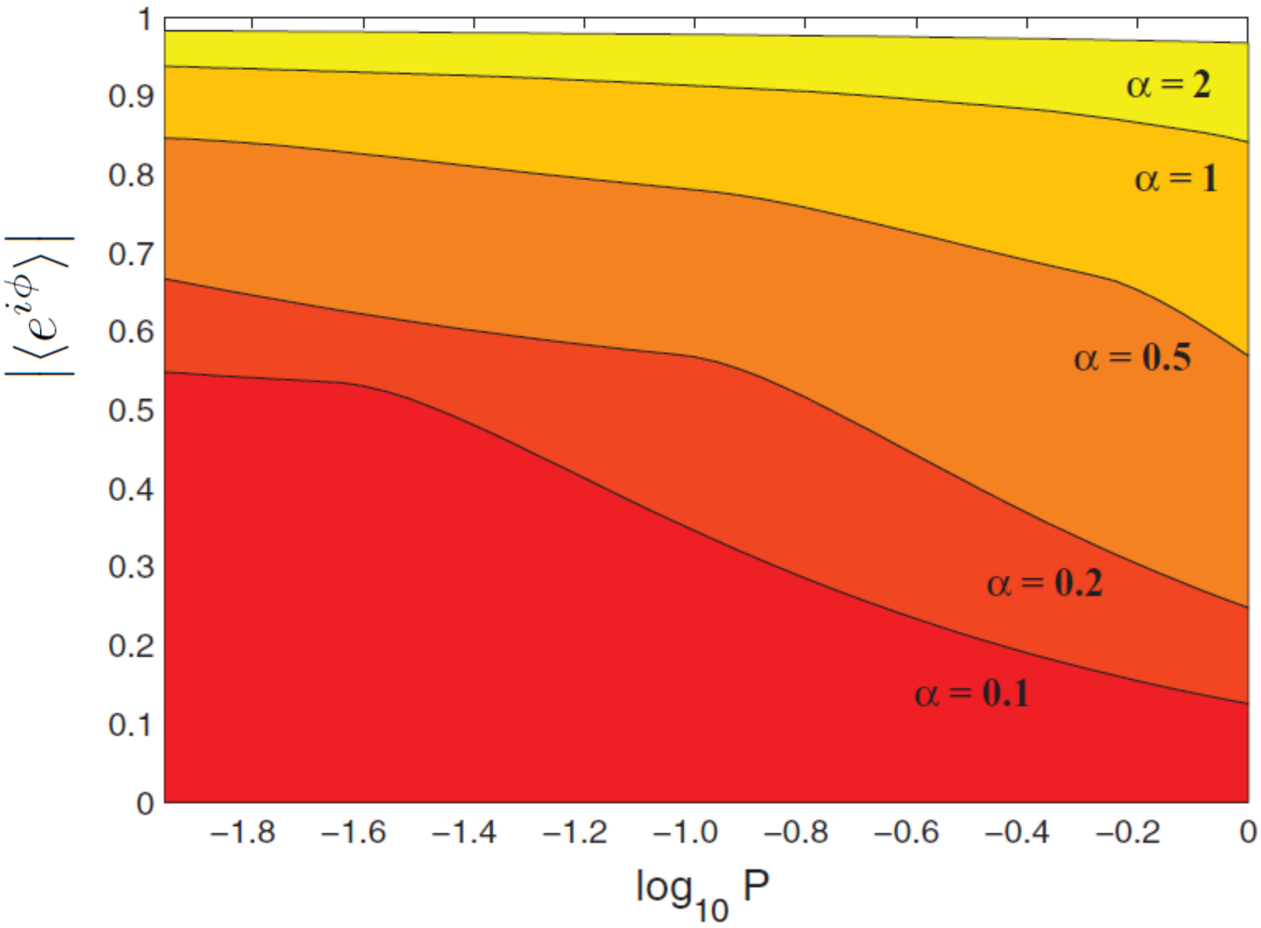
$\alpha = 0.5$





For each probability, there is only
a finite number of relevant filter parameters

Measurement improvement for different states



In conclusion

- The optimal probabilistic measurement gives a bound on how much phase information can be obtained at the cost of reducing success rate
- It can be represented by a trace decreasing filter and the optimal phase measurement

- Phys. Rev. A **88**, 045802 (2013)

Thank you for the attention!