

# Optimizing the rotating point spread function by SLM aided spiral phase modulation



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# Outline

**1. Introduction**

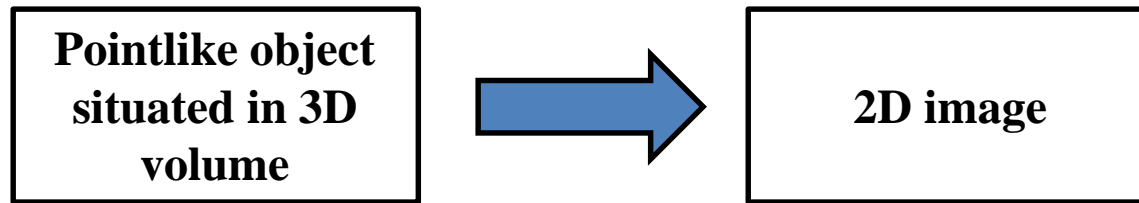
2. New design of the rotating PSF

3. Properties of the rotating PSF

4. Experiment

5. Conclusion

# 3D particle localization and tracking



**How to obtain 3D object position from 2D image?**

4Pi microscopy

*S. Hell, et al., Confocal microscopy with an increased detection aperture: type-B 4Pi confocal microscopy, Optics Letters 19, 222-224 (1994)*

Detailed analysis of defocused image

*M. Speidel, et al., Three-dimensional tracking of fluorescent nanoparticles with subnanometer precision by use of off-focus imaging, Optics Letters 28, 69-71 (2003)*

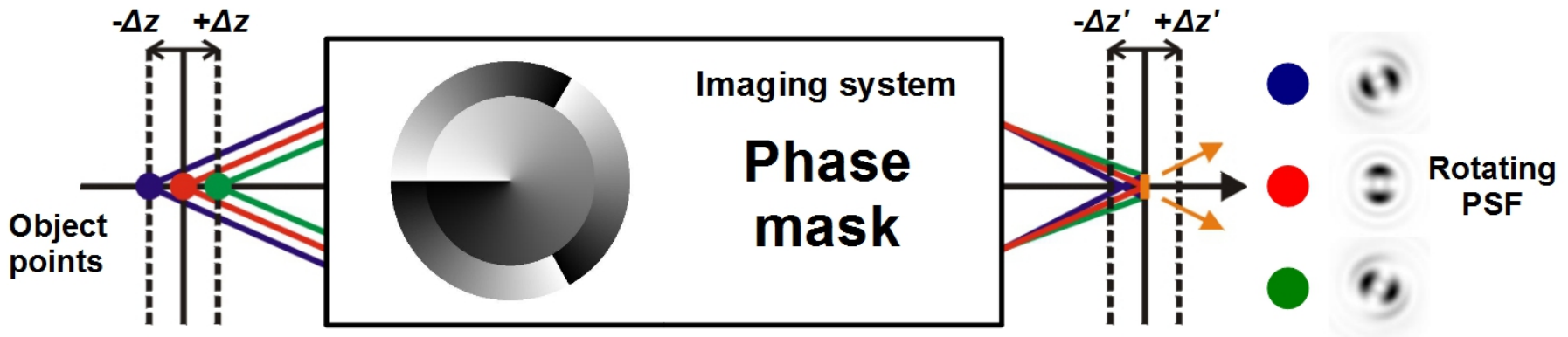
Introducing of astigmatism by cylindrical lens

*H. Kao, et al., Tracking of single fluorescent particles in three dimensions: use of cylindrical optics to encode particle position, Biophysical Journal 67, 1291-1300 (1994)*

**Rotating point spread function (PSF)**

*A. Greengard, et al., Depth from diffracted rotation, Optics Letters 31, 181-183 (2006)*

# Core idea of the rotating PSF



- Image of the object point rotates with defocusing
- Axial position of the pointlike object can be determined from the angle of PSF rotation
- Defocus-induced rotation of PSF can be implemented to standard imaging system by phase modulation

# The various methods of rotating PSF implementation

## 1. Phase mask composed of L-G modes

### Double-helix PSF

*S. Pavani, et al., Three dimensional tracking of fluorescent microparticles using a photon-limited double-helix response system, Optics Express 16, 22048-22057 (2008)*

### Corkscrew PSF

*M. Lew, et al., Corkscrew point spread function for far-field three-dimensional nanoscale localization of pointlike objects, Optics Letters 36, 202-204 (2011)*

## 2. Sampled spiral phase mask

### Azimuthal sampling

*M. Baranek, et al., Rotating vortex imaging implemented by a quantized spiral phase modulation, J. Europ. Opt. Soc. Rap. Public 8, 13017 (2013)*

### Radial sampling

*S. Prasad, Rotating point spread function via pupil-phase engineering, Optics Letters 38, 585-587 (2013)*

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**2. New design of the rotating PSF**

3. Properties of the rotating PSF

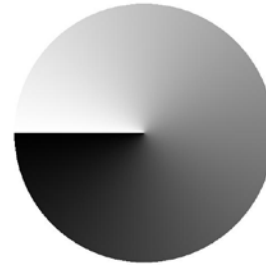
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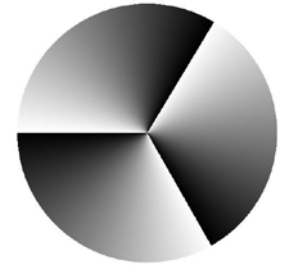
# Spiral phase mask parameters

Topological charge  $-l$

$l = 1$



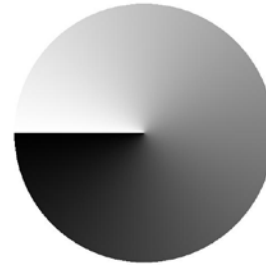
$l = 3$



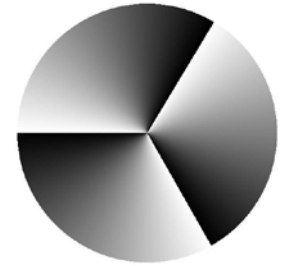
# Spiral phase mask parameters

Topological charge –  $l$

$l = 1$



$l = 3$



Phase segments number –  $M$

$l = 1$   
 $M = 8$



$l = 3$   
 $M = 9$

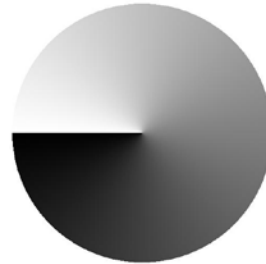




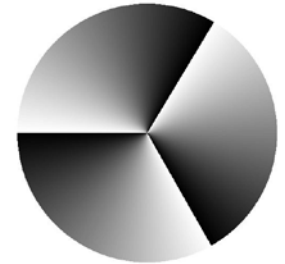
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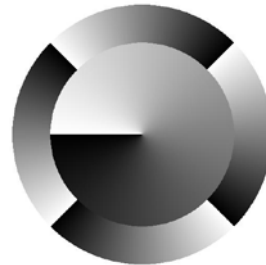


$l = 3$   
 $M = 9$



**Radial zones number –  $N$**

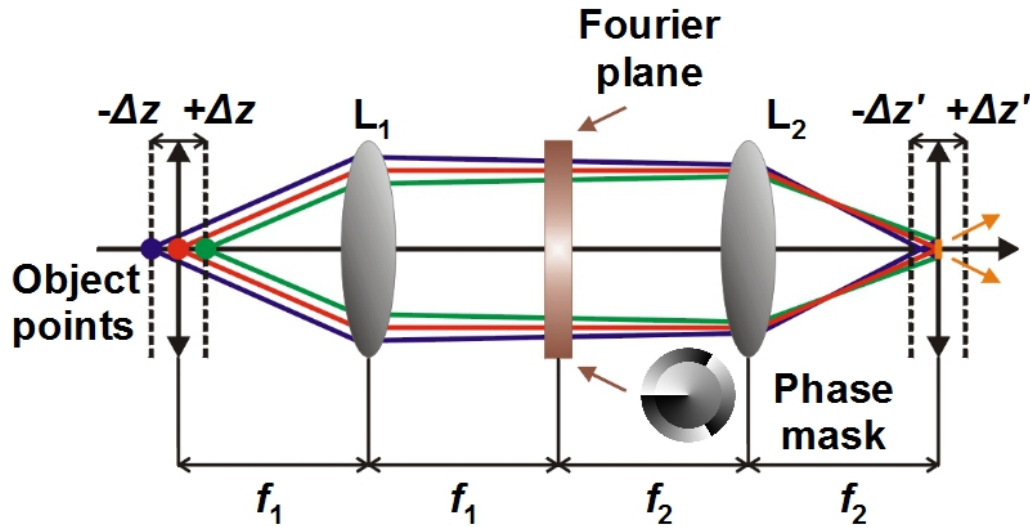
**Topological charge  
in  $n$ -th radial zone –  $l_n$**



**Phase segments number  
in  $n$ -th radial zone –  $M_n$**

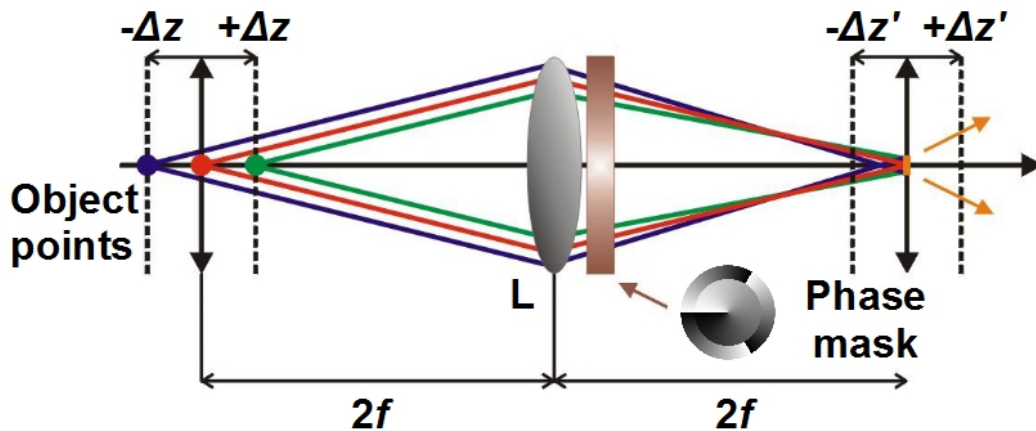


# Theoretical model



## 4-f optical system

modulation of frequency spectrum



## Vortex lens

modulation of complex amplitude

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# Computation – continuous azimuthal profile

Optical field in  
the image plane



Mask  
transparency



Defocus



Fourier transform  
term



$$U \propto \int \int_{-\infty}^{+\infty} S(\mathbf{r}_{\perp}) \exp \left[ -i \frac{k \Delta \Phi}{2} |\mathbf{r}_{\perp}|^2 \right] \exp [i 2 \pi \mathbf{r}_{\perp} \cdot \mathbf{F}'_{\perp}] d\mathbf{r}_{\perp},$$

**Radial zones approximated to rings**



*S. Prasad, Rotating point spread function via pupil-phase engineering, Optics Letters 38, 585-587 (2013)*

$$I \propto |A|^2 \sum_{n=1}^N J_{l_n}^2 + 2|A|^2 \sum_{n=1, n < n'}^N \sum_{n'=1}^N J_{l_{n'}} J_{l_n} \rightarrow \text{Bessel functions}$$

$$\alpha = \frac{k R^2 \Delta \Phi}{4N} \times \cos \left[ (l'_{n'} - l_n) \left( \psi + \frac{\pi}{2} \right) - 2\alpha(n' - n) \right]$$

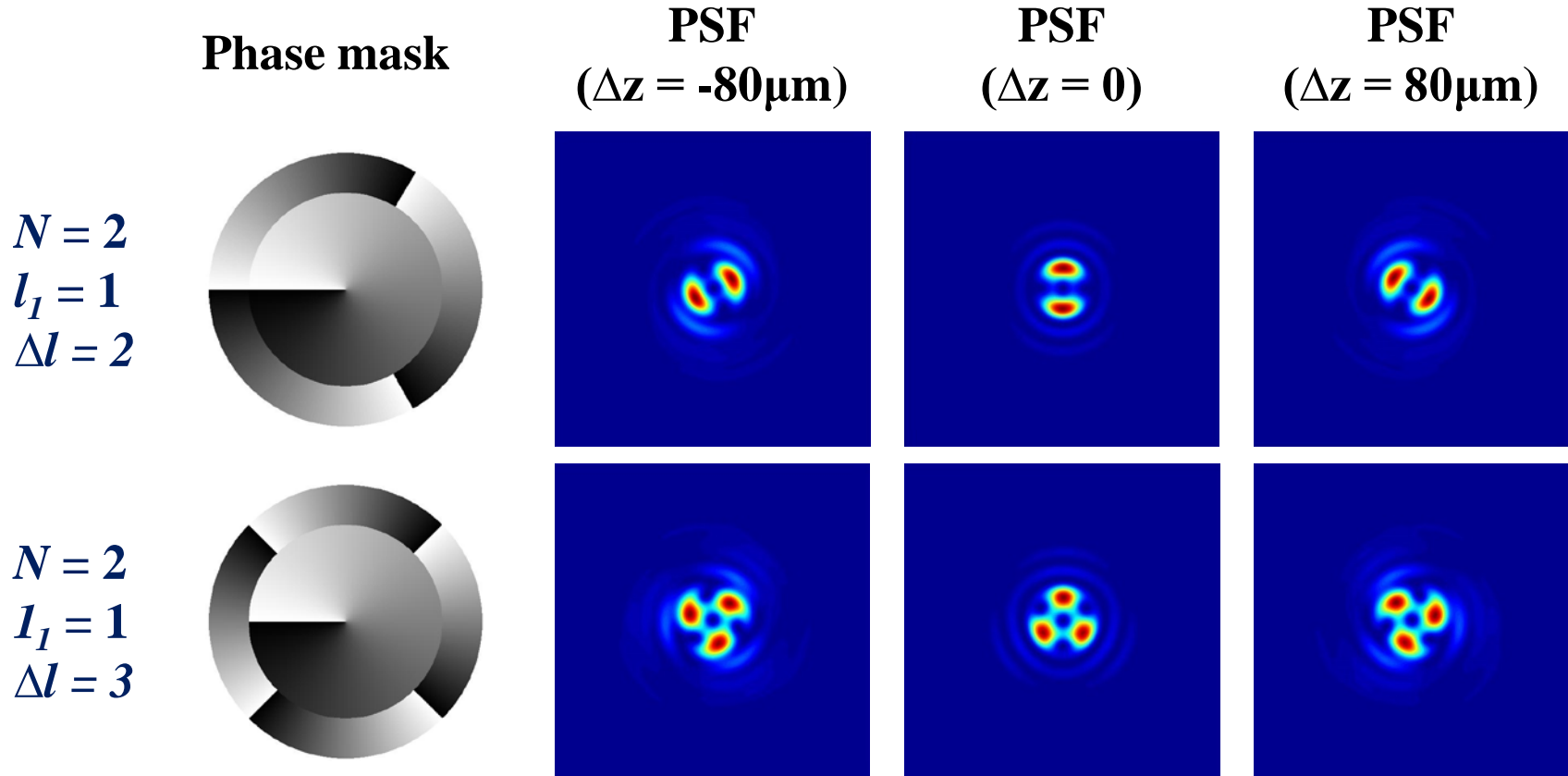


Rotation rate associated  
to each couple of radial  
zones

$$\frac{d\psi}{d\alpha} = \frac{2(n' - n)}{l_{n'} - l_n}$$

**Topological charge difference  
between each adjacent radial  
zones has to be constant**

# Number of lobes in intensity spot



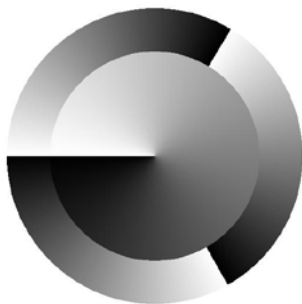
Number of lobes in intensity spot is fully determined by parameter  $\Delta l$

$$I \propto |A|^2 \sum_{n=1}^N J_{l_n}^2 + 2|A|^2 \sum_{n=1, n < n'}^N \sum_{n'=1}^N J_{l_{n'}} J_{l_n} \times \cos \left[ (l'_{n'} - l_n) \left( \psi + \frac{\pi}{2} \right) - 2\alpha(n' - n) \right]$$

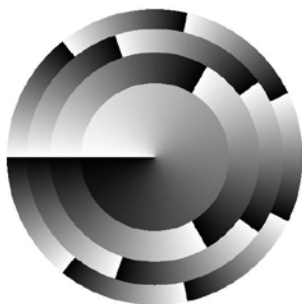
# Rotation rate

Phase mask

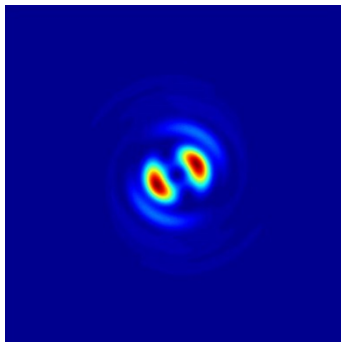
$$N = 2$$
$$l_l = 1$$
$$\Delta l = 2$$



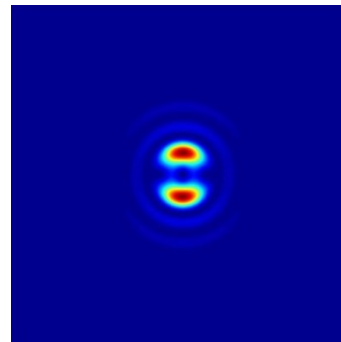
$$N = 4$$
$$l_l = 1$$
$$\Delta l = 2$$



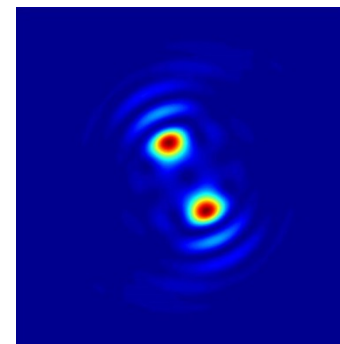
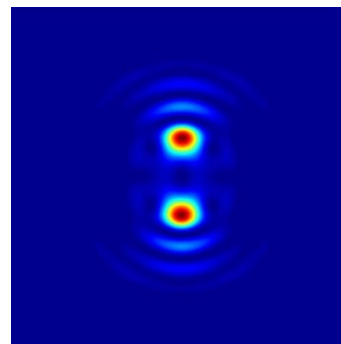
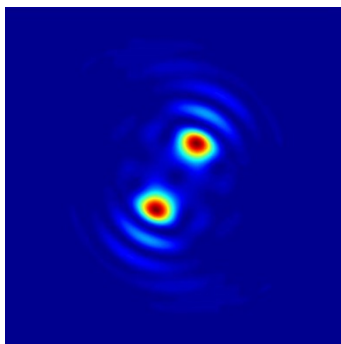
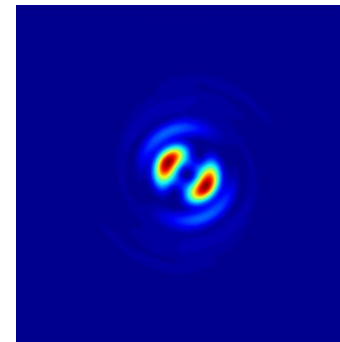
PSF  
( $\Delta z = -80\mu\text{m}$ )



PSF  
( $\Delta z = 0$ )



PSF  
( $\Delta z = 80\mu\text{m}$ )



Rotation rate can be controlled by spiral mask parameters  $N$  and  $\Delta l$

$$\frac{d\psi}{d\Delta z} = \frac{\pi N \Delta l}{\lambda}$$

# Computation – sampled azimuthal profile

Optical field in  
the image plane



Mask  
transparency



Defocus



Fourier transform  
term



$$U \propto \int \int_{-\infty}^{+\infty} S(\mathbf{r}_{\perp}) \exp \left[ -i \frac{k \Delta \Phi}{2} |\mathbf{r}_{\perp}|^2 \right] \exp [i 2 \pi \mathbf{r}_{\perp} \cdot \mathbf{F}'_{\perp}] d\mathbf{r}_{\perp},$$

**Discrete spiral mask expressed as  
combination of continuous masks**



*Ch. S. Guo, et al., Optimal phase steps  
of multi-level spiral phase plates, Opt.  
Commun. 268, 235-239 (2006)*

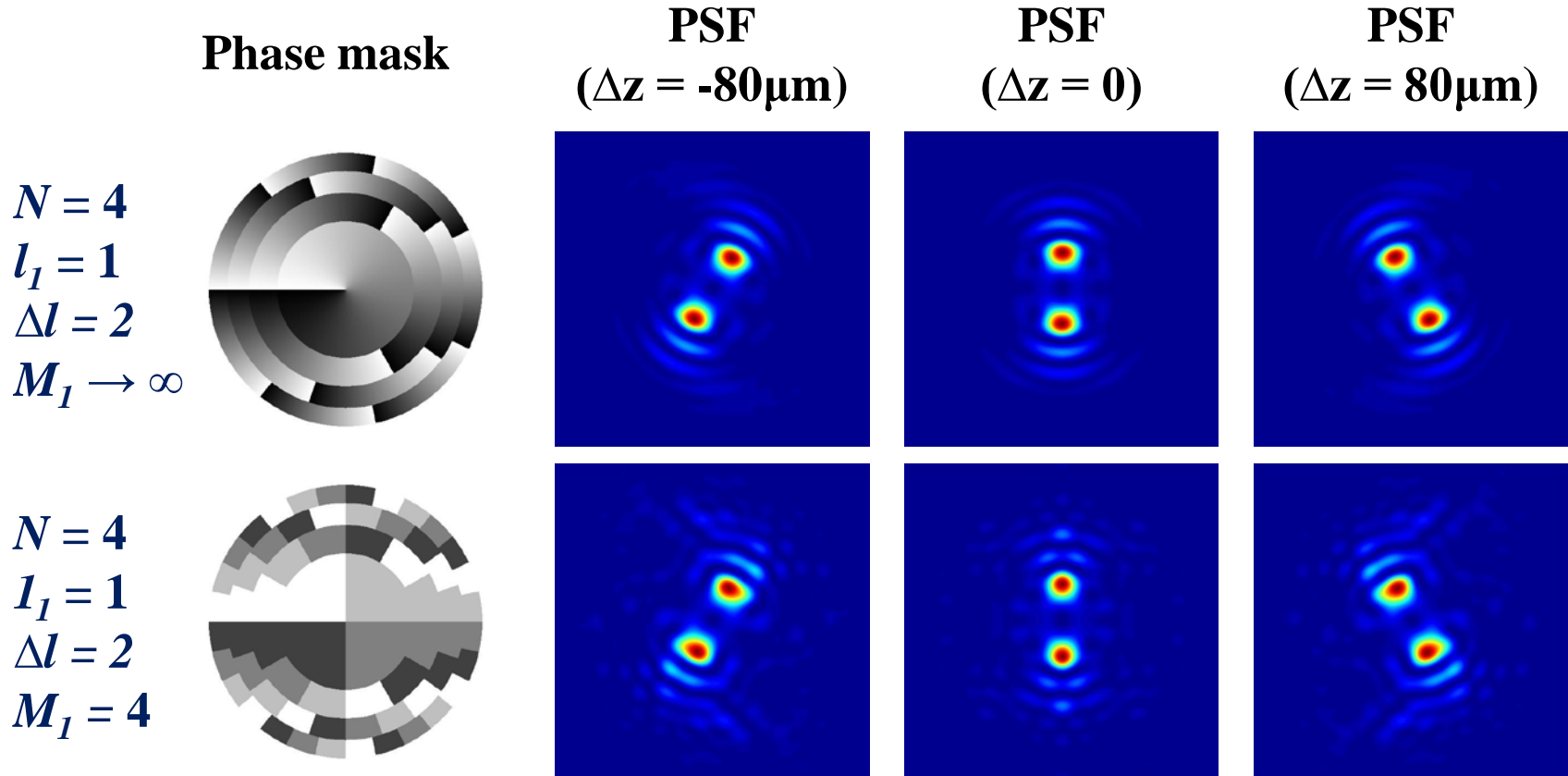
$$I \propto |A|^2 \sum_{n=1}^N |c_{l_n}|^2 J_{l_n}^2 + 2|A|^2 \sum_{n=1, n < n'}^N \sum_{n'=1}^N |c_{l_n} c_{l_{n'}}| J_{l_n} J_{l_{n'}} \\ \times \cos \left[ \left( \psi + \frac{\pi}{2} \right) (l_{n'} - l_n) + \pi \left( \frac{l_{n'}}{M_{n'}} - \frac{l_n}{M_n} \right) - 2\alpha(n' - n) \right]$$



**Condition  $M_n = M_1(l_n/l_1)$  has to  
fulfilled to elimination of  
additional rotation**

Additional rotating

# Influence of azimuthal sampling



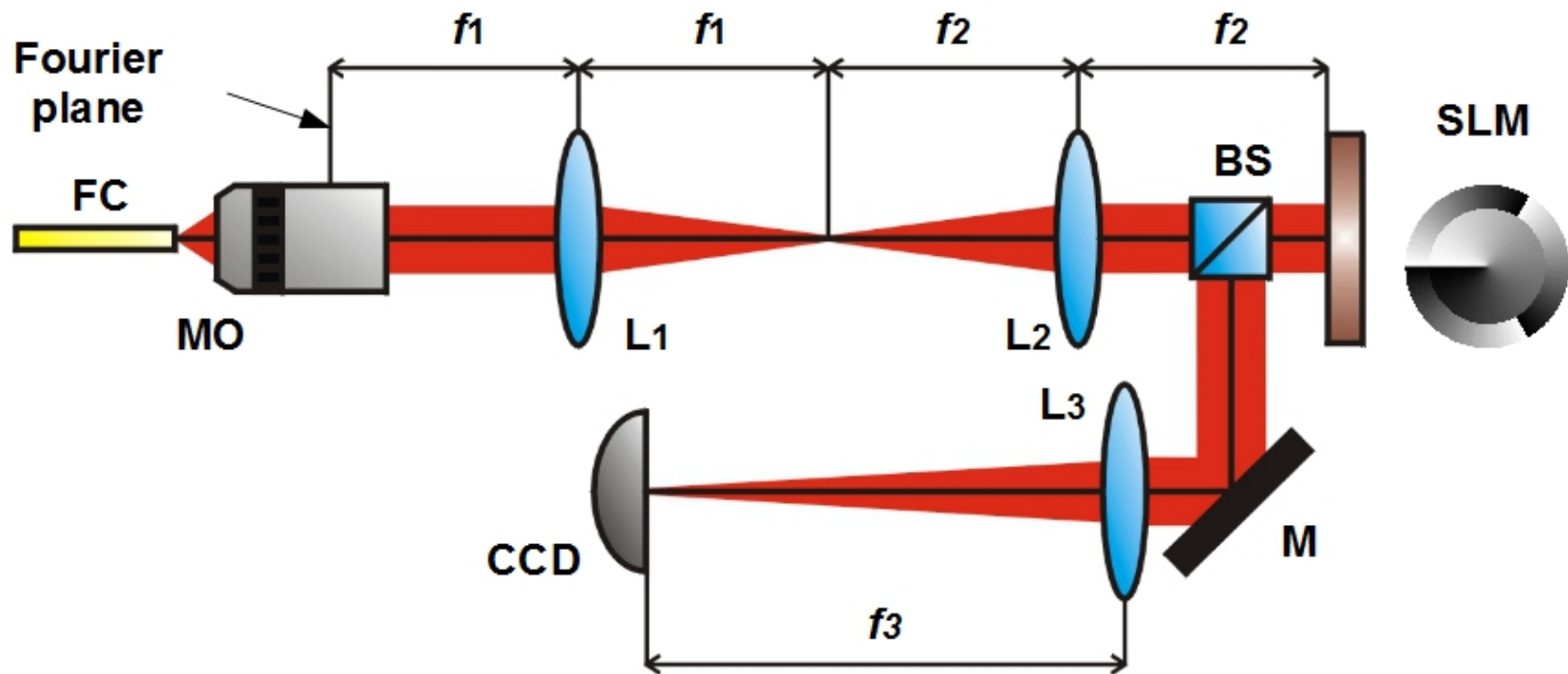
Azimuthal sampling does not significantly affect key rotating PSF properties, if condition  $\frac{l_n}{l_1} = \frac{M_n}{M_1}$  is valid



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# Experimental setup



## Experiment:

He-Ne laser (20 mW, 632.8 nm);

MO – microobjektive (Melles Griot-OVI, 50x, NA = 0.55);

SLM – Boulder (512x512 px);

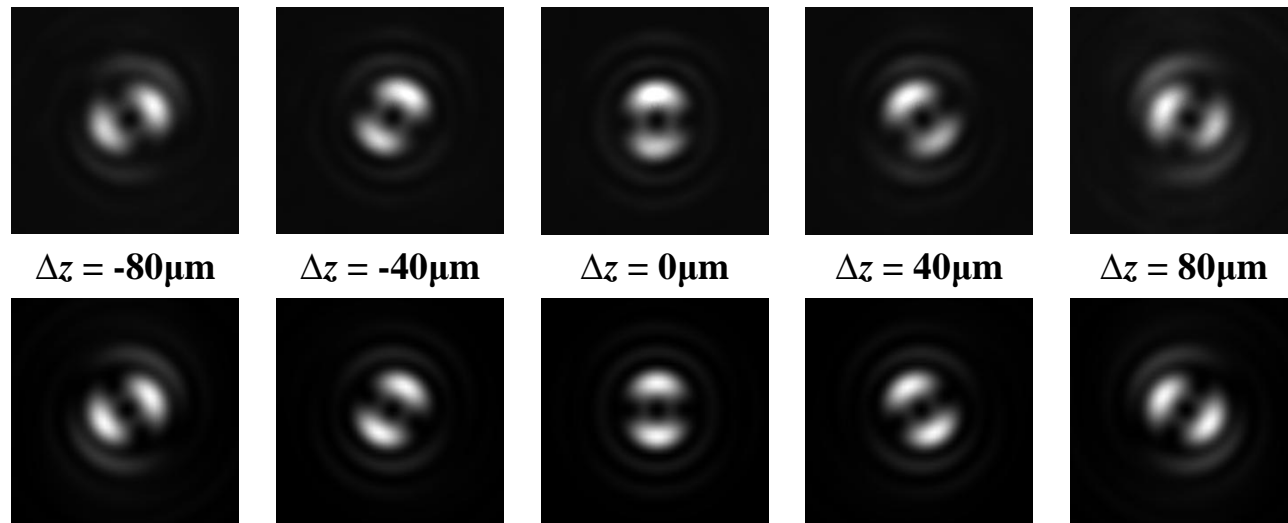
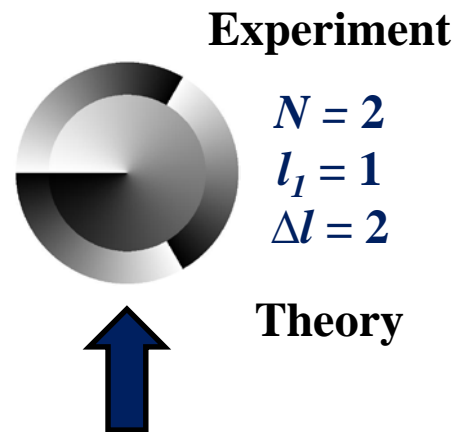
$L_1, L_2, L_3$  – lenses ( $f_1 = 200$  mm,  $f_2 = 200$  mm,  $f_3 = 400$  mm)

FC – fiber core (NA = 0.1)

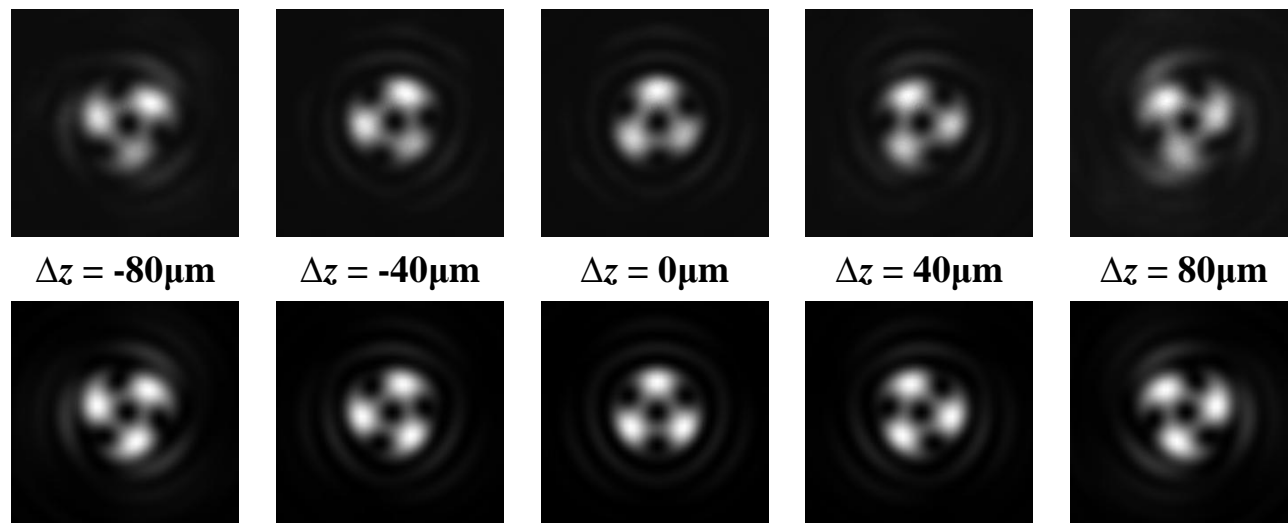
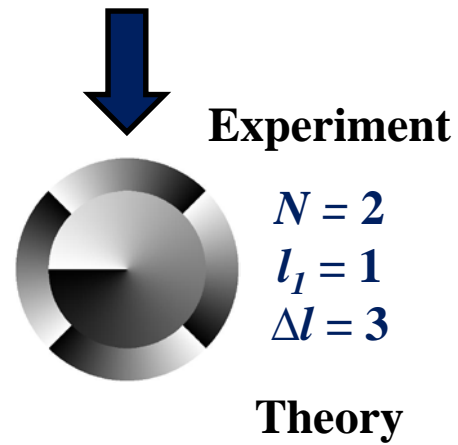
BS – beam splitter

M – mirror

# Experimental results



**Phase mask**



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# Conclusion

## Summary

We presented the new method for rotating PSF generation

- Presented technique has high energy efficiency and can be easily implemented to standard imaging systems
- The PSF was described mathematically in dependence on the parameters of the spiral mask
- PSF transverse profile and rotation rate can be controlled by two independent parameters of the phase mask
- Continuous azimuthal change of the helical phase profile can be satisfactorily replaced by just a few phase levels used in practical implementation

## Outlook

Our future research is focused on the application potential of designed rotating PSF

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**Thank You for Your attention**