

QUANTUM OPERATIONS WITH LIGHT

Radim Filip, Petr Marek, Kimin Park
Department of Optics, Palacky University

Czech-Japan collaboration (Furusawa Lab)



TEAM:

Radim Filip

**Quantum Coherence
and Nonclassicality**

**Miroslav Gavenda
Petr Marek**

**Students:
Lukáš Lachman**

**Measurement-Induced
Operations**

**Petr Marek
Kimin Park**

**Students:
Petr Zapletal
Vojta Kupčik**

**Quantum Key
Distribution**

**Vladyslav Usenko
Lazslo Ruppert**

**Students:
Ivan Derkač**

**Quantum
Optomechanics**

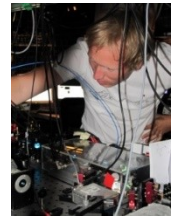
Andrey Rakhubovsky

**Students:
Nikita Vostrosablin**

**Interaction of Light
with Atoms**

**Lukáš Slodička
Petr Marek**

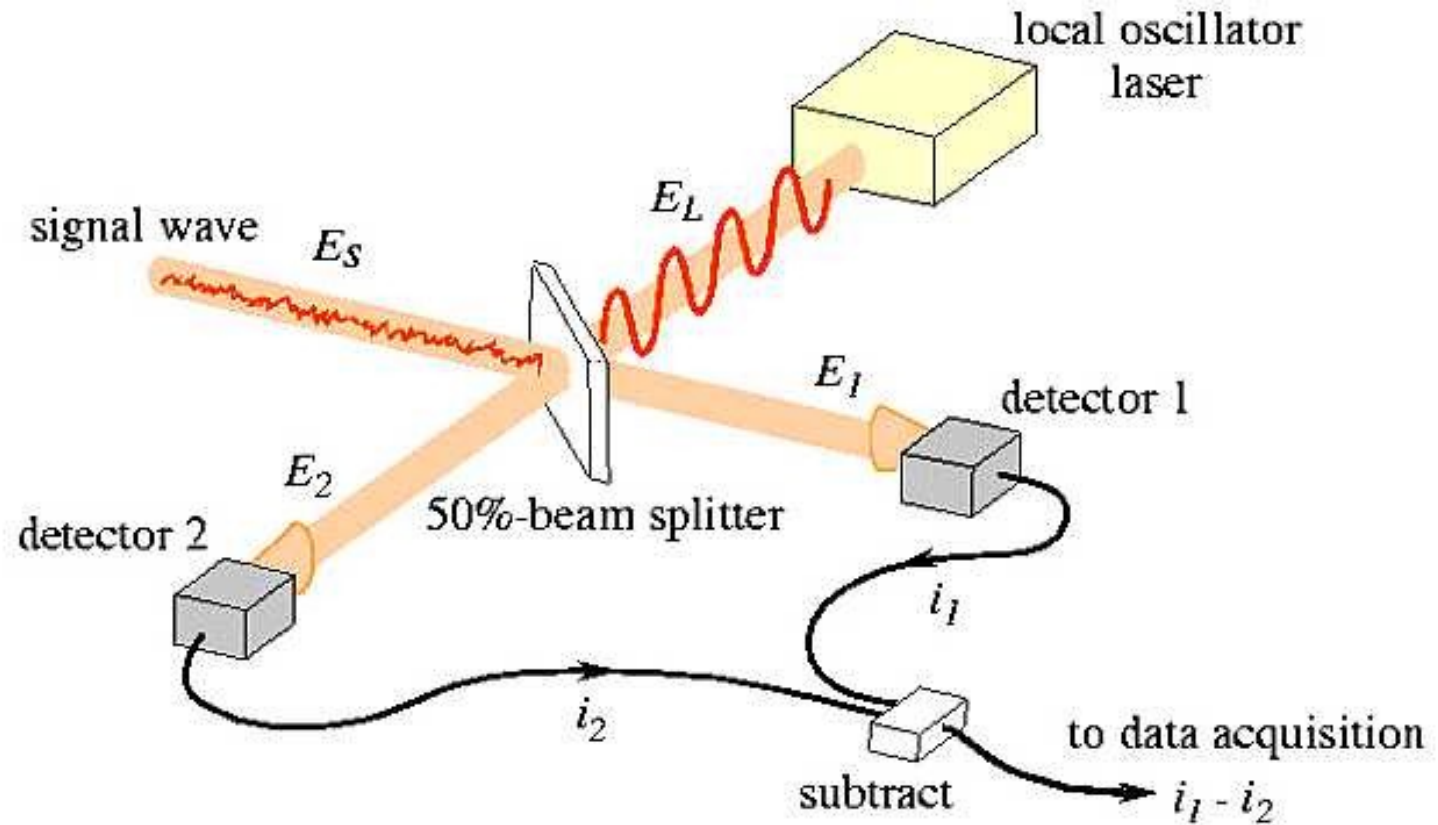
**Students:
Petr Obšil**





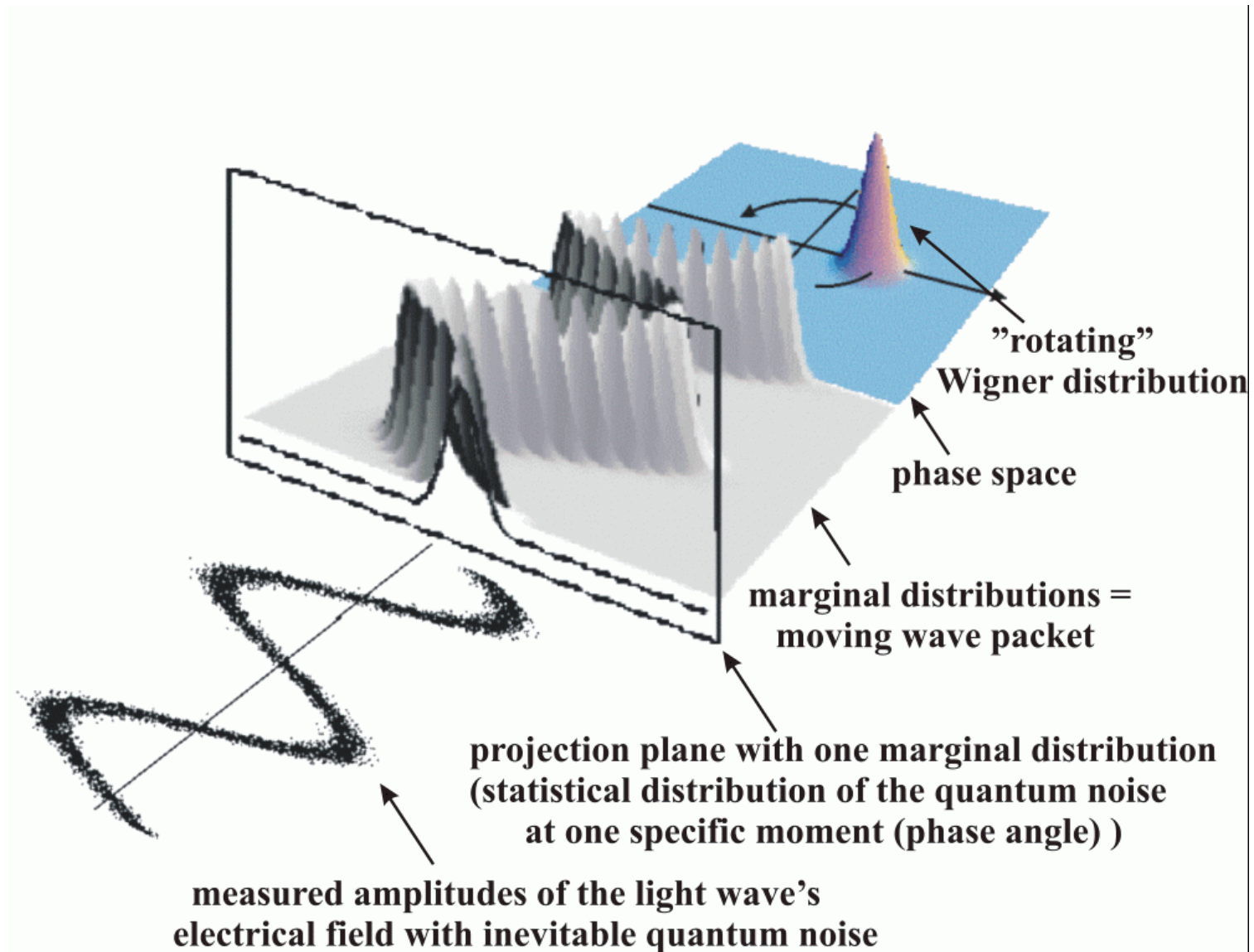


CV QUANTUM NOISE

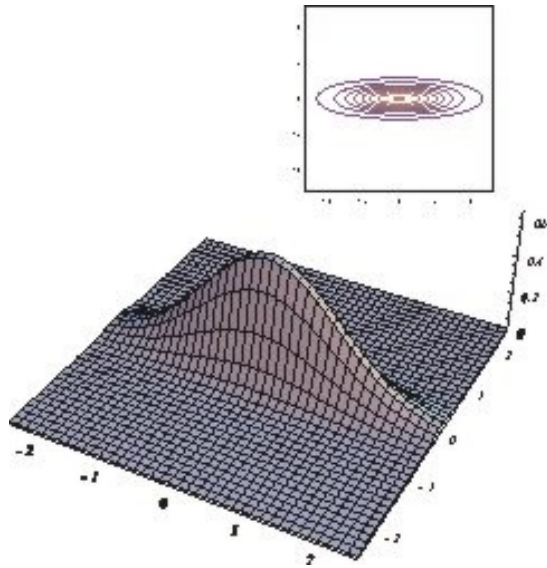


homodyne detection

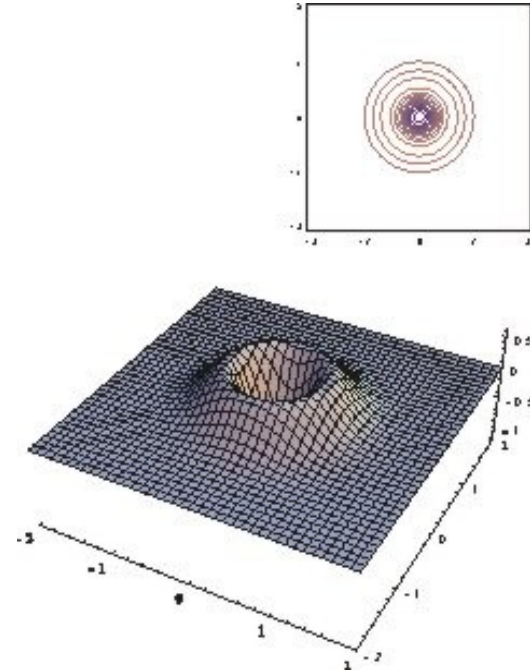
CV QUANTUM NOISE



NONCLASSICAL QUANTUM RESOURCES:



<http://qis.ucalgary.ca/quantech/wiggallery.php>



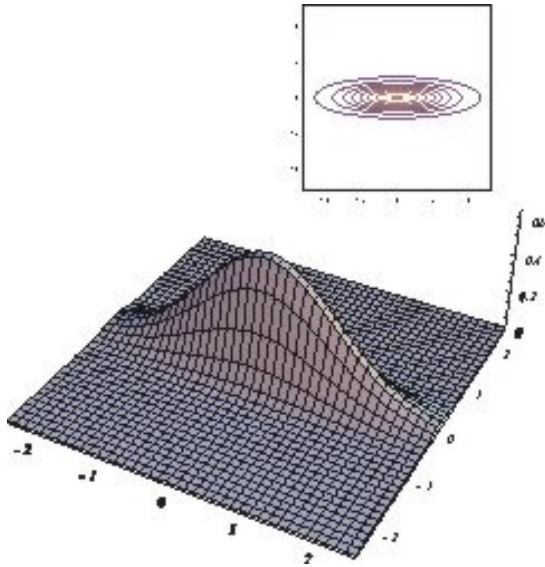
Gaussian squeezed state:

- positive Wigner function
- single quadrature variance below vacuum level

non-Gaussian Fock state:

- negative Wigner function
- photon number variance reduced

NONCLASSICAL QUANTUM RESOURCES:



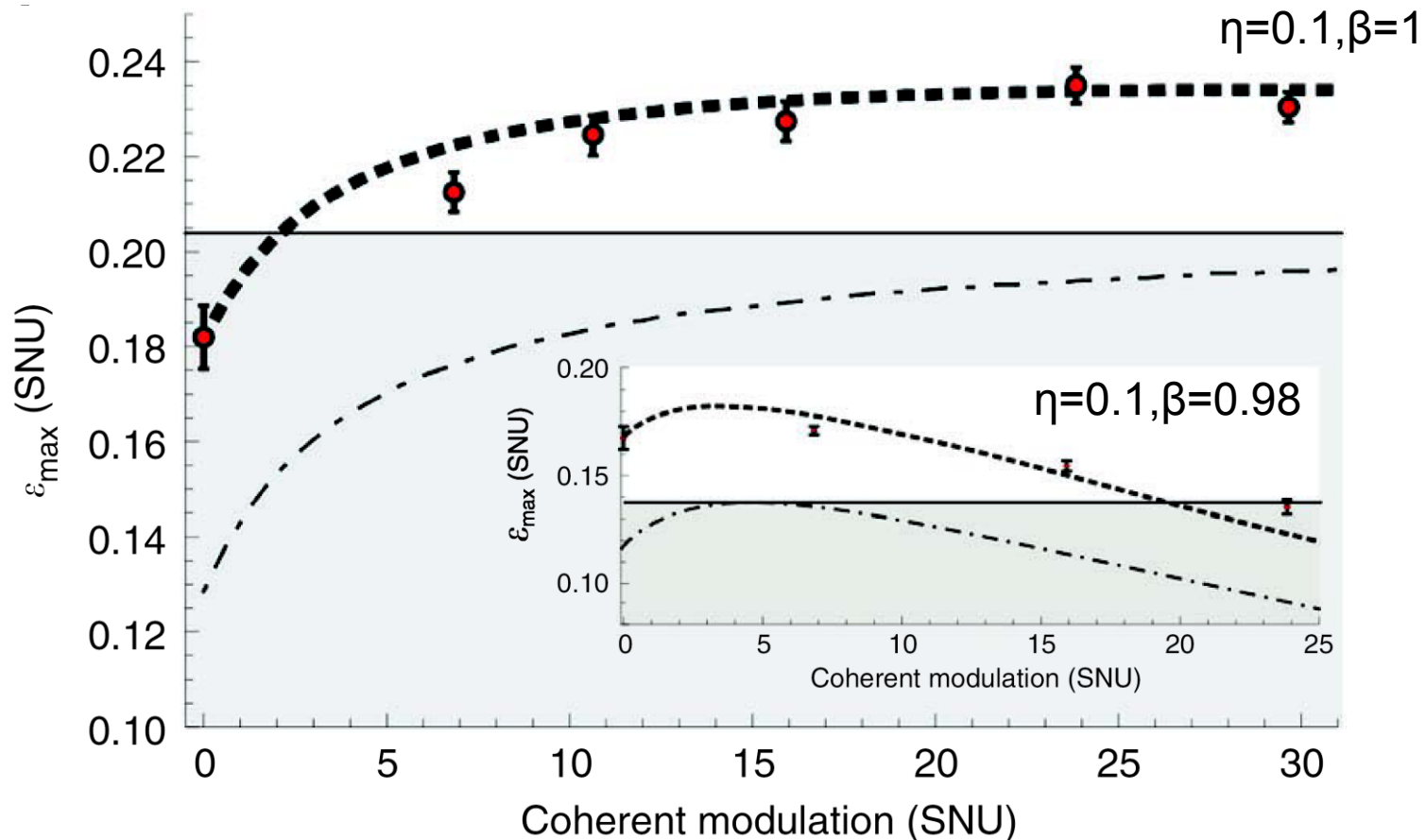
<http://qis.ucalgary.ca/quantech/wiggallery.php>

Gaussian squeezed state:

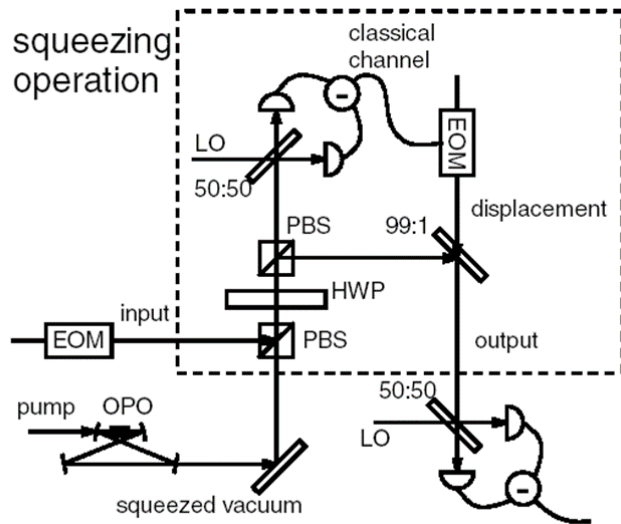
- positive Wigner function
- single quadrature variance below vacuum level

SQUEEZED STATE QKD

Overcoming coherent state protocol: arbitrary weak squeezing is useful to make protocol more robust.



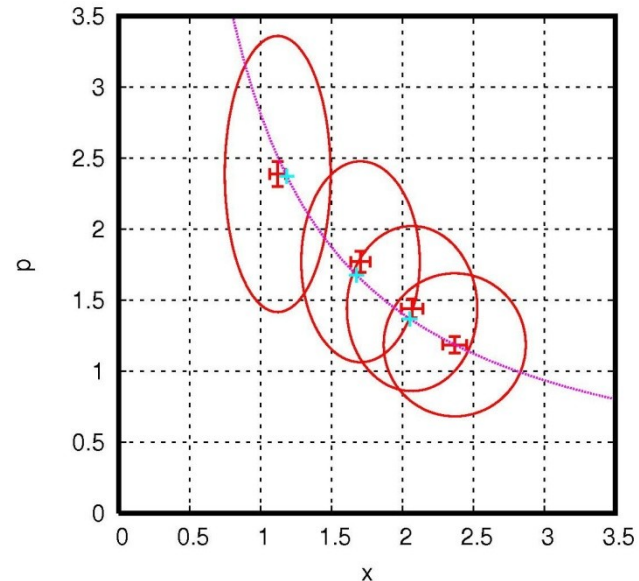
SQUEEZED STATE = RESOURCE FOR SQUEEZER



$$\hat{x}_{\text{out}} = \frac{1}{\sqrt{T}} \hat{x}_{\text{in}},$$

$$\hat{p}_{\text{out}} = \sqrt{T} \hat{p}_{\text{in}} + \sqrt{1-T} \hat{p}_{\text{vac}} e^{-r}$$

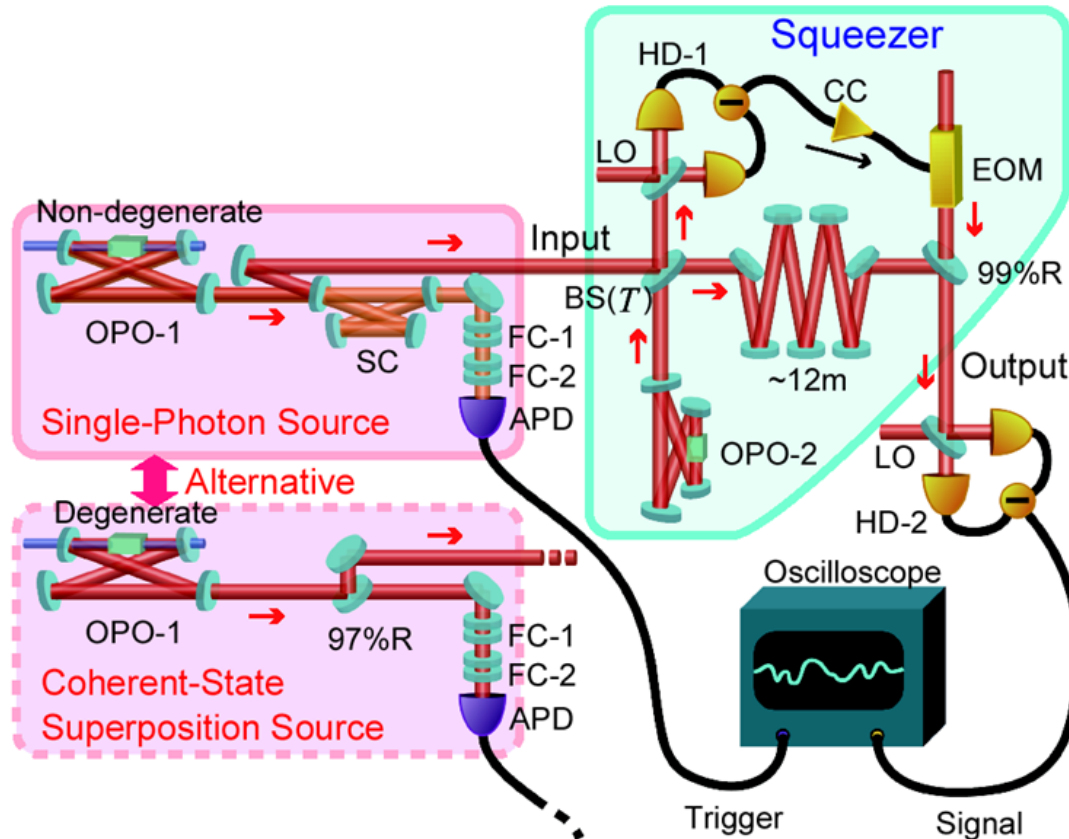
Off-line squeezed state ->
On-line squeezer



R. Filip, P. Marek and U.L. Andersen,
Phys. Rev. A 71, 042308 (2005).

J. Yoshikawa et al., Phys. Rev. A 76,
060301(R) (2007)

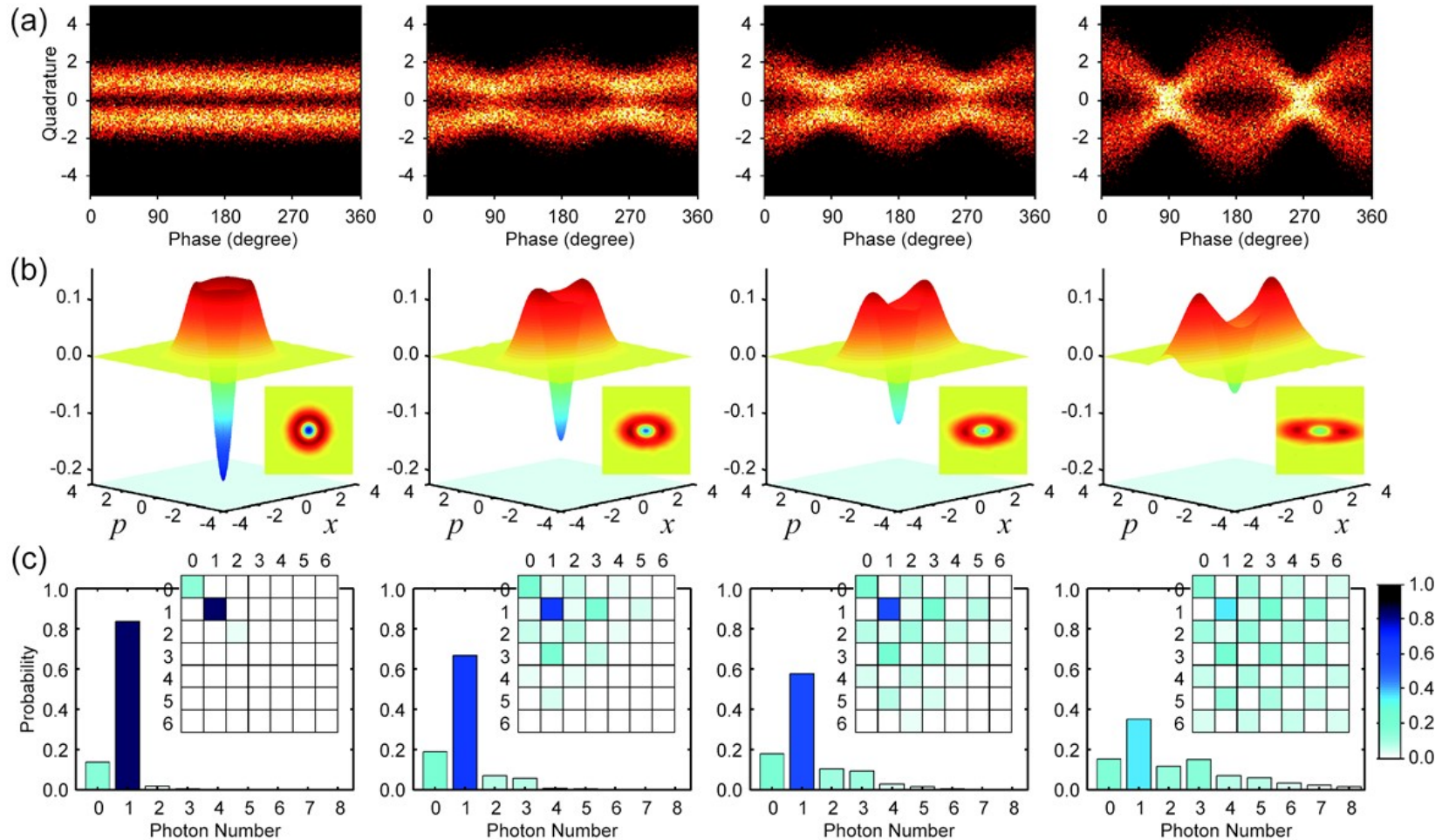
SQUEEZING OF SINGLE PHOTON



$$\hat{S}(\gamma) = e^{\gamma(\hat{a}^{\dagger 2} - \hat{a}^2)/2}$$

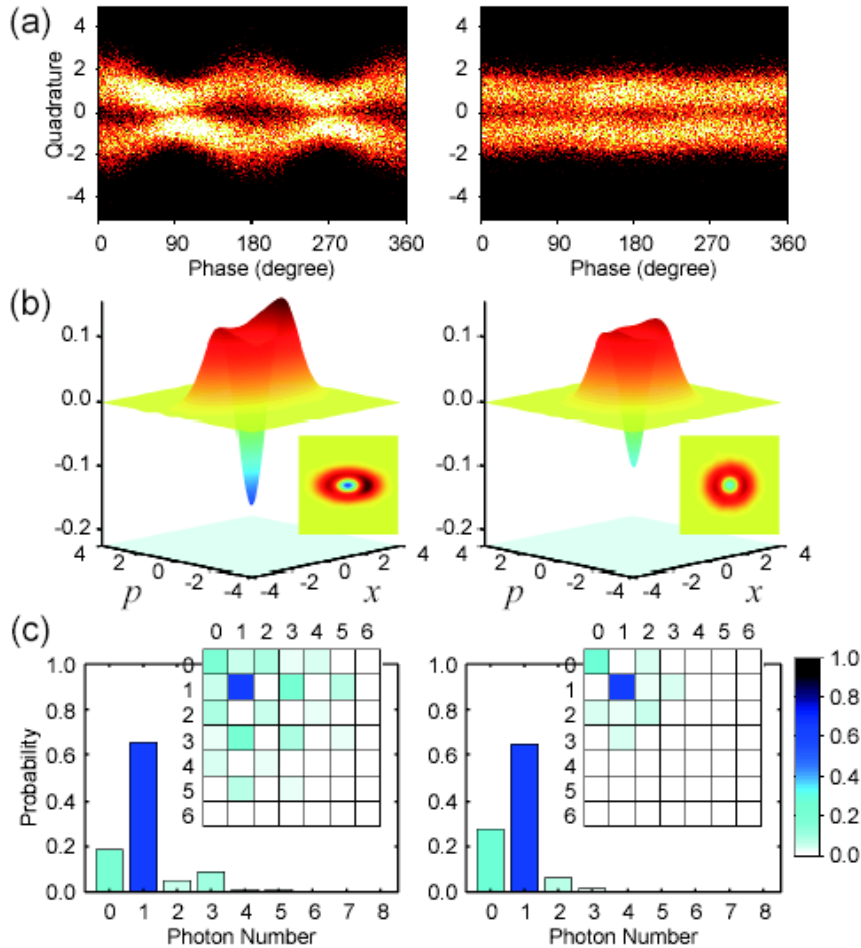
Yoshichika Miwa, Jun-ichi Yoshikawa, Noriaki Iwata, Mamoru Endo, Petr Marek, Radim Filip, Peter van Loock, and Akira Furusawa, Phys. Rev. Lett. 113, 013601 (2014).

SQUEEZING OF SINGLE PHOTON



Yoshichika Miwa, Jun-ichi Yoshikawa, Noriaki Iwata, Mamoru Endo, Petr Marek, Radim Filip, Peter van Loock, and Akira Furusawa, Phys. Rev. Lett. 113, 013601 (2014).

UNSCQUEEZING OF SQUEEZED PHOTON

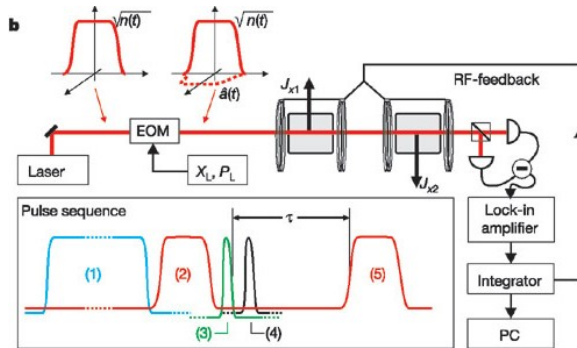


Outcomes:

- reversible squeezer
- preserves negative Wigner function

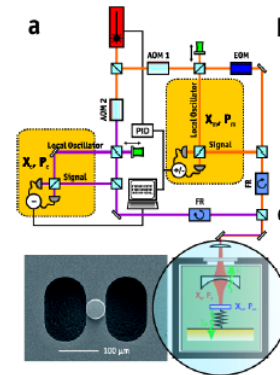
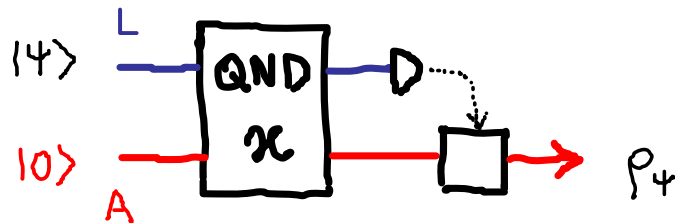
Yoshichika Miwa, Jun-ichi Yoshikawa, Noriaki Iwata, Mamoru Endo, Petr Marek, Radim Filip, Peter van Loock, and Akira Furusawa, Phys. Rev. Lett. 113, 013601 (2014).

QUANTUM INTERFACES



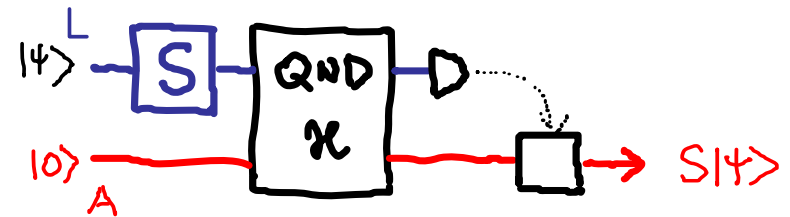
Quantum memory

UNITY GAIN



Quantum opto-mechanics

NON-UNITY GAIN

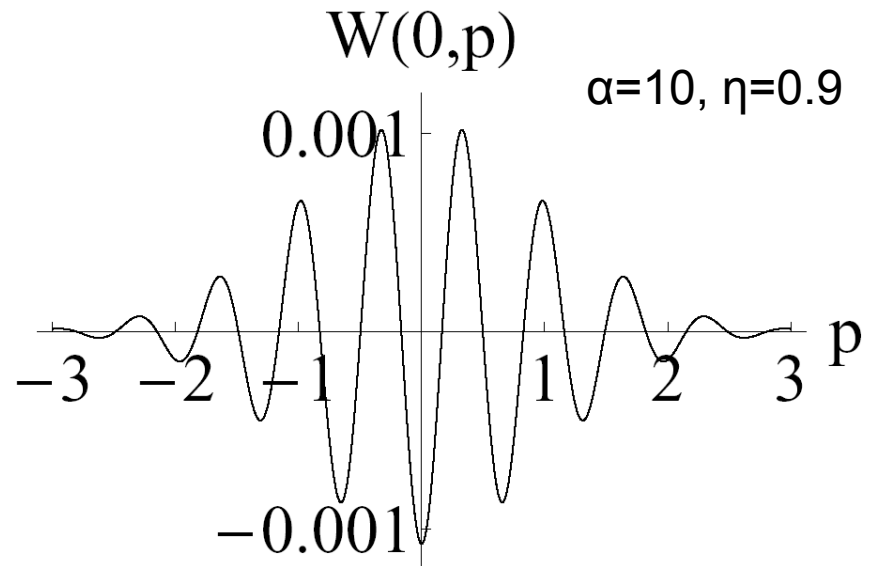
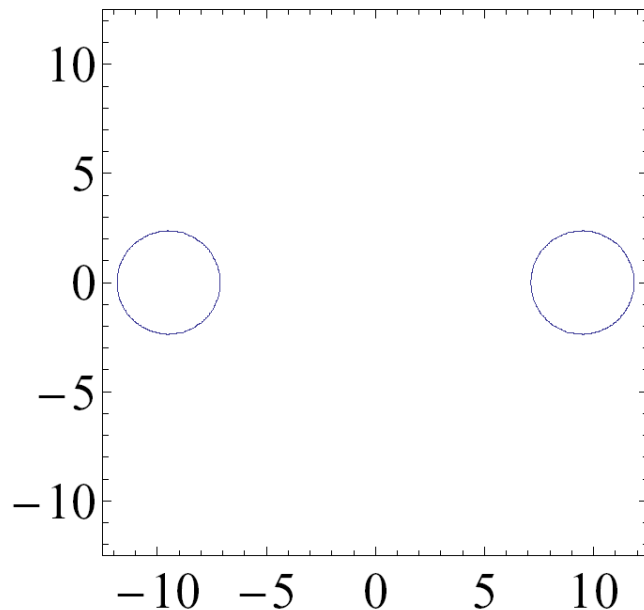


- Pre-squeezing effectively enhances interaction of light with matter.
- Transfer is limited only by optical loss.

QUANTUM DECOHERENCE

$$(|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$

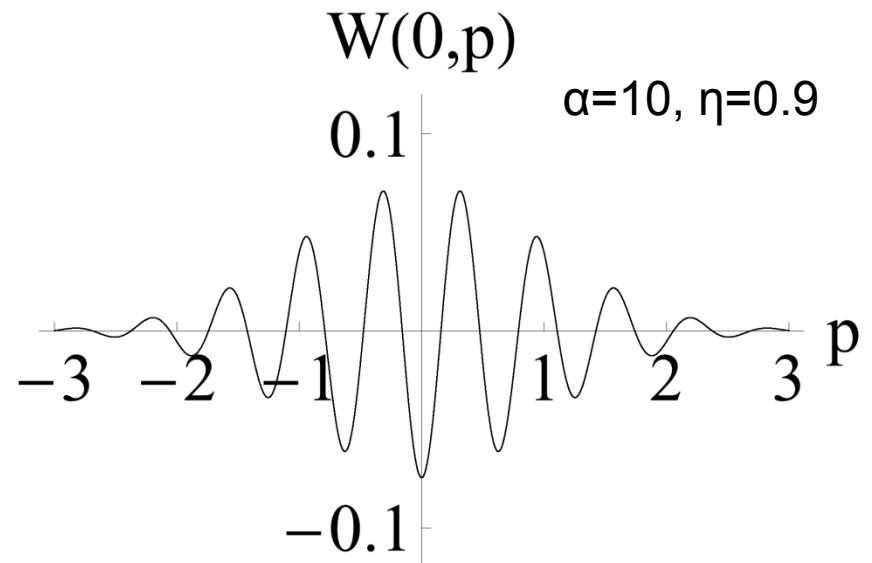
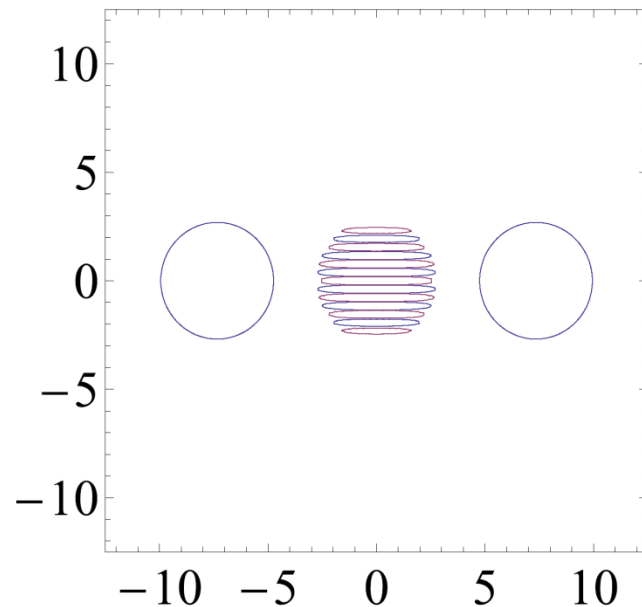
Losses with $\eta > 0.5$ do not vanish the oscillations, but they are hardly visible.



ERROR CORRECTION

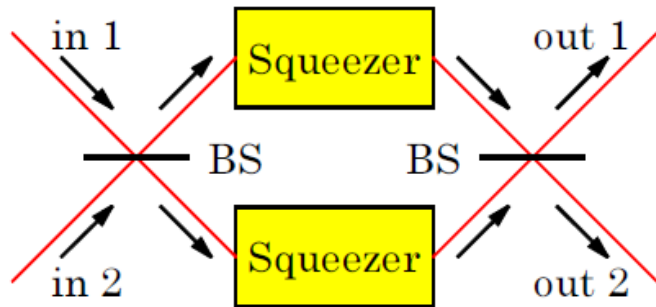
$$(|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$

For $\eta > 0.5$, visibility of the oscillations is significantly improved by **pre/post squeezing**.



QUANTUM GAUSSIAN OPERATIONS

Quantum operations based on online squeezers:



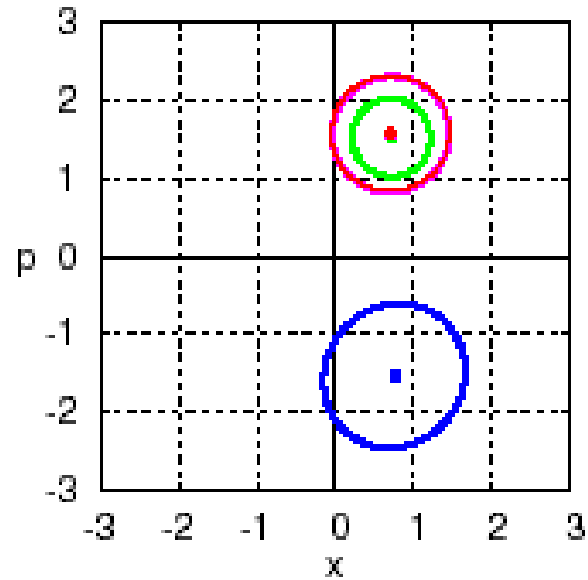
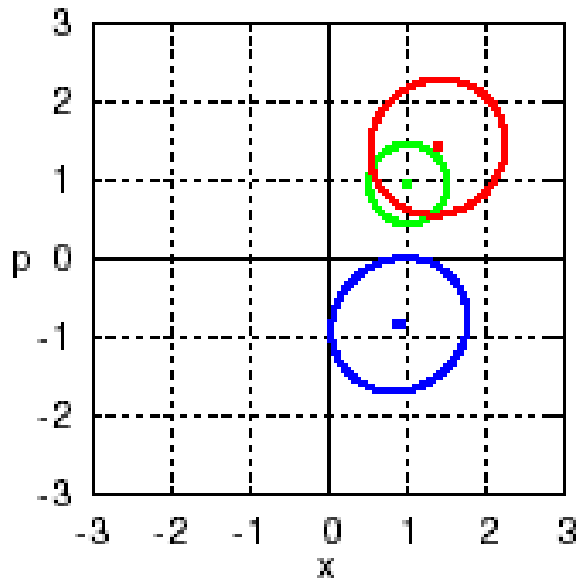
S.L. Braunstein, Phys.Rev. A71, 055801 (2005).



QND interaction: J. Yoshikawa et al., Phys. Rev. Lett. 101, 250501 (2008).

Phase-insensitive amplifier: J. Yoshikawa, Y. Miwa, R. Filip, A. Furusawa, Phys. Rev. A 83, 052307 (2011).

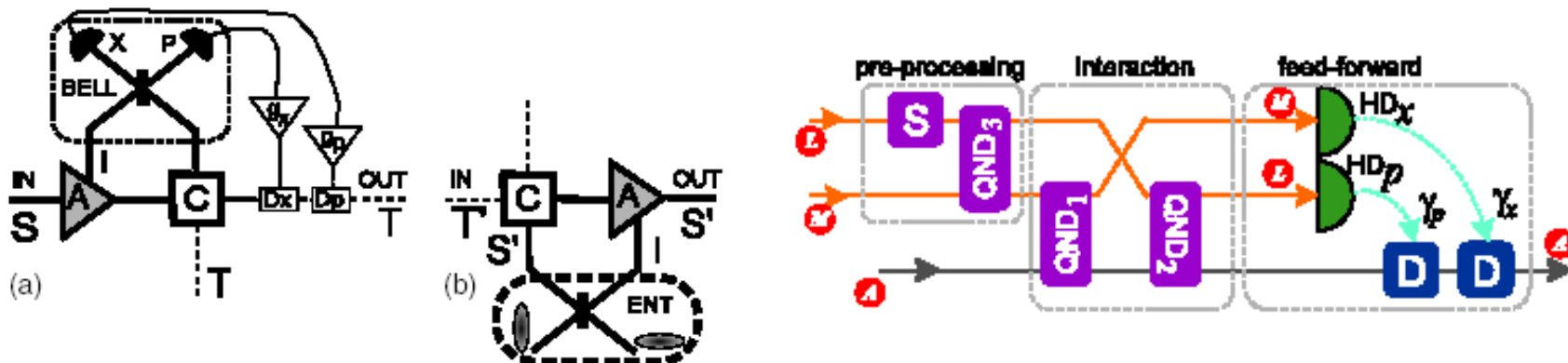
REVERSIBLE QUANTUM AMPLIFIER AND CLONER



R. Filip, J. Fiurasek , P. Marek,
PRA 69 , 012314 (2004).

J. Yoshikawa, Y. Miwa, R. Filip, A. Furusawa, *Demonstration of reversible phase-insensitive optical amplifier*, Phys. Rev. A 83, 052307 (2011)

QUANTUM INTERFACES BETWEEN LIGHT AND MATTER

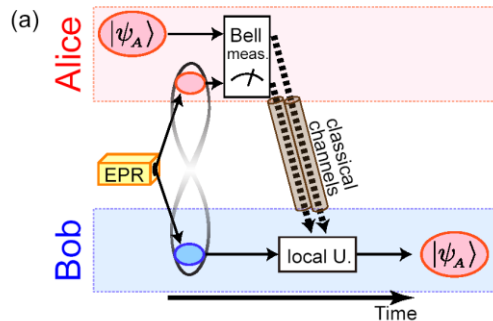


R. Filip, Phys. Rev. A 80, 022304 (2009); P. Marek and R. Filip, Phys. Rev. A 81, 042325 (2010).

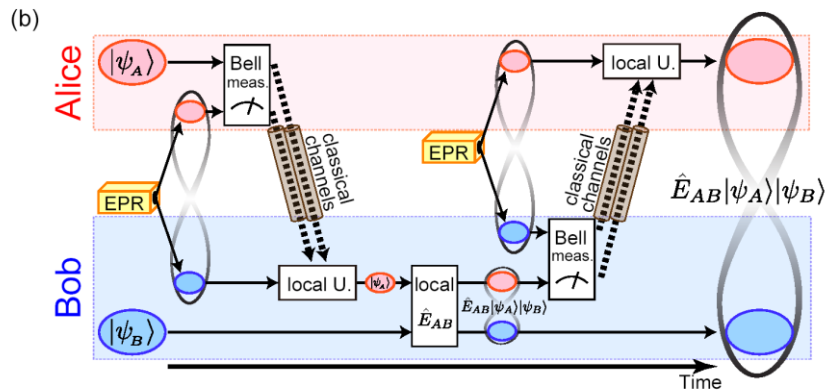
- Quantum pre-amplification and feed-forward control **perfectly transfer** any quantum state to **noisy** system through arbitrarily **weak** coupling.
- **Full quantum linear amplifier and QND interaction are useful tool for quantum pre-processing!**

NONLOCAL QND OPERATION

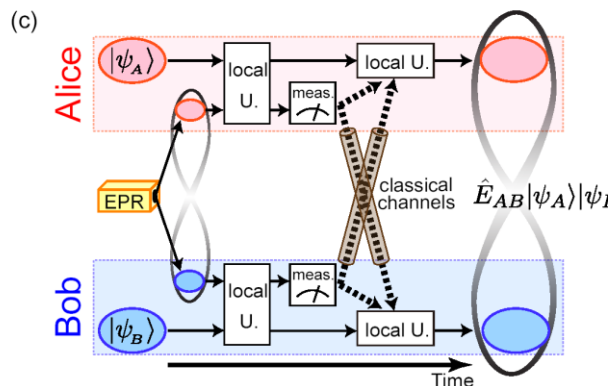
teleportation



double teleportation



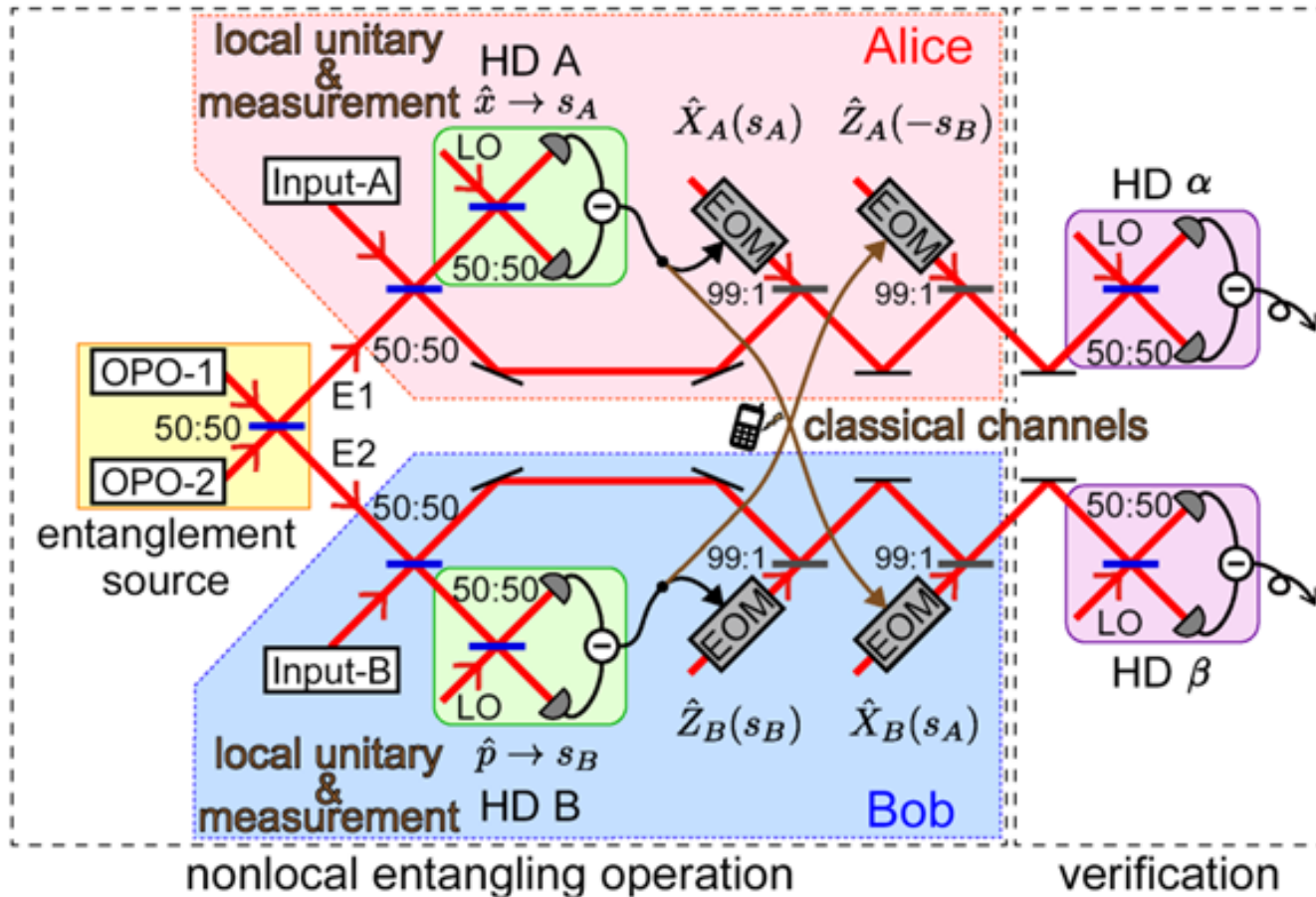
nonlocal parallel operation



$$\hat{\Sigma}_{AB} = e^{-2i\hat{x}_A\hat{p}_B}$$

$$\hat{\Sigma}_{AB}|x_A\rangle_A \otimes |x_B\rangle_B = |x_A\rangle_A \otimes |x_B + x_A\rangle_B$$

NONLOCAL QND OPERATION



Shota Yokoyama, Ryuji Ukai, Jun-ichi Yoshikawa, Petr Marek, Radim Filip, and Akira Furusawa, Phys. Rev. A 90, 012311 (2014).

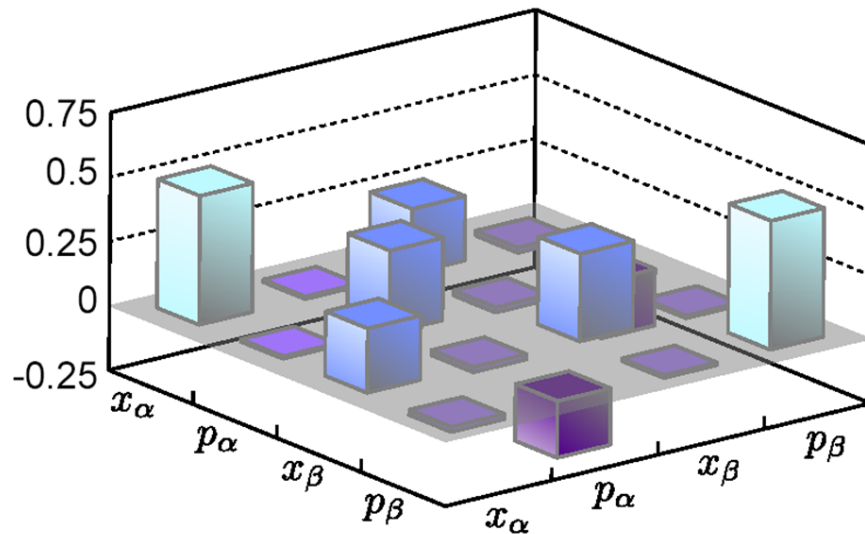
NONLOCAL QND OPERATION

$$\hat{\xi}_{\alpha\beta} = \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 1 & \sqrt{2} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix} \hat{\xi}_{AB} + \hat{\delta}$$

$$\equiv \hat{E}_{AB}^\dagger \hat{\xi}_{AB} \hat{E}_{AB} + \hat{\delta},$$

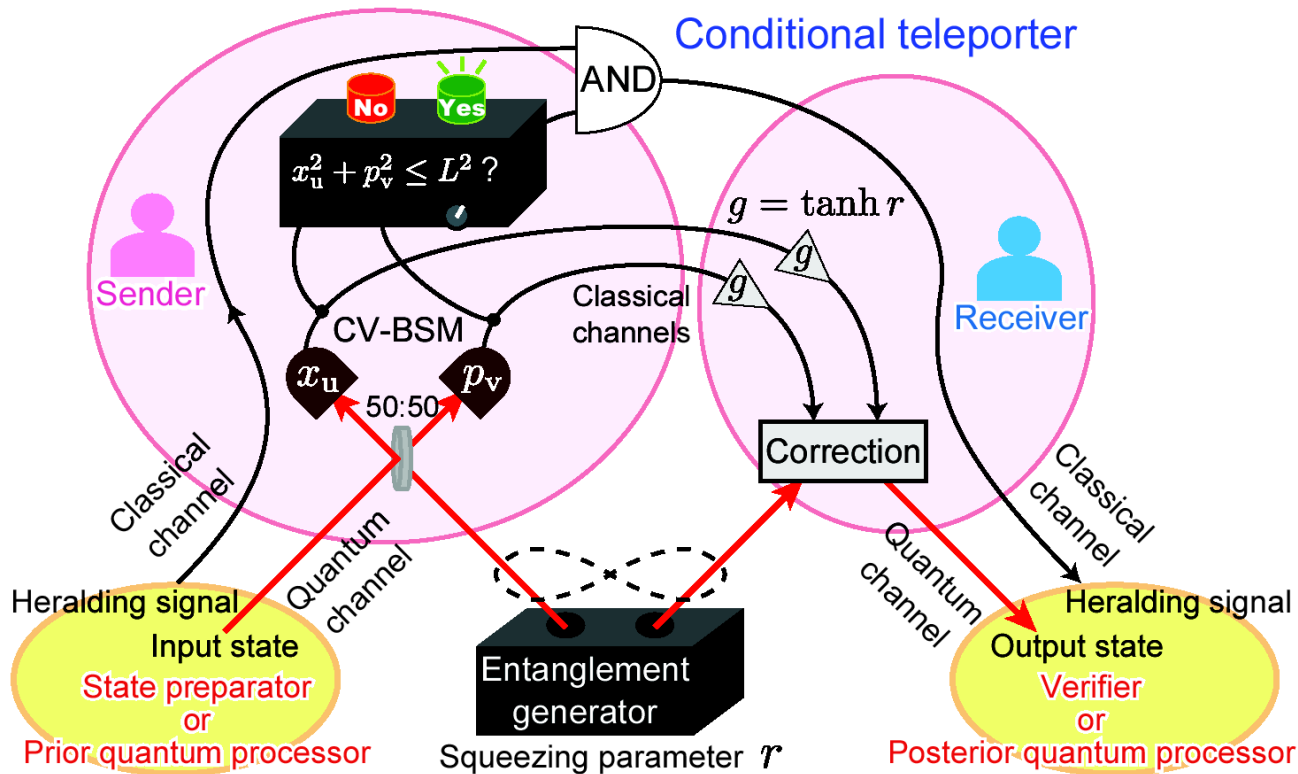
$$\hat{\xi}_{AB} = (\hat{x}_A, \hat{p}_A, \hat{x}_B, \hat{p}_B)^T, \quad \hat{\xi}_{\alpha\beta} = (\hat{x}_\alpha, \hat{p}_\alpha, \hat{x}_\beta, \hat{p}_\beta)^T$$

$$\hat{\delta} = (0, e^{-r} \hat{p}_2^{(0)}, e^{-r} \hat{x}_1^{(0)}, 0)^T$$



QND entanglement!

CONDITIONAL TRANSFER OF SINGLE PHOTON



T. Ide, H. F. Hofmann, T. Kobayashi, and A. Furusawa, Phys. Rev. A 65, 012313 (2002).

Ladislav Mišta, Jr., Radim Filip, and Akira Furusawa, Phys. Rev. A 82, 012322 (2010)

LOSSY TELEPORTATION OF SINGLE PHOTON

$$|\psi\rangle \rightarrow \sum_{k=0}^{\infty} \frac{1}{k! \sinh^{2k} r} \hat{a}^k [(\tanh r)^{\hat{n}} |\psi\rangle \langle\psi| (\tanh r)^{\hat{n}}] \hat{a}^{\dagger k}$$

NOISELESS TELEPORTATION OF SINGLE PHOTON

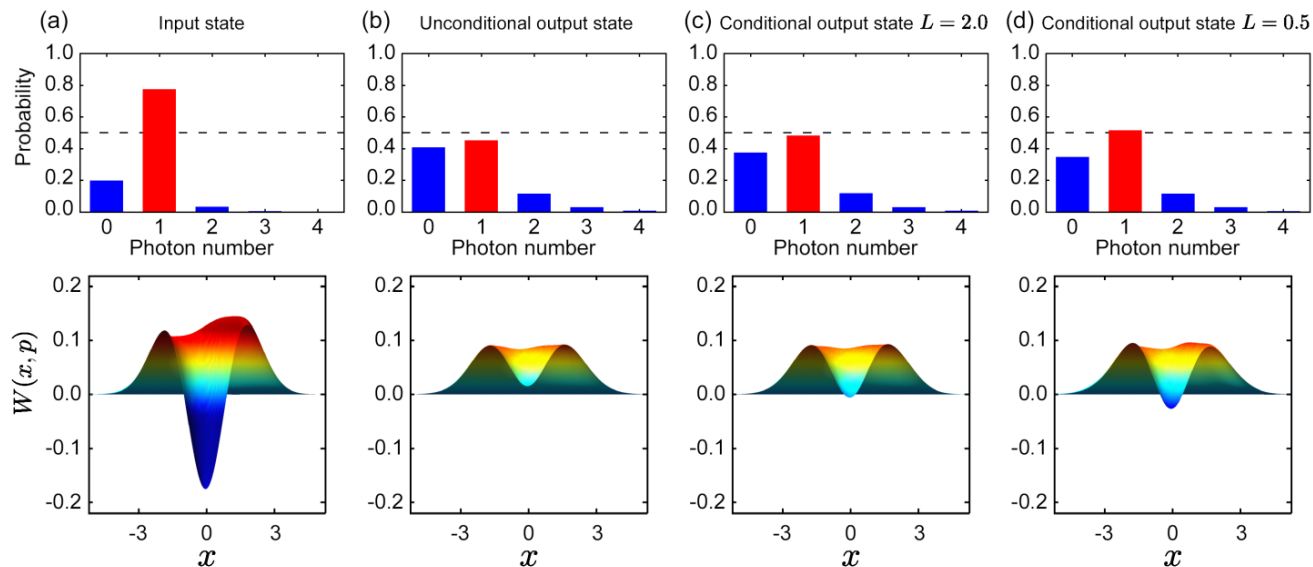
$$|\psi\rangle \rightarrow \sum_{k=0}^{\infty} \frac{1}{\sqrt{2^k}} \left[(\tanh r)^{\hat{n}} |\psi\rangle \langle\psi| (\tanh r)^{\hat{n}} \right] \otimes |0\rangle$$

$$|\psi\rangle \rightarrow (\tanh r)^{\hat{n}} |\psi\rangle$$

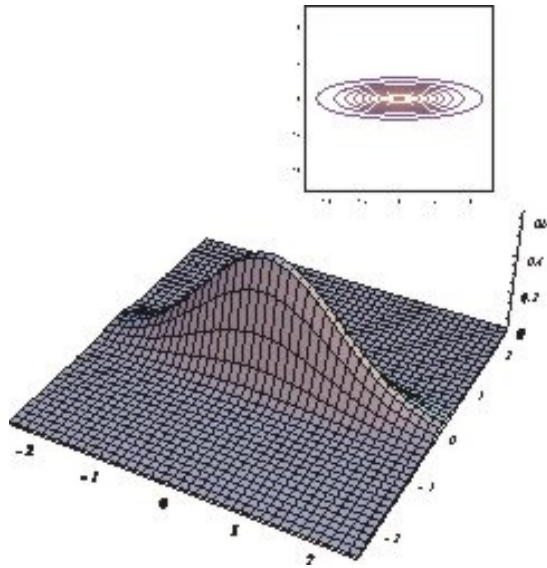
NOISELESS TELEPORTATION OF SINGLE PHOTON

$$|\psi\rangle \rightarrow \sum_{k=0}^{\infty} \frac{1}{\sqrt{k!}} \left[(\tanh r)^{\hat{n}} |\psi\rangle \langle\psi| (\tanh r)^{\hat{n}} \right] \frac{1}{\sqrt{k!}}$$

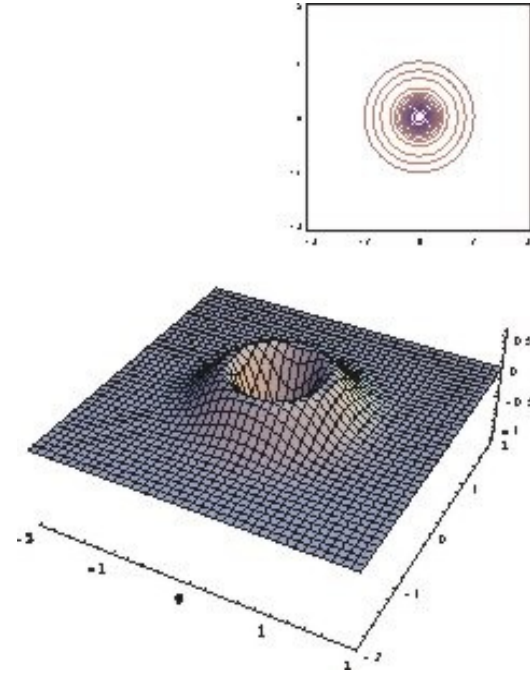
$$|\psi\rangle \rightarrow (\tanh r)^{\hat{n}} |\psi\rangle$$



NONCLASSICAL QUANTUM RESOURCES:



<http://qis.ucalgary.ca/quantech/wiggallery.php>



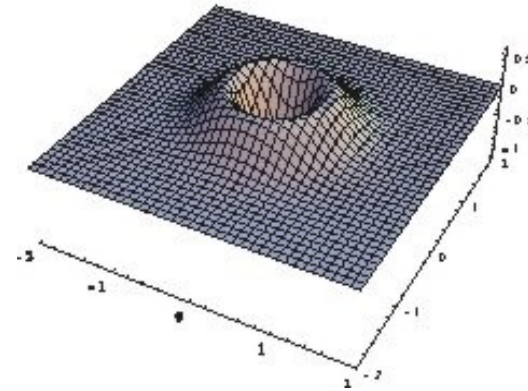
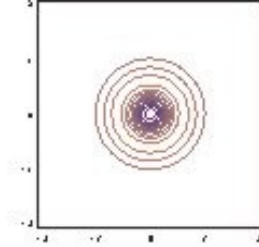
Gaussian squeezed state:

- positive Wigner function
- single quadrature variance below vacuum level

non-Gaussian Fock state:

- negative Wigner function
- all quadrature variance above vacuum level

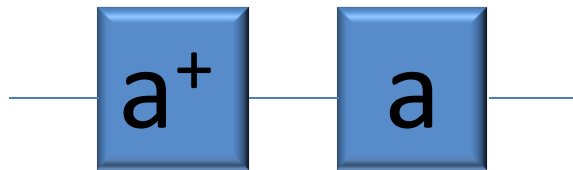
NONCLASSICAL QUANTUM RESOURCES:



non-Gaussian Fock state:

- negative Wigner function
- all quadrature variance above vacuum level

NOISELESS AMPLIFIER BY aa^\dagger



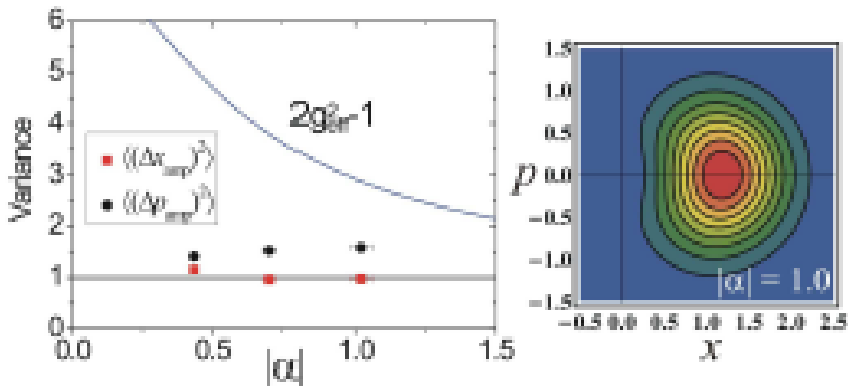
$$|\alpha\rangle = |0\rangle + \alpha|1\rangle + \dots$$

$$a^\dagger |\alpha\rangle = |1\rangle + 2^{1/2} \alpha |2\rangle + \dots$$

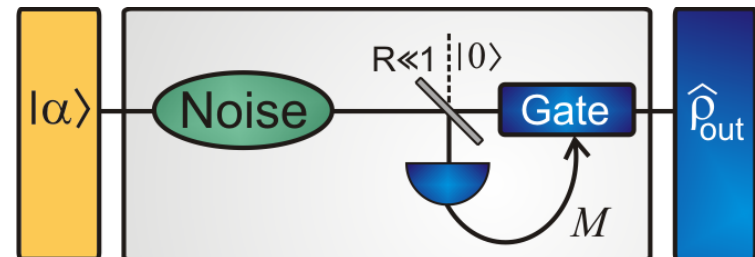
$$aa^\dagger |\alpha\rangle = |0\rangle + 2\alpha |1\rangle + \dots$$

P. Marek and R. Filip, Phys. Rev. A 81, 022302 (2010).

A. Zavatta, J. Fiurášek, M. Bellini, Nature Phot. 5, 52 (2011)

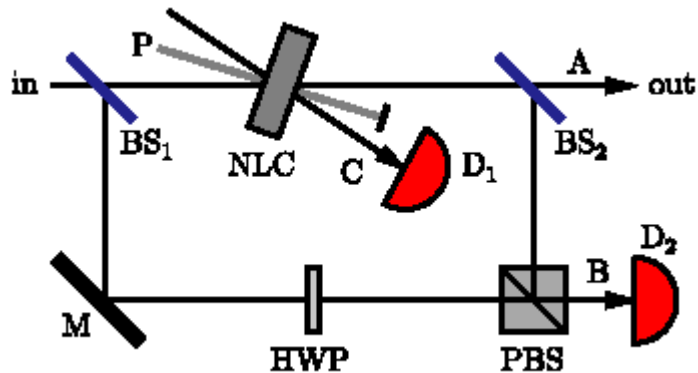


M.A. Usuga, Ch. R. Müller, Ch. Wittmann, P. Marek, R. Filip, Ch. Marquardt, G. Leuchs, U.L. Andersen, Nature Phys. 6, 767–771 (2010)



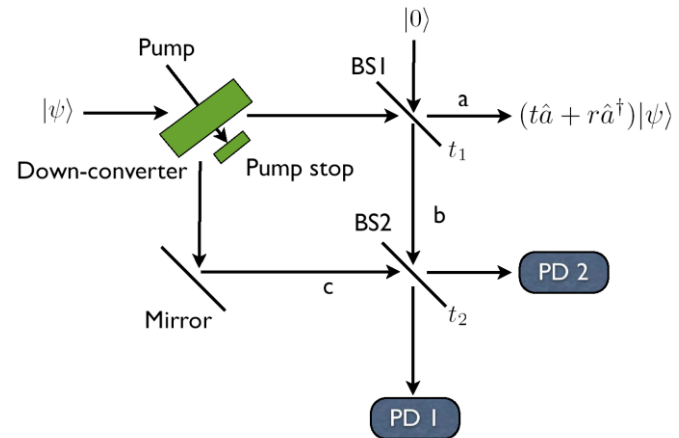
SUPERPOSITION of a, a^\dagger

Probabilistic Kerr effect



J. Fiurášek, Phys. Rev. A 80, 053822 (2009).

Implementation of $(a+a^\dagger)|in\rangle$



Su-Yong Lee, Hyunchul Nha, Phys. Rev. A 82, 053812 (2010)

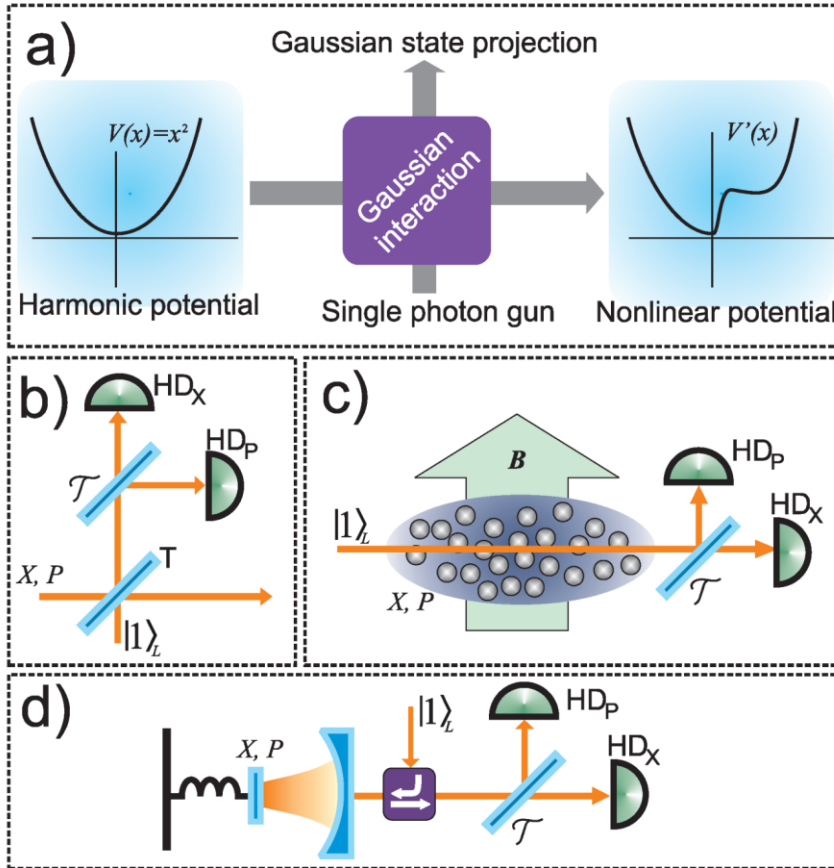
NONLINEAR EFFECTS



$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + V(\hat{X})$$

$$U(\hat{X}, \tau) = e^{-\frac{i}{\hbar} V(\hat{X})\tau}$$

CONDITIONAL X-gate



$$U(\hat{X}, \tau) = \sum_{k=0}^{\infty} \frac{U^{(k)}(\bar{X})}{k!} (\hat{X} - \bar{X})^k$$

$$U(\hat{X}, \tau) = \prod_{k=0}^N (1 + \lambda_k \hat{X})$$

b) Linear optical implementation

$$\hat{A}_{BS} = (T'T)\hat{n}(A^* + 2B^*R^*\hat{a} + R\hat{a}^\dagger)$$

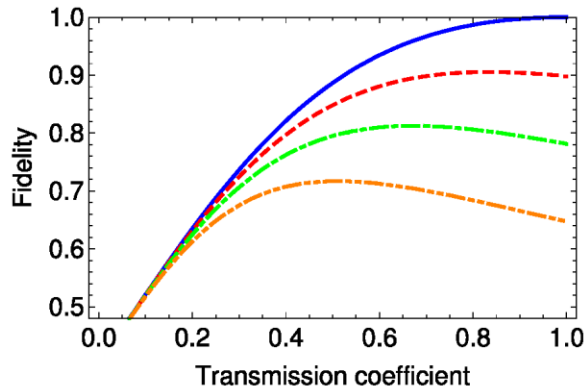
$$A = \sqrt{2}(x\mathcal{T} - ip\mathcal{R}) \quad B = 2^{-1}(\mathcal{R}^2 - \mathcal{T}^2)$$

c) Implementation with atomic ensemble

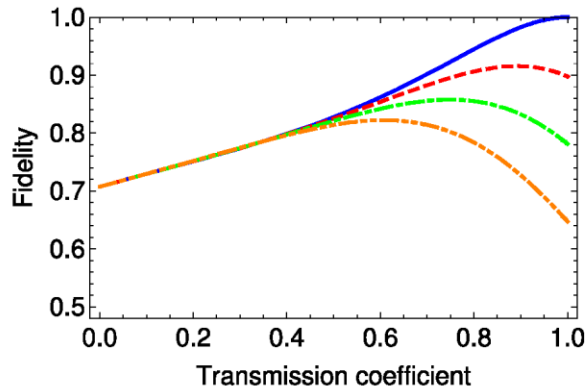
d) opto-mechanical implementation

$$\begin{aligned} {}_L \langle x_0 = 0 | U_{\text{QND}} f(\hat{X}_L) | 0 \rangle_L &= {}_L \langle x_0 = 0 | f(-\kappa \hat{X}) U_{\text{QND}} | 0 \rangle \\ &= f(-\kappa \hat{X})_L \langle x_0 = 0 | U_{\text{QND}} | 0 \rangle = F(\hat{X}) \exp\left[-\frac{1}{2} \kappa^2 \hat{X}^2\right]. \end{aligned}$$

QUALITY OF SINGLE PHOTON

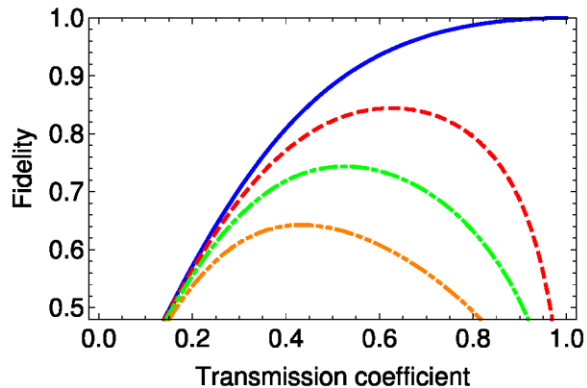


(a) $|\beta = 0.1\rangle$

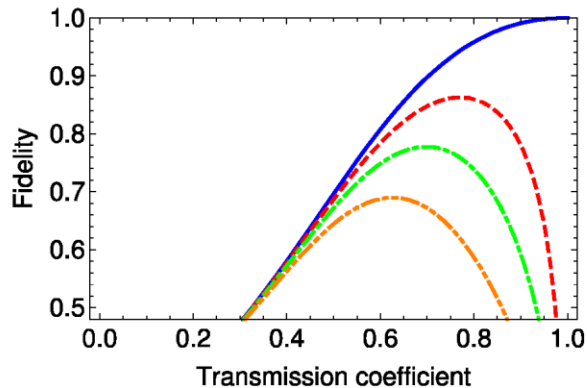


(b) $|\beta = 1\rangle$

$p(1)=1$
 $p(1)=0.8$
 $p(1)=0.6$
 $p(1)=0.4$

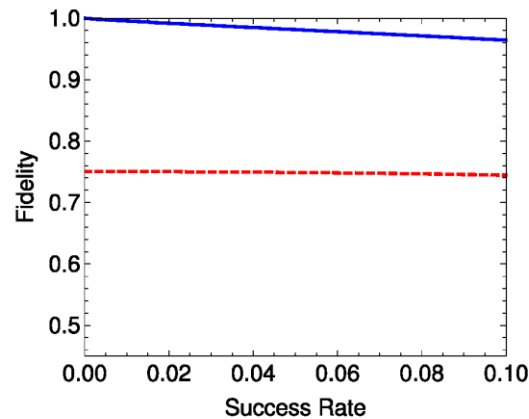


(c) $|\xi = 0.1\rangle$

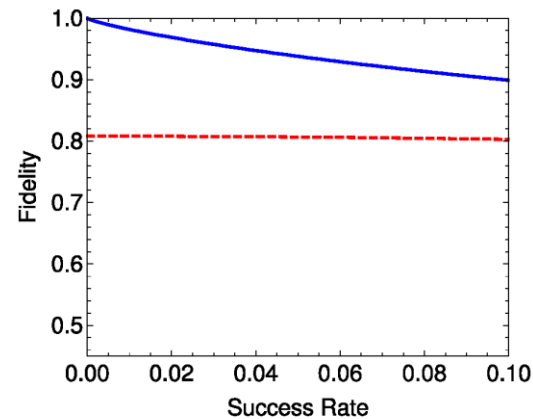


(d) $|1\rangle$

FIDELITY VERSUS SUCCESS



(a) Single photon input $|1\rangle$



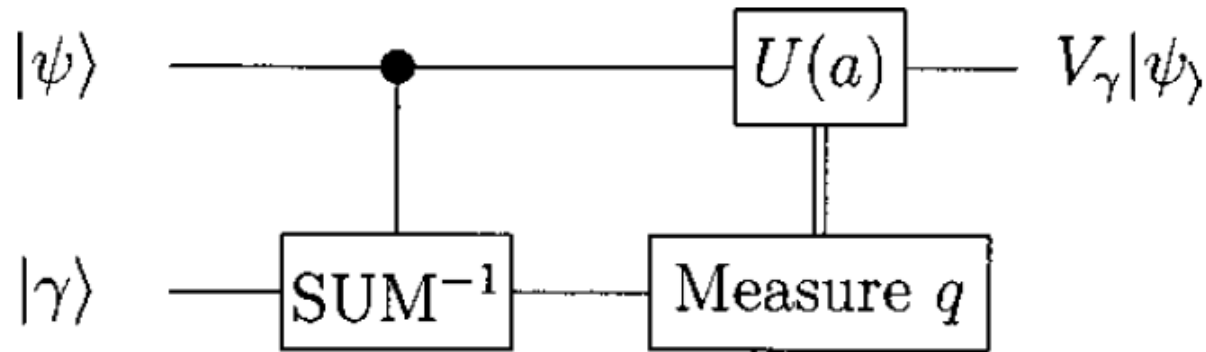
(b) Coherent state input $|\beta = 1\rangle$

EXAMPLE: CUBIC NONLINEARITY

$$\begin{aligned} \exp[i\chi\hat{X}^3] &\approx 1 + i\chi\hat{X}^3 - \frac{\chi^2}{2}\hat{X}^6 \propto \\ &(1 - (\frac{\chi}{-1+i})^{1/3}\hat{X})(1 + (\frac{\chi}{1-i})^{1/3}\hat{X}) \\ &(1 - (-1)^{-2/3}(\frac{\chi}{-1+i})^{1/3}\hat{X})(1 - (\frac{\chi}{1+i})^{1/3}\hat{X}) \\ &(1 + (\frac{\chi}{-1-i})^{1/3}\hat{X})(1 - (-1)^{-2/3}(\frac{\chi}{1+i})^{1/3}\hat{X}) \end{aligned}$$

CUBIC NONLINEARITY = KEY TO NONLINEAR WORLD

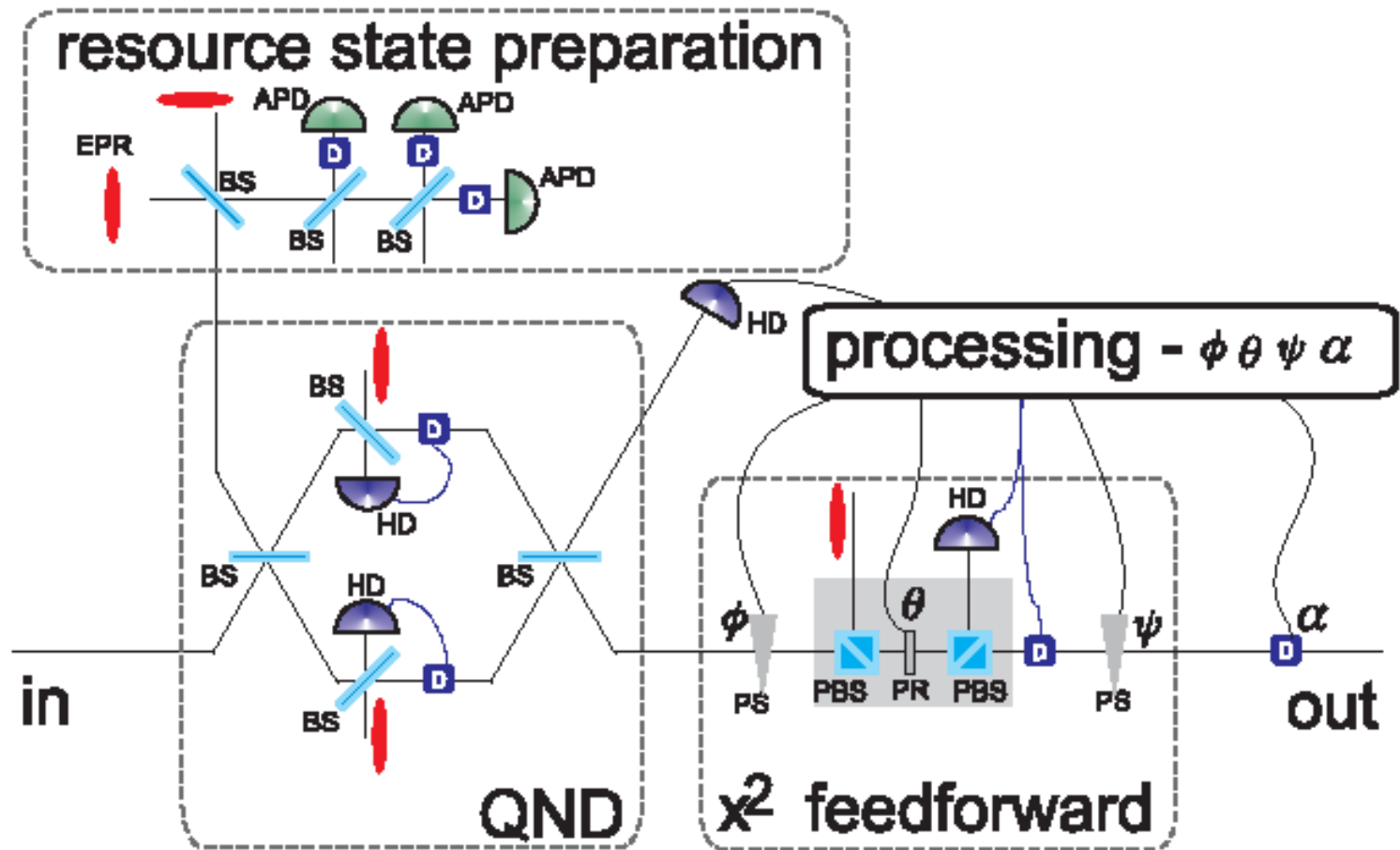
$$\hat{H}_3 = \omega_3 \hat{x}^3 \quad |\gamma\rangle = \int e^{i\chi x^3} |x\rangle dx$$



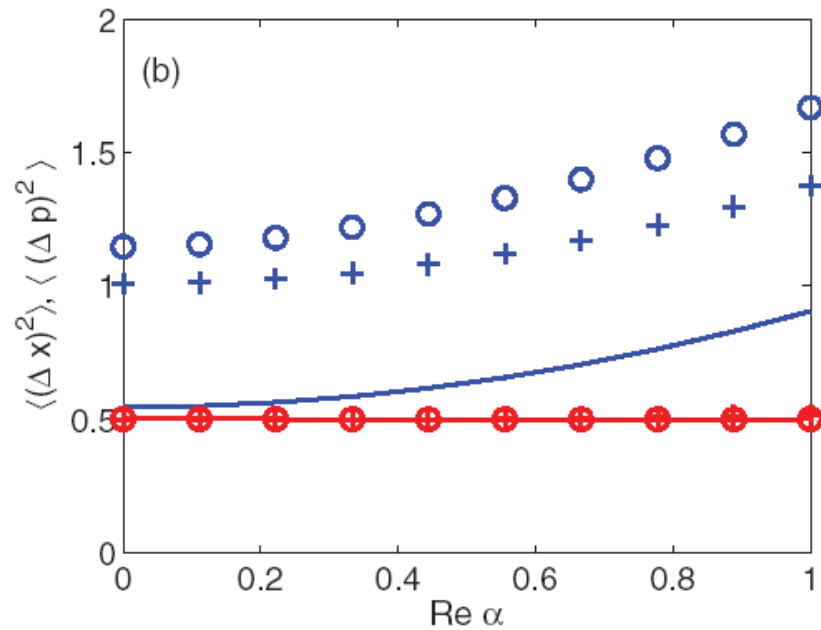
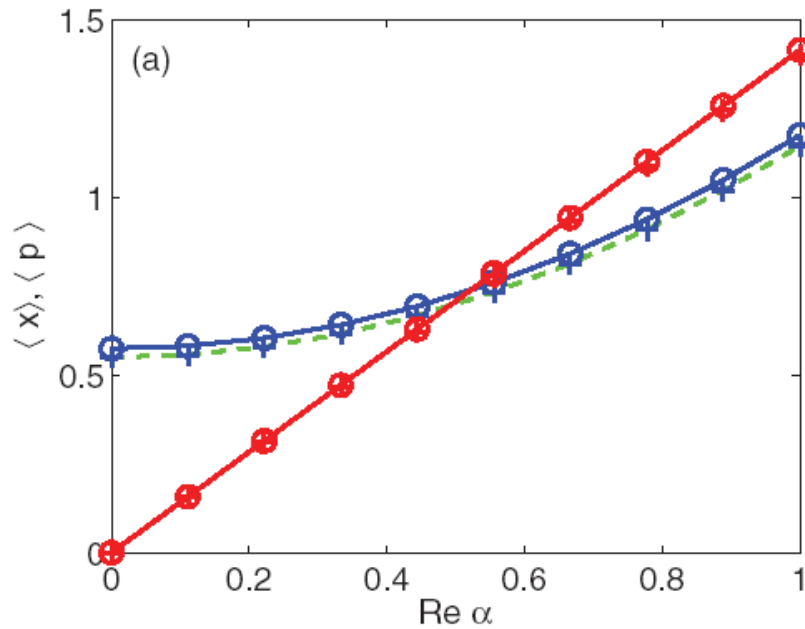
[Gottesman and Preskill, PRA 64
012310 (2001)]

We have QND gate, we need cubic state, X^2 feed-forward correction techniques and then do it.

FEASIBLE CUBIC INTERACTION



X^3 NONLINEAR EFFECT



P. Marek, R. Filip and A. Furusawa, Phys. Rev. A 84, 053802 (2011).

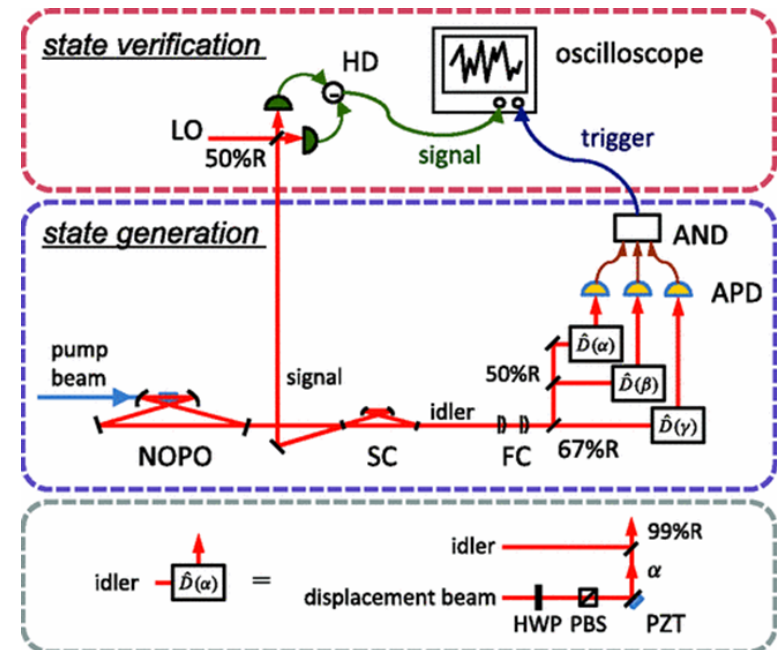
CUBIC (X^3) STATE

simplest approximation:

$$e^{i\chi\hat{x}^3} \hat{S}|0\rangle$$

$$(1 + i\chi\hat{x}^3) \hat{S}|0\rangle$$

$$\hat{S} \left(|0\rangle + \chi' \frac{3}{2\sqrt{2}} |1\rangle + \chi' \frac{\sqrt{3}}{2} |3\rangle \right)$$



P. Marek, R. Filip, and A. Furusawa,
Phys. Rev. A 84, 053802 (2011).

Mitsuyoshi Yukawa, Kazunori Miyata,
Hidehiro Yonezawa, Petr Marek, Radim
Filip, and Akira Furusawa,
Phys. Rev. A 88, 053816 (2013).

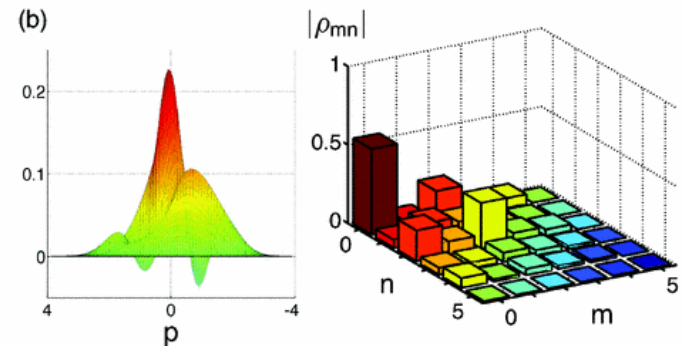
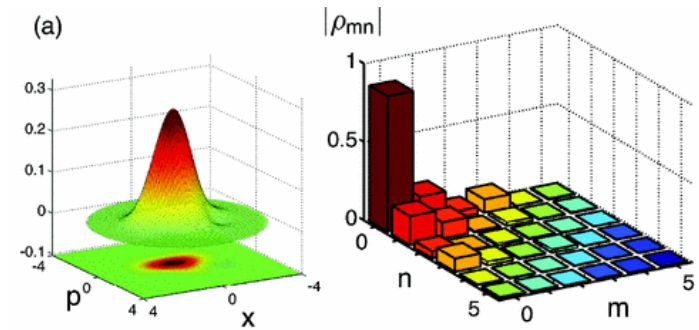
CUBIC (X^3) STATE

simplest approximation:

$$e^{i\chi\hat{x}^3} \hat{S}|0\rangle$$

$$(1 + i\chi\hat{x}^3) \hat{S}|0\rangle$$

$$\hat{S} \left(|0\rangle + \chi' \frac{3}{2\sqrt{2}} |1\rangle + \chi' \frac{\sqrt{3}}{2} |3\rangle \right)$$



P. Marek, R. Filip, and A. Furusawa,
Phys. Rev. A 84, 053802 (2011).

Mitsuyoshi Yukawa, Kazunori Miyata,
Hidehiro Yonezawa, Petr Marek, Radim
Filip, and Akira Furusawa,
Phys. Rev. A 88, 053816 (2013).

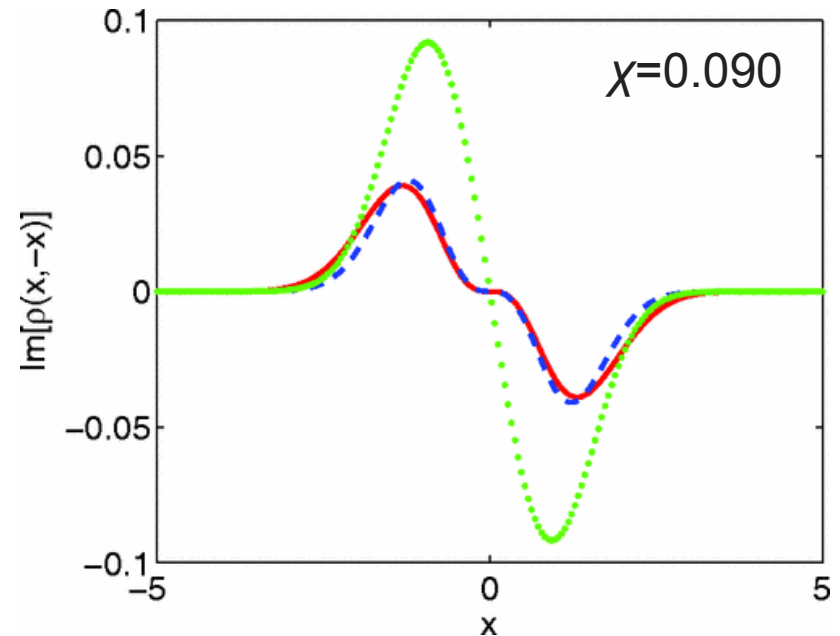
CUBIC (X^3) STATE

simplest approximation:

$$e^{i\chi\hat{x}^3} \hat{S}|0\rangle$$

$$(1 + i\chi\hat{x}^3) \hat{S}|0\rangle$$

$$\hat{S} \left(|0\rangle + \chi' \frac{3}{2\sqrt{2}} |1\rangle + \chi' \frac{\sqrt{3}}{2} |3\rangle \right)$$



P. Marek, R. Filip, and A. Furusawa,
Phys. Rev. A 84, 053802 (2011).

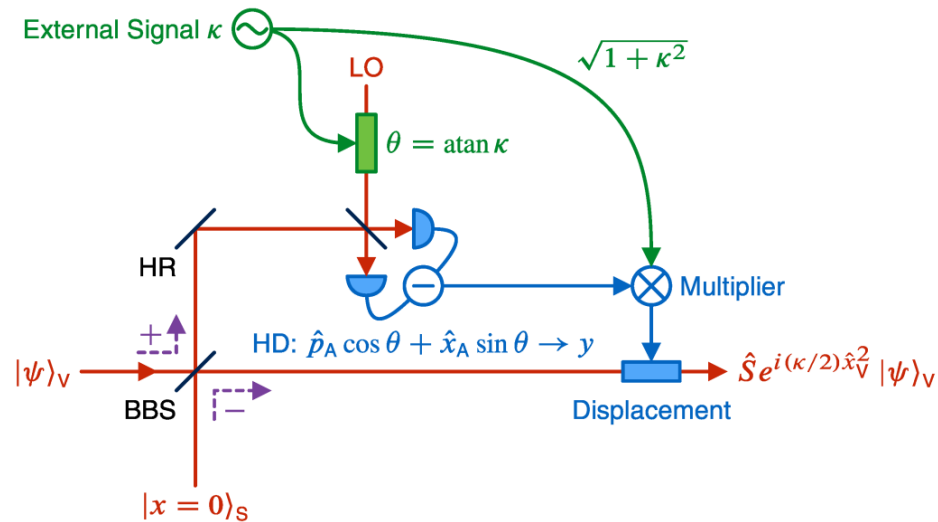
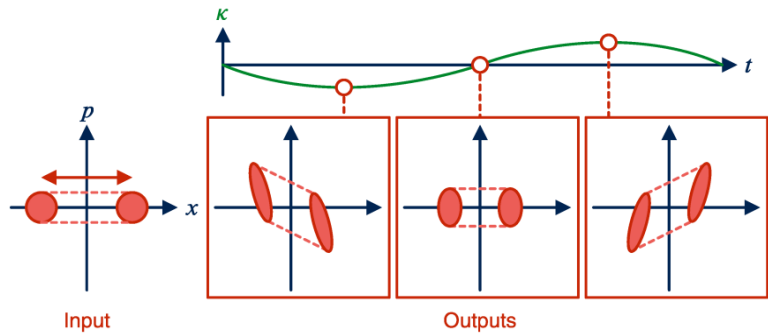
Mitsuyoshi Yukawa, Kazunori Miyata,
Hidehiro Yonezawa, Petr Marek, Radim
Filip, and Akira Furusawa,
Phys. Rev. A 88, 053816 (2013).

QUADRATIC (X^2) FEEDFORWARD

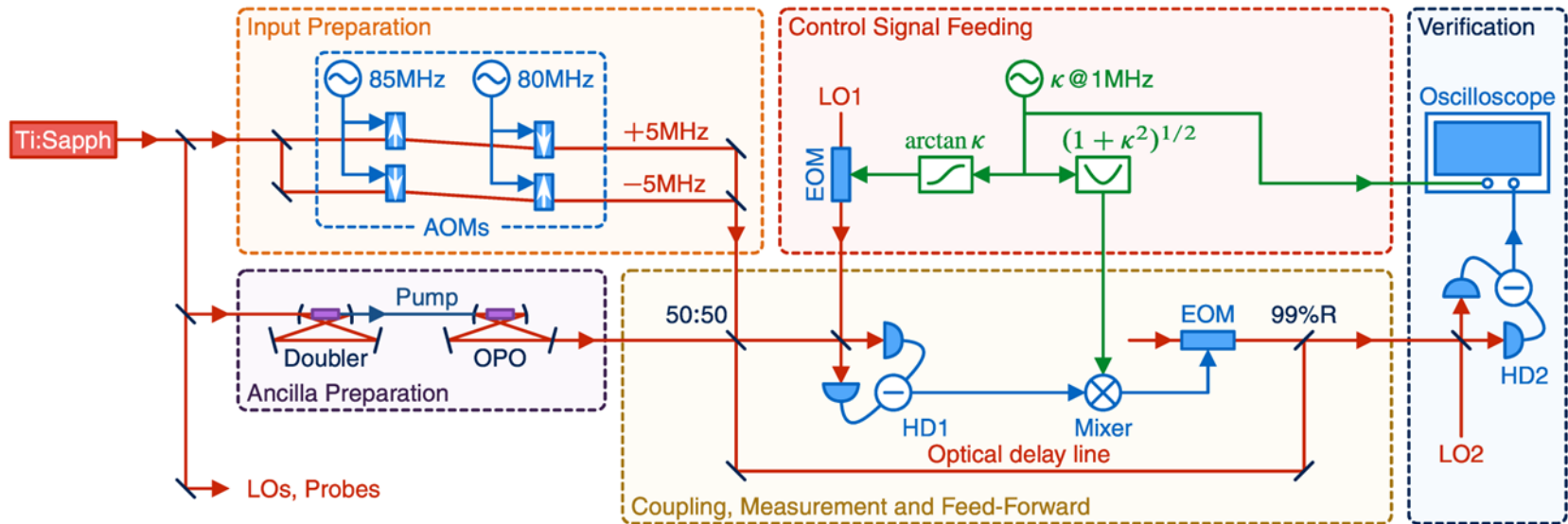
$$X \rightarrow X, P \rightarrow P + \kappa X$$

$$\hat{x} = \frac{1}{\sqrt{2}} \hat{x}_V - \frac{1}{\sqrt{2}} \hat{x}_S^{(0)},$$

$$\hat{p} = \sqrt{2} \left(\hat{p}_V + \frac{\kappa}{2} \hat{x}_V \right) + \frac{\kappa}{\sqrt{2}} \hat{x}_S^{(0)}$$

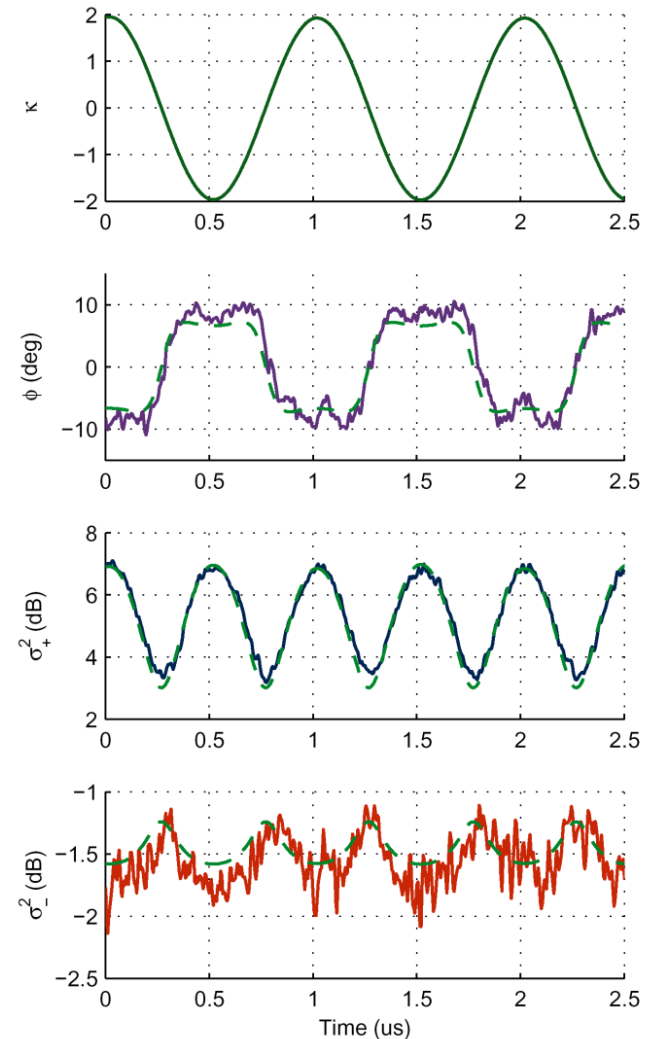
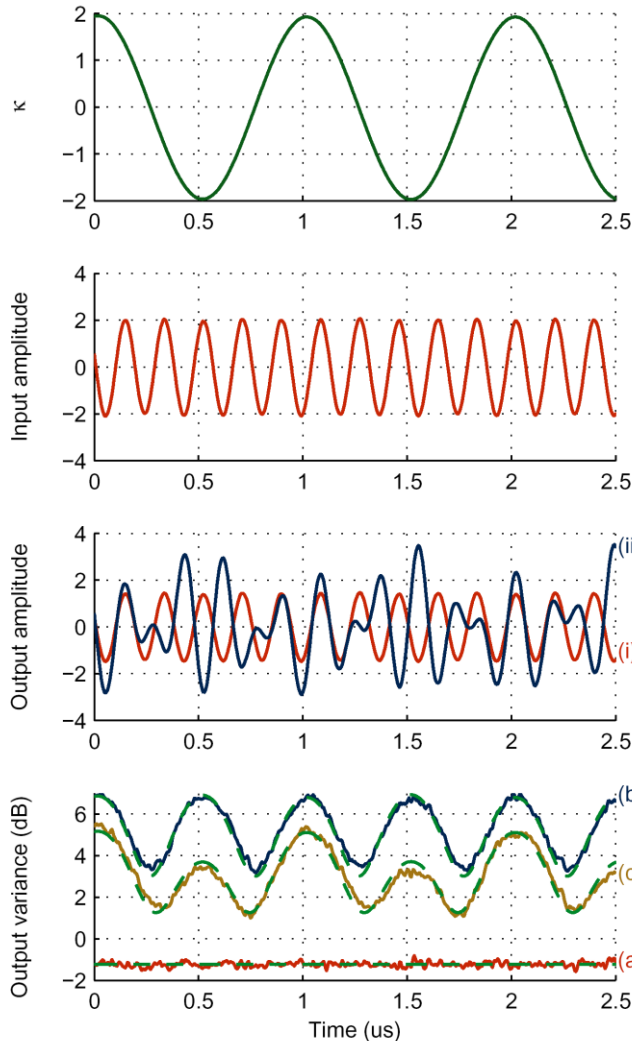


QUADRATIC (X^2) FEEDFORWARD



Kazunori Miyata, Hisashi Ogawa, Petr Marek, Radim Filip, Hidehiro Yonezawa, Jun-ichi Yoshikawa, and Akira Furusawa, in preparation.

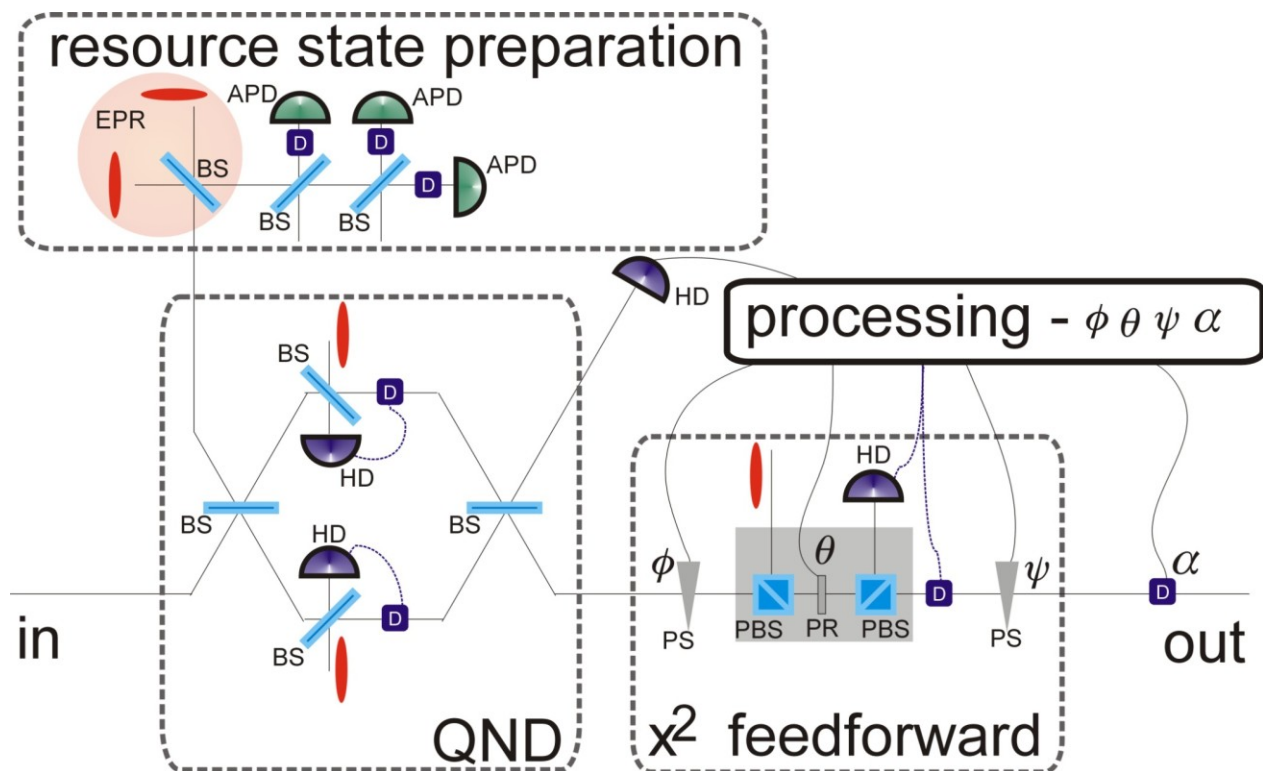
QUADRATIC (X^2) FEEDFORWARD



Kazunori Miyata, Hisashi Ogawa, Petr Marek, Radim Filip, Hidehiro Yonezawa, Jun-ichi Yoshikawa, and Akira Furusawa, in preparation.

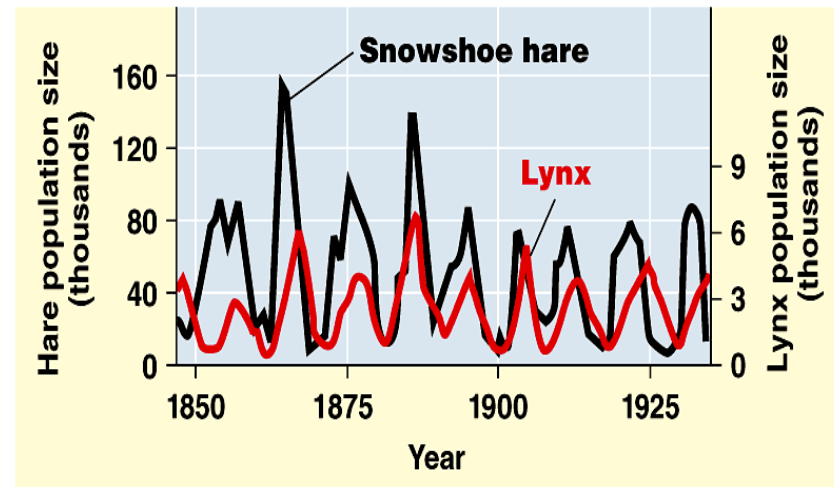
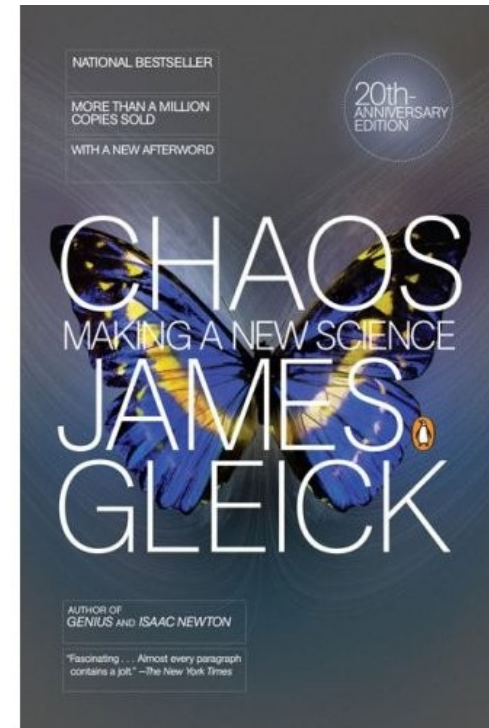
FURTHER OPTIMIZATIONS

We jointly optimized resource state, coupling gate and feed-forward to reduce experimental imperfections.



WHERE WE GO?

HIGHER QUANTUM NONLINEARITY



CENTER OF EXCELENCE FOR CLASSICAL AND QUANTUM INTERACTIONS IN NANOWORLD



Department of Optics, Palacky University, Olomouc
(RF, Lukáš Slodička)

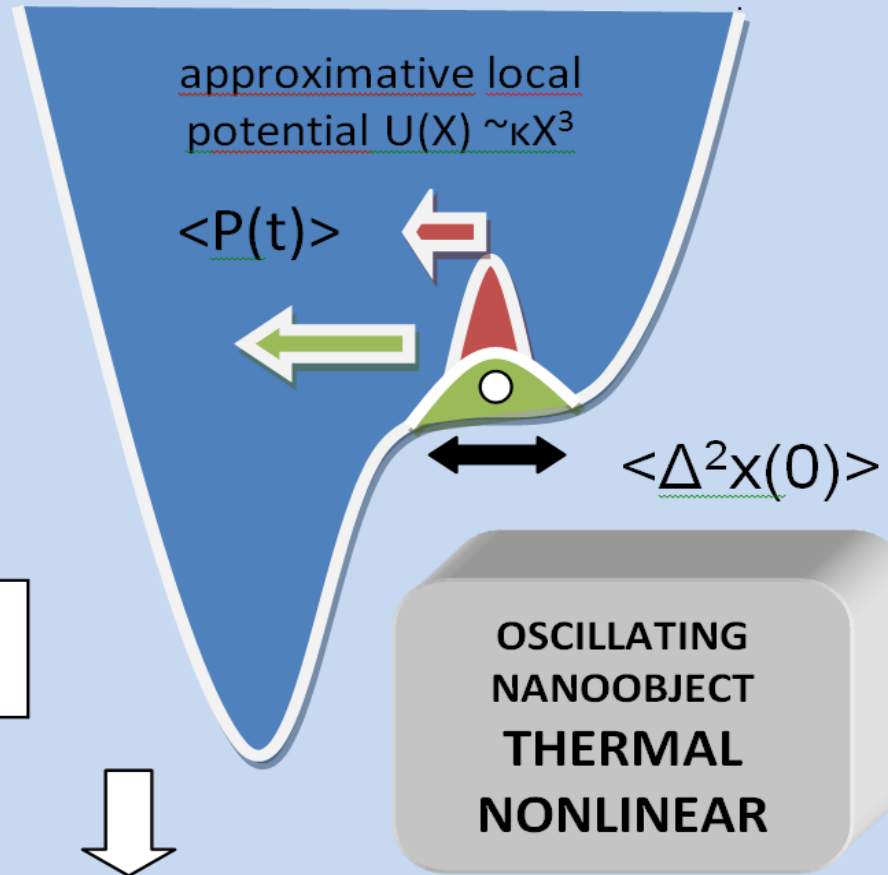
Institute of Scientific Instruments of the ASCR, Brno
(Pavel Zemánek, Ondřej Číp)

CLASSICAL TO QUANTUM WITH NANOOBJECT

Nonlinear noise-to-signal transfer:
 $\langle P(t) \rangle \sim \kappa t \langle \Delta^2 x(0) \rangle$

Noise: variance in initial position around $\langle x(0) \rangle = 0$

Signal: displacement in mean $\langle P(t) \rangle$ of momentum after evolution time t



NONLINEAR DYNAMICS WITH TRAPPED COOLED NANOOBJECT TOWARDS NONLINEAR QUANTUM OPTOMECHANICS

QUANTUM TO CLASSICAL WITH TRAPPED ION

QUANTUM
MECHANICAL
OSCILLATOR

THERMAL

QUANTUM INTERACTIONS
BETWEEN MECHANICAL
OSCILLATOR AND TWO-
LEVEL SYSTEM

INTERACTION
HAMILTONIANS:

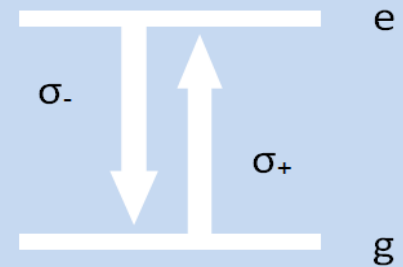
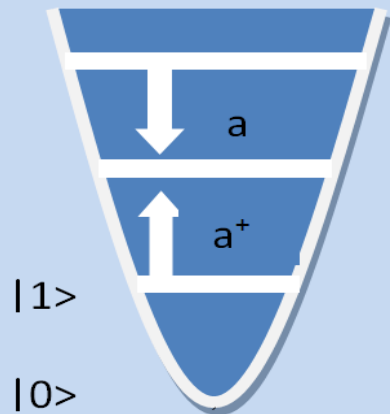
$$H_{rsb} = \hbar\kappa(a\sigma_+ + a^+\sigma_-)$$

$$H_{bsb} = \hbar\kappa(a^+\sigma_+ + a\sigma_-)$$

$$H_d = i\hbar\kappa(\sigma_+ + \sigma_-)(a - a^+)$$

QUANTUM
TWO-LEVEL
SYSTEM

GROUND
STATE



QUANTUM ENTANGLEMENT WITH THERMALLY EXCITED MECHANICAL OSCILLATOR