State transformations and nonlinear effects in two-mode toroidal BEC

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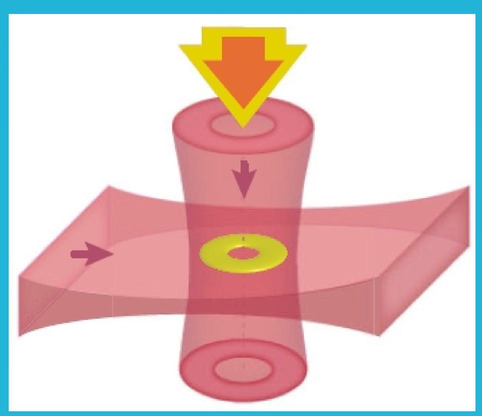
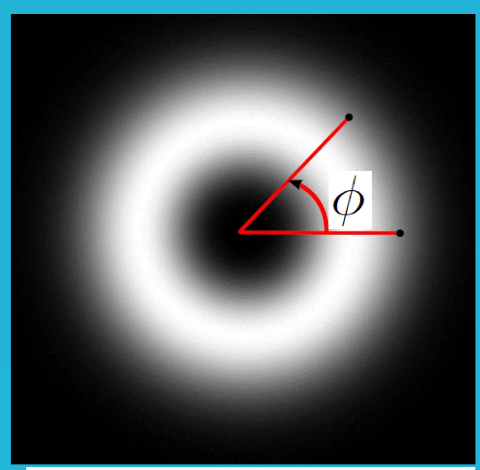
Recent advances in cold atoms trapping allow for creating a multiply connected (toroidal) Bose-Einstein condensate (BEC) by means of all-optical dipole traps. These traps can be created, e.g., by perpendicular crossing of two laser beams, one of these being a Laguerre-Gaussian (LG) [1] beam. Such a trap provides a quasi 1D (circular) smooth enough potential allowing for excitation of the atomic superfluid persistent (long-lived) current [2]. We propose to use optical potential created by two counter-propagating LG beams with opposite winding numbers. These beams create a periodic potential closed around the circle instead of the single peaked potential of [2]. This optical lattice acts as a circular Bragg grating for atomic superfluid states. When the amplitude of the lattice is low compared to the kinetic energy per atom and the Bragg (resonance) condition is fulfilled the lattice couples states with the same magnitude but opposite sign of the orbital angular momentum (OAM), hence restricting the evolution to the two dimensional Hilbert space. An analog of double well/spin half BEC [3] is created this way.

Trapped BEC:

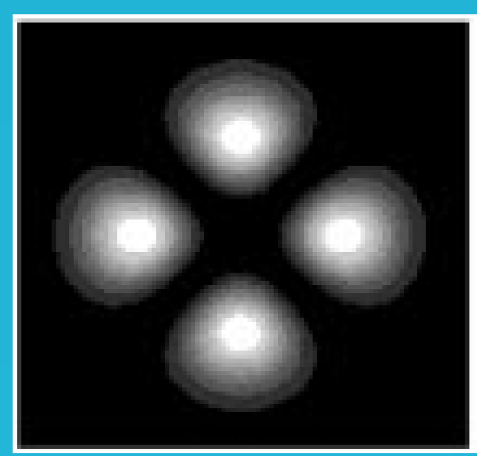


Laguerre-Gaussian beam:



$E_l^{LG} \propto \exp(il\phi)$

Circular optical lattice:



$$V(\phi) \propto |\exp(il\phi) + \exp(-il\phi)|^2 \propto u_0 \cos^2(l\phi),\ l = 2$$

Notation:

$\hat{u}^\dagger|0\rangle = |1_{\rm up}, 0_{\rm down}\rangle,\ i.e.,$ one atom with OAM pointing up

$\hat{d}^\dagger|0\rangle = |0_{\rm up}, 1_{\rm down}\rangle,\ i.e.,$ one atom with OAM pointing down

Pseudospin operators:

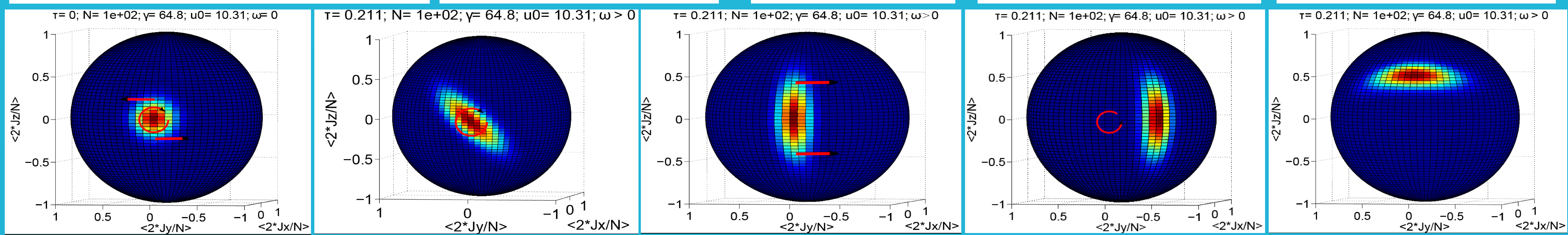
$$\hat{J}_x = \frac{\hat{u}^\dagger\hat{d} + \hat{u}\hat{d}^\dagger}{2},\quad \hat{J}_y = \frac{\hat{u}^\dagger\hat{d} - \hat{u}\hat{d}^\dagger}{2i},\quad \hat{J}_z = \frac{\hat{u}^\dagger\hat{u} - \hat{d}^\dagger\hat{d}}{2}$$

The final Hamiltonian for trapped BEC in two-mode approximation:

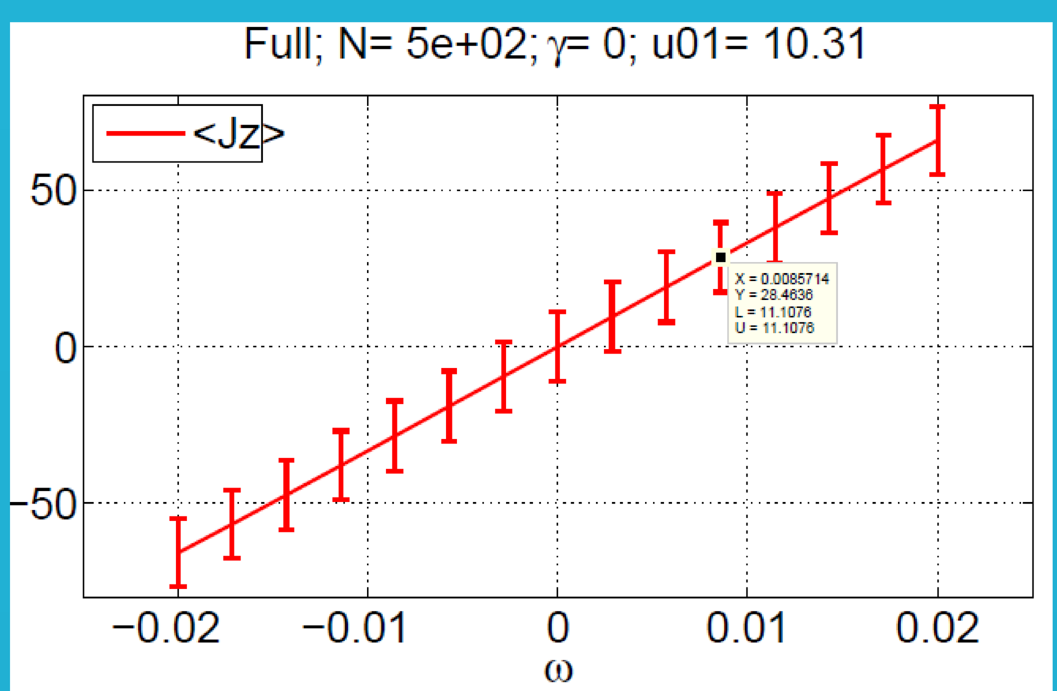
$$\hat{H} = -2\omega l\hat{J}_z + \frac{u_0}{2}\hat{J}_x - \frac{\gamma}{2\pi N}\hat{J}_z^2$$

Ramsey interferometry with squeezing [4]:

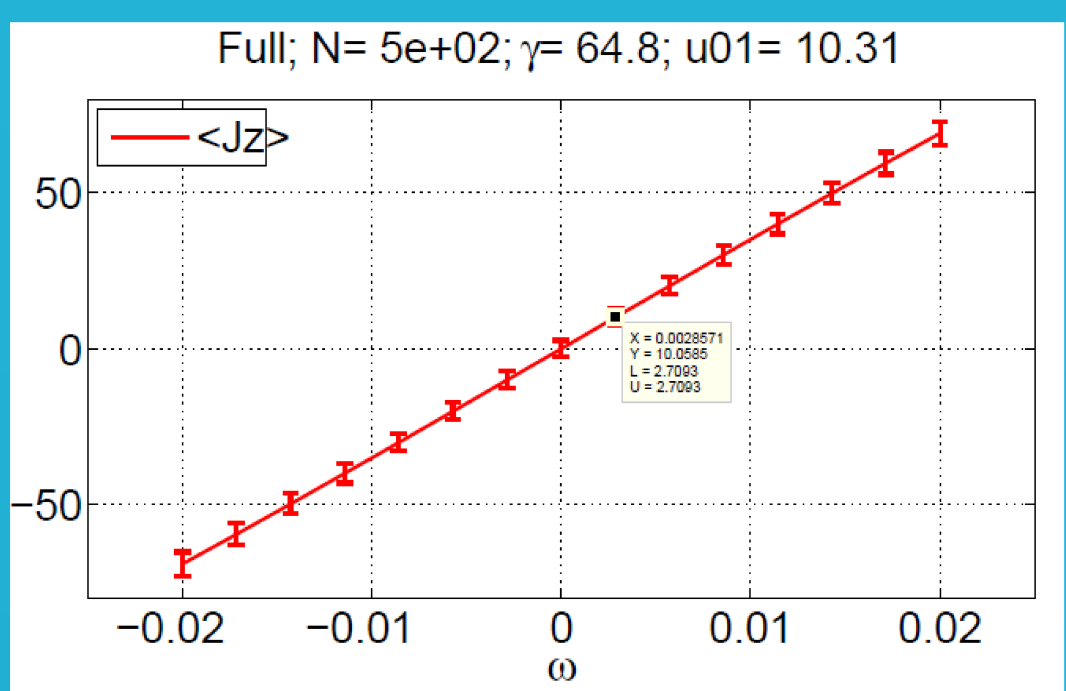
$$\hat{H} = -2\omega l\hat{J}_z + \frac{u_0}{2}\hat{J}_x - \frac{\gamma}{2\pi N}\hat{J}_z^2 \qquad \hat{H} = -2\omega l\hat{J}_z + \frac{u_0}{2}\hat{J}_x \qquad \hat{H} = -2\omega l\hat{J}_z \qquad \hat{H} = -2\omega l\hat{J}_z + \frac{u_0}{2}\hat{J}_x \qquad \hat{H} = -2\omega l\hat{J}_z + \frac{u_0}{2}\hat{J}_x$$



Linear interferometer:



Nonlinear interferometer:



Sensitivity improvement over the linear case:

$$\Delta\omega_{\rm nonlin} \approx \exp\left[-\frac{\gamma\tau_{\rm squeeze}}{4\pi}\right]\Delta\omega_{\rm lin}$$

$$\begin{aligned} \tau_{\rm unit} &= 38\ {\rm ms} \\ \omega_{\rm unit} &= 2\pi\cdot 4{\rm Hz} \\ m &= 23\cdot 1.66\cdot 10^{-27}\ {\rm kg}\ (^{23}{\rm Na}) \\ R &= 10\ \mu{\rm m} \\ a_s &= 2.7\cdot 10^{-9}\ {\rm m} \end{aligned}$$

References:

[1] L. Allen *et al.*, PRA 45, 8185-8189 (1992), M.P. McDonald *et al.*, Science 296, 1101 (2002).

[2] A. Ramanathan *et al.*, PRL 106, 130401 (2011).

[3] S. Levy *et al.*, Nature 449, 579-583 (2007), T. Zibold *et al.*, PRL 105, 204101 (2010).

[4] B. Julia-Diaz *et al.*, PRA 86, 023615 (2012).