An Algorithm for the Multi-Relational Boolean Factor Analysis based on Essential Elements

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Previous Seminar

- Krmelova M., Trnecka M.: Boolean Factor Analysis of Multi-Relational Data. In: M. Ojeda-Aciego, J. Outrata (Eds.): CLA 2013: Proceedings of the 10th International Conference on Concept Lattices and Their Applications, 2013, pp. 187–198.
- Multi-Relational Data = data composed from many tables interconnected via relations between objects or attributes of these data tables.
- Basic problem settings: Two Boolean data tables C_1 and C_2 interconnected with relation $\mathcal{R}_{C_1C_2}$ $\rightarrow$ multi-relational factors.
- Notion of Multi-Relational Factor, i.e. pair (or tuple) of classic factors from data tables.
- Algorithm for computing Multi-Relational factors is missing!
- Our goal: propose an algorithm for Multi-Relation Boolean Factor Analysis.

Satisfyng Relation

- In previous work were introduced three approaches:
  - Narrow approach
  - Wide approach
  - α-approach
- We use the most natural approach = narrow approach.
- Idea of the narrow approach: we connect two factors $F_i^{C_1}$ and $F_j^{C_2}$ if the non-empty set of attributes (if such exist), which are common (in the relation $\mathcal{R}_{C_1C_2}$) to all objects from the first factor $F_i^{C_1}$, is the subset of attributes of the second factor $F_j^{C_2}$.

Naive Algorithm

Table: C_1

	a	b	c	d
1		×	×	×
2	×		×	
3		×		×
4	×	×	×	×

Table: C_2

	e	f	g	h
5	×			×
6		×	×	
7	×	×	×	
8			×	×

Table: $\mathcal{R}_{C_1C_2}$

	e	f	g	h
1		×	×	
2	×		×	
3	×	×		×
4	×	×	×	×

- Factors of data table C_1:
  $F_1^{C_1} = \langle\{1, 4\}, \{b, c, d\}\rangle$, $F_2^{C_1} = \langle\{2, 4\}, \{a, c\}\rangle$, $F_3^{C_1} = \langle\{1, 3, 4\}, \{b, d\}\rangle$
- Factors of table C_2:
  $F_1^{C_2} = \langle\{6, 7\}, \{f, g\}\rangle$, $F_2^{C_2} = \langle\{5\}, \{e, h\}\rangle$, $F_3^{C_2} = \langle\{5, 7\}, \{e\}\rangle$, $F_4^{C_2} = \langle\{8\}, \{g, h\}\rangle$.
- These factors form two multi-relational factors $\langle F_1^{C_1}, F_1^{C_2}\rangle$ and $\langle F_3^{C_1}, F_1^{C_2}\rangle$.

Naive Algorithm

- Usually is problematic to connect all factors from each data table = small number of connections between them.
- Poor quality multi-relational factors.
- Naive algorithm = bad algorithm.
- We can do it better.
- We need help $\rightarrow$ essential elements.
- Notion of the Essential Elements was introduced in: Belohlavek R., Trnecka M.: From-Below Approximations in Boolean Matrix Factorization: Geometry and New Algorithm. `http://arxiv.org/abs/1306.4905`, 2013.

Not All Elements Are Equal

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Zeros - not interesting

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
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1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Essential Elements

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & \mathbf{1} & 0 & 0 & 0 & 0 \\
0 & 1 & \mathbf{1} & \mathbf{1} & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \mathbf{1} & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \mathbf{1} & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & \mathbf{1} & \mathbf{1} & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{1} & \mathbf{1} & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & \mathbf{1} \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & \mathbf{1} & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & \mathbf{1} & \mathbf{1} & 1 & \mathbf{1} & 1 & 1 & 0 & \mathbf{1} & 0 & 1 & 0 & 0 & \mathbf{1} \\
0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Essential Elements

- Entries in Boolean data table which are sufficient for covering the whole data table by factors (concepts).
- Formally, essential elements in the data table $\langle X, Y, C \rangle$ are defined via minimal intervals in the concept lattice. The entry C_{ij} is essential iff interval bounded by formal concepts $\langle i^{\uparrow\downarrow}, i^{\uparrow} \rangle$ and $\langle j^{\downarrow}, j^{\downarrow\uparrow} \rangle$ is non-empty and minimal w.r.t. $\subseteq$.
- If the table entry C_{ij} is essential, then interval $\mathcal{I}_{ij}$ represents the set of all formal concepts (factors) which cover this entry.
- It is sufficient take only one arbitrary concept from each interval to create exact Boolean decomposition of $\langle X, Y, C \rangle$.
- Effective algorithm for construction of the essential part.

Idea of Algorithm

Table: C_1

	a	b	c	d
1		×	×	×
2	×		×	
3		×		×
4	×	×	×	×

Table: $Ess(C_1)$

	a	b	c	d
1			×	
2	×			
3		×		×
4				

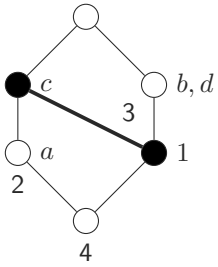
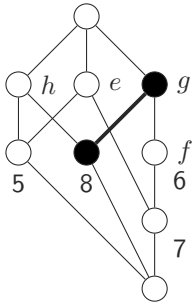


Table: C_2

	e	f	g	h
5	×			×
6		×	×	
7	×	×	×	
8			×	×

Table: $Ess(C_2)$

	e	f	g	h
5	×			×
6		×		
7	×			
8			×	×



Idea of Algorithm

- If we take highlighted intervals, we obtain possibly four connections.
- First highlighted interval contains two concepts $c_1 = \langle\{1, 2, 4\}, \{c\}\rangle$ and $c_2 = \langle\{1, 4\}, \{b, c, d\}\rangle$. Second consist of concepts $d_1 = \langle\{6, 7, 8\}, \{g\}\rangle$ and $d_2 = \langle\{8\}, \{g, h\}\rangle$.
- Only two connections (c_1 with d_1 and c_1 with d_2) satisfy relation $\mathcal{R}_{C_1C_2}$, i.e. can be connected.
- Search space reduction: If we are not able to connect concept $\langle A, B\rangle$ with concept $\langle C, D\rangle$, we are not able to connect $\langle A, B\rangle$ with any concept $\langle E, F\rangle$, where $\langle C, D\rangle \subseteq \langle E, F\rangle$.
- Moreover if we are not able to connect concept $\langle A, B\rangle$ concept $\langle E, F\rangle$, we are not able connect any concept $\langle C, D\rangle \subseteq \langle A, B\rangle$, with concept $\langle E, F\rangle$.

Heuristic

- Search in intervals is still time consuming.
- Heuristic: take attribute concepts in intervals of the second data table. In intervals of the first data table take greatest concepts which can be connected via relation (set of common attributes in relation is non-empty).
- The idea behind this heuristic: a bigger set of objects possibly have a smaller set of common attributes in a relation = bigger probability to connect this factor with some factor from the second data table.

- Applying heuristic on the example, we obtain three factors in the first data table, $F_1^{C_1} = \langle\{2,4\},\{a,c\}\rangle$, $F_2^{C_1} = \langle\{1,3,4\},\{c,d\}\rangle$, $F_3^{C_1} = \langle\{1,2,4\},\{c\}\rangle$ and four factors $F_1^{C_2} = \langle\{5\},\{e,h\}\rangle$, $F_2^{C_2} = \langle\{6,7\},\{f,g\}\rangle$, $F_3^{C_2} = \langle\{7\},\{e,f,g\}\rangle$, $F_4^{C_2} = \langle\{8\},\{g,h\}\rangle$ from the second one.
- Between this factors, there are six connections satisfying the relation.
- Naive algorithm - only two connections.

	$F_1^{C_2}$	$F_2^{C_2}$	$F_3^{C_2}$	$F_4^{C_2}$
$F_1^{C_1}$			$\times$	
$F_2^{C_1}$		$\times$	$\times$	
$F_3^{C_1}$		$\times$	$\times$	$\times$

Final Algorithm for MBMF

Input: Boolean matrices C_1, C_2 and relation $R_{C_1C_2}$ between them and $p \in [0, 1]$
Output: set $\mathcal{M}$ of multi-relational factors

$E_{C_1} \leftarrow Ess(C_1)$
$E_{C_2} \leftarrow Ess(C_2)$
$U_{C_1} \leftarrow C_1$
$U_{C_2} \leftarrow C_2$
while $(|U_{C_1}| + |U_{C_2}|)/(|C_1| + |C_2|) \geq p$ **do**
    foreach *essential element* $(E_{C_1})_{ij}$ **do**
        compute the best candidate $\langle a, b\rangle$ from interval $\mathcal{I}_{ij}$
    end
    $\langle A, B\rangle \leftarrow$ select one from set of candidates which maximize cover of C_1
    select non-empty row i in E_{C_2} for which is $A^{\uparrow R_{C_1C_2}} \subseteq (C_2)_{i\_}^{\downarrow\uparrow C_2}$ and which maximize cover of C_1 and C_2
    $\langle C, D\rangle \leftarrow \langle (C_2)_{i\_}^{\uparrow\downarrow C_2}, (C_2)_{i\_}^{\uparrow C_2}\rangle$
    if *value of cover function for* C_1 *and* C_2 *is equal to zero* **then**
        break
    end
    add $\langle\langle A, B\rangle, \langle C, D\rangle\rangle$ **to** $\mathcal{M}$
    set $(U_{C_1})_{ij} = 0$ where $i \in A$ and $j \in B$
    set $(U_{C_1})_{ij} = 0$ where $i \in C$ and $j \in D$
end
return $\mathcal{F}$

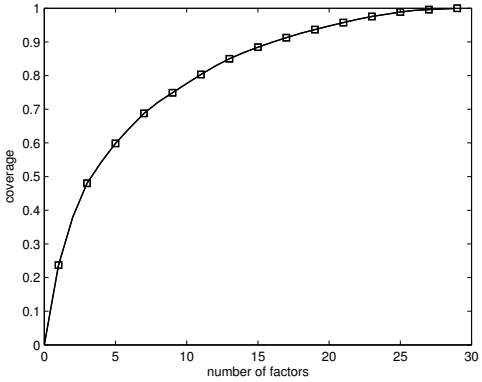
Remarks

- In each step we connect factors, that cover the biggest still uncovered part.
- Firstly, we obtain multi-relational factor $\langle F_2^{C_1}, F_2^{C_2} \rangle$ which covers 50 percent of the data. Then we obtain factor $\langle F_3^{C_1}, F_4^{C_2} \rangle$ which covers together with first factor 75 percent of the data and last we obtain factor $\langle F_1^{C_1}, F_3^{C_2} \rangle$.
- These factors cover 90 percent of input.
- Adding of other factors we do not obtain better coverage.
- These factors cover the same part of input data as six connections from previous table.
- Multi-relational factors are not always able to explain the whole data. This is due to nature of data.
- Relation is over objects of first data table C_1 and attributes of second data table C_2.

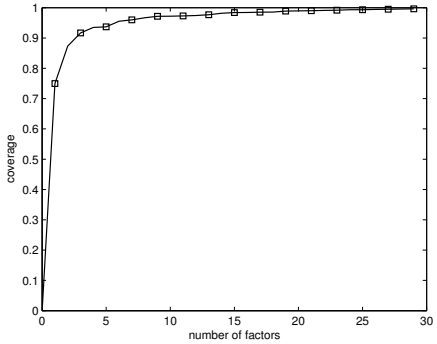
MovieLens Dataset

- http://grouplens.org/datasets/movielens/
- Two data tables: set of users and their attributes (e.g. gender, age, ... occupation) and a set of movies and their attributes (e.g. genre).
- Relation between data tables (contains 1000209 anonymous ratings of 3952 movies made by 6040 MovieLens users who joined to MovieLens in 2000).
- Each user has at least 20 ratings.
- Ratings are made on a 5-star scale (values 1-5, 1 means, that user does not like a movie and 5 means that he likes a movie).
- We convert the ordinal relation in to binary one and we make restriction to 3000 users (users, who rate movies the most).
- We use three different scaling:
  - User rates a movie.
  - User does not like a movie (he rates movie with 1-2 stars).
  - User likes a movie (rates 4-5).

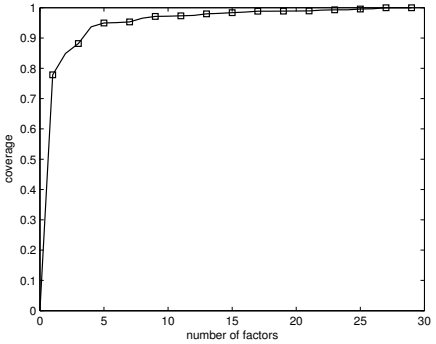
Cumulative Coverage of Input (“User rates a movie")



Coverage of Input Data Tables (“User rates a movie")



(a) Coverage of Users data table



(b) Coverage of Movies data table

Results

The most important factors are:

- Males rate new movies (movies from 1991 to 2000).
- Young adult users (ages 25-34) rate drama movies.
- Females rate comedy movies.
- Youth users (18-24) rate action movies.

Another interesting factors are:

- Old users (from category 56+) rates movies from 1941 to 1950.
- Users in age range 50-55 rate children's movies.
- K-12 students rate animation movies.

Reconstruction Error

- In case of MovieLens we are able to reconstruct input data tables almost wholly for each three relations.
- Q: Can we reconstruct relation between data tables?
- A: Yes, we can.
- Multi-relational factor carry information about the relation between data tables.
- We can reconstruct it with some error (result of narrow approach).
- Reconstruction error of relation can be minimize (if we take this error into account in phase of computing coverage).

Conclusion and Future Research

- We present new algorithm for multi-relational Boolean matrix factorization.
- The most important factors (factors which explain the biggest portion of data) are computed first.
- Algorithm is applicable for usually large datasets.

Future research:

- Generalization of the algorithm for ordinal data.
- Construction of algorithm which takes into account reconstruction error of the relation between data tables.
- Test the potential of this method in recommendation systems.
- Create not crisp operator for connecting classic factors into multi-relational factors.

Thank you