Partial suppression of nonadiabatic transitions

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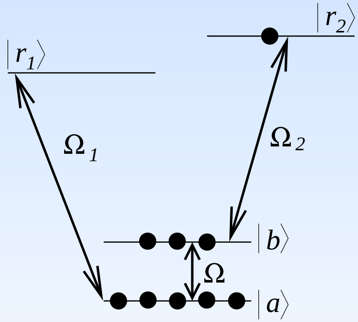
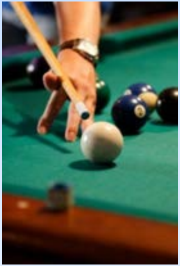
Partial suppression of nonadiabatic transitions

Outlook

- Adiabatic processes, exact compensation
- Partial compensation of nonadiabatic transitions
- Examples: interacting spins, two bosons in two wells, expanding potential well, atoms with Rydberg blockade
- Discussion: further prospects of the method



Dynamic vs. adiabatic transport



Adiabatic processes and exact compensation

Evolution under time-dependent Hamiltonian

$$i\hbar\frac{d}{dt}|\Psi\rangle = H_0(t)|\Psi\rangle \tag{1}$$

expand

$$|\Psi(t)\rangle = \sum_n a_n(t)e^{i\theta_n(t)}|n(t)\rangle \tag{2}$$

with instantaneous eignestates

$$H_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle \tag{3}$$

$$\theta_n(t) = -\int_0^t \frac{E_n(t')}{\hbar}dt' \tag{4}$$

Adiabatic processes and exact compensation

Evolution under time-dependent Hamiltonian

$$i\hbar \sum_n \dot{a}_n e^{i\theta_n} |n\rangle + \sum_n a_n E_n e^{i\theta_n} |n\rangle + i\hbar \sum_n a_n e^{i\theta_n} |\dot{n}\rangle = \sum_n a_n E_n e^{i\theta_n} |n\rangle \quad (5)$$

multiplying with $\langle k|$:

$$\dot{a}_k = -\sum_n a_n e^{i(\theta_n - \theta_k)} \langle k|\dot{n}\rangle \quad (6)$$

Adiabatic processes and exact compensation

Evolution under time-dependent Hamiltonian

Time derivative of $H_0|n\rangle = E_n|n\rangle$:

$$\dot{H}_0|n\rangle + H_0|\dot{n}\rangle = \dot{E}_n|n\rangle + E_n|\dot{n}\rangle \tag{7}$$

Multiply with $\langle k|$ with $k \neq n$:

$$\begin{aligned} \langle k|\dot{H}_0|n\rangle + \langle k|H_0|\dot{n}\rangle &= E_n\langle k|\dot{n}\rangle, && (8)\\ \langle k|\dot{H}_0|n\rangle + E_k\langle k|\dot{n}\rangle &= E_n\langle k|\dot{n}\rangle, && (9)\\ \langle k|\dot{n}\rangle &= \frac{\langle k|\dot{H}_0|n\rangle}{E_n - E_k} && (10) \end{aligned}$$

Therefore

$$\dot{a}_k = -a_k\langle k|\dot{k}\rangle - \sum_{n\neq k} a_n e^{i(\theta_n - \theta_k)} \frac{\langle k|\dot{H}_0|n\rangle}{E_n - E_k} \tag{11}$$

Adiabatic processes and exact compensation

Evolution under time-dependent Hamiltonian
Adiabatic approximation:

$$\left|\frac{\langle k|\dot{H}_0|n\rangle}{E_n - E_k}\right| T \ll 1, \quad (12)$$

$$\dot{a}_k \approx -a_k \langle k|\dot{k}\rangle \quad (13)$$

$$a_k(t) \approx a_k(0)e^{i\gamma(t)} \quad (14)$$

Geometric phase γ:

$$\gamma(t) = i\int_0^t \langle k(t')|\dot{k}(t')\rangle dt' \quad (15)$$

Adiabatic processes and exact compensation

Evolution under time-dependent Hamiltonian
Apply an additional Hmiltonian
[Demirplak and Rice, J. Phys. Chem. A **107,** 9937 (2003); Berry, J. Phys. A Math. Theor. **42,** 365303 (2009)] :

$$H_B = i\hbar \sum_n \left(|\dot{n}\rangle\langle n| - |n\rangle\langle n|\dot{n}\rangle\langle n| \right) \tag{16}$$

System starting in eigenstate $|n\rangle$ of H_0, evolving under $H = H_0 + H_B$ stays **exactly** in eigenstate $|n\rangle$ of H_0.

Partial compensation

What if H_B is not available in the lab?

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Task: Keep system as close as possible to eigenstate $|0(t)\rangle$ of $H_0(t)$.
Choose suitable $\alpha_1(t), \alpha_2(t)$ ..., the system evolves under Hamiltonian

$$H = H_0 + \alpha_1 L_1 + \alpha_2 L_2 \cdots = H_0 + H_C \tag{17}$$

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Minimize norm of vector $(H_C - H_B)|0\rangle$, i.e., minimize

$$\langle 0| \left(\sum_{k=1}^{K} \alpha_k L_k - H_B \right) \left(\sum_{k'=1}^{K} \alpha_{k'} L_{k'} - H_B \right) |0\rangle. \tag{18}$$

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Partial compensation

Minimising the quadratic form by solving linear equations:

$$\sum_{k=1}^{K} A_{m,k}\alpha_k = C_m, \tag{19}$$

where

$$\begin{aligned} A_{m,k} &= \langle L_m L_k + L_k L_m \rangle, & (20) \\ C_k &= \langle L_k H_B + H_B L_k \rangle, & (21) \end{aligned}$$

(mean values calculated in state $|0(t)\rangle$)

T. Opatrný and K. Mølmer, New J. Phys. **16** 015025 (2014).

Example: two spin-$\frac{1}{2}$ particles

$$H_0 = -B(\sigma_x^{(1)} + \sigma_x^{(2)}) + J\sigma_z^{(1)}\sigma_z^{(2)} \tag{22}$$

Eigenstates:
$|\rightarrow\rightarrow\rangle$ (for $|B/J| \gg 1$)
$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ (for $|B/J| \ll 1$).

$$H_0 = \begin{pmatrix} J & -B & -B & 0 \\ -B & -J & 0 & -B \\ -B & 0 & -J & -B \\ 0 & -B & -B & J \end{pmatrix}. \tag{23}$$

Example: two spin-$\frac{1}{2}$ particles

With parametrization

$$J = A \sin\varphi \tag{24}$$

$$B = \frac{A}{2} \cos\varphi \tag{25}$$

eigenvectors

$$|\phi_1\rangle = \frac{1}{2\sqrt{1+\sin\varphi}} \begin{pmatrix} \cos\varphi \\ 1+\sin\varphi \\ 1+\sin\varphi \\ \cos\varphi \end{pmatrix}, \qquad |\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix},$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \qquad |\phi_4\rangle = \frac{1}{2\sqrt{1+\sin\varphi}} \begin{pmatrix} 1+\sin\varphi \\ -\cos\varphi \\ -\cos\varphi \\ 1+\sin\varphi \end{pmatrix}. \tag{26}$$

Berry Hamiltonian

$$H_B = i\hbar|\dot{\phi_1}\rangle\langle\phi_1| + i\hbar|\dot{\phi_4}\rangle\langle\phi_4|, \tag{27}$$

$$H_B = \frac{\hbar\dot{\varphi}}{4}\begin{pmatrix} 0 & -i & -i & 0 \\ i & 0 & 0 & i \\ i & 0 & 0 & i \\ 0 & -i & -i & 0 \end{pmatrix} = \frac{\hbar\dot{\varphi}}{4}\left(\sigma_y^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_y^{(2)}\right). \tag{28}$$

Example: two spin-$\frac{1}{2}$ particles

Partial compensation

$$L = p\sigma_y^{(1)}\sigma_z^{(2)} + q\sigma_z^{(1)}\sigma_y^{(2)}. \tag{29}$$

$$\begin{aligned} \langle\phi_1|L^2|\phi_1\rangle &= (q+p)^2, & (30)\\ \langle\phi_1|LH_B + H_BL|\phi_1\rangle &= (q+p)\hbar\dot{\varphi}, & (31)\\ & & (32) \end{aligned}$$

$$\alpha = \frac{\langle\phi_1|LH_B + H_BL|\phi_1\rangle}{2\langle\phi_1|L^2|\phi_1\rangle} = \frac{\hbar\dot{\varphi}}{2(q+p)}. \tag{33}$$

$$H_C = \alpha L = \hbar\dot{\varphi}\frac{p\sigma_y^{(1)}\sigma_z^{(2)} + q\sigma_z^{(1)}\sigma_y^{(2)}}{2(q+p)}. \tag{34}$$

Example: four spin-$\frac{1}{2}$ particles

Original Hamiltonian:

$$H_0 = -B(t) \sum_{j=1}^{4} \sigma_x^{(j)} + J_0 \sum_{j=1}^{3} \sigma_z^{(j)} \sigma_z^{(j+1)}. \tag{35}$$

Parameter change: $B(t) = B_0 \exp\left(-2.4t/t_0\right)$

State change from $|\rightarrow\rightarrow\rightarrow\rightarrow\rangle$ to $|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle$

For adiabatic transition: t_0 must be large.

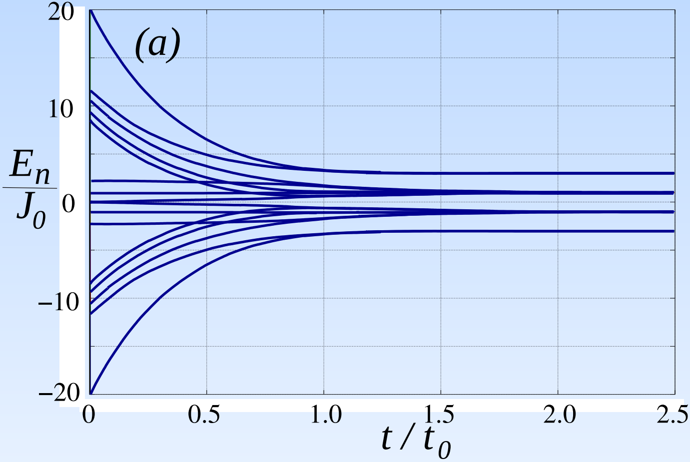


Figure: Eigenvalues of the Hamiltonian (35).

Example: four spin-$\frac{1}{2}$ particles

Possible choice of compensating operators:

$$
\begin{aligned}
L_1 &= \sigma_y^{(1)}\sigma_z^{(2)} + \sigma_z^{(3)}\sigma_y^{(4)}, && (36)\\
L_2 &= \sigma_z^{(1)}\sigma_y^{(2)} + \sigma_y^{(3)}\sigma_z^{(4)}, && (37)\\
L_3 &= \sigma_y^{(2)}\sigma_z^{(3)} + \sigma_z^{(2)}\sigma_y^{(3)}, && (38)\\
L_4 &= \sigma_z^{(1)}\sigma_y^{(4)} + \sigma_y^{(1)}\sigma_z^{(4)}. && (39)
\end{aligned}
$$

Example: four spin-$\frac{1}{2}$ particles

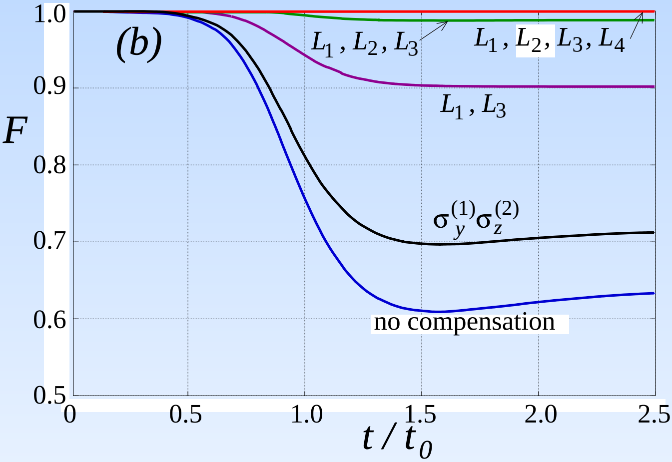


Figure: Fidelity of the evolved state

Example: four spin-$\frac{1}{2}$ particles

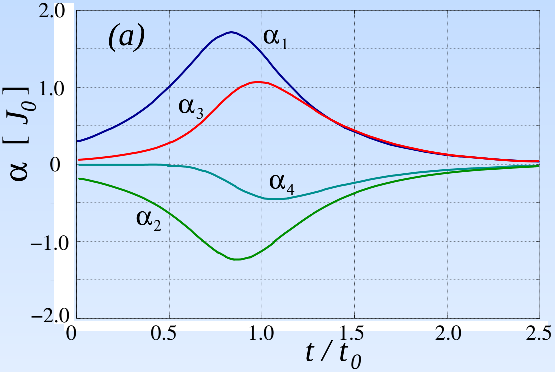


Figure: Compensation parameters α_1–α_4 of the scheme with 4 operators.

Example: four spin-$\frac{1}{2}$ particles

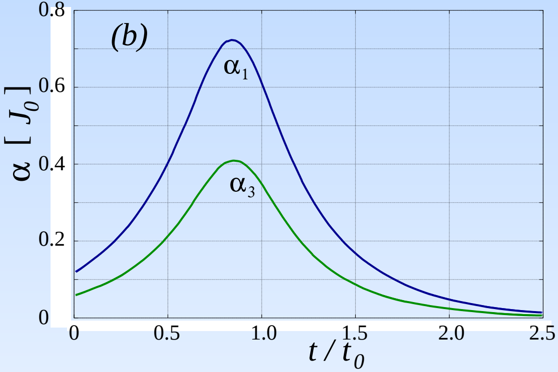


Figure: Compensation parameters α_1 and α_3 of the scheme with 2 operators.

Example: four spin-$\frac{1}{2}$ particles

Further prospects:

- More spins, more general interactions: any general rules?
- Applications in trapped-ion experiments: any particular sets of compensating operators?
- Systematic approach to generate compensating operators: H. Saberi, A. del Campo, T. Opatrný, K. Mølmer "Adiabatic Tracking of Quantum Many-Body Dynamics"

Example: more spin-$\frac{1}{2}$ particles

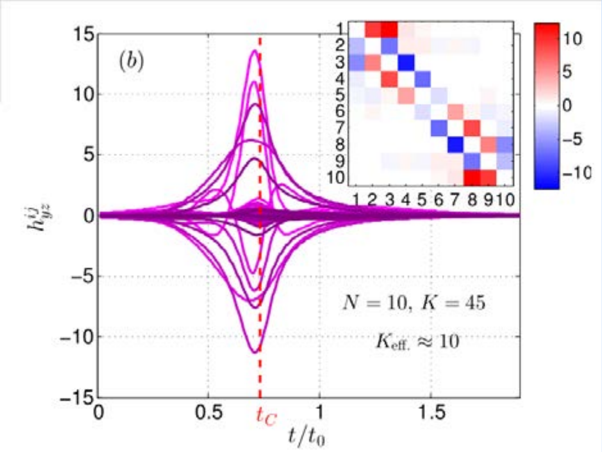


Figure: Compensation parameters with 10 spins.

Example: more spin-$\frac{1}{2}$ particles

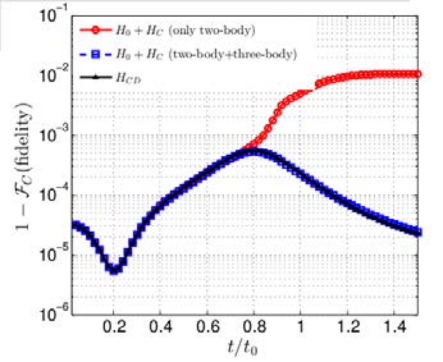


Figure: Compensation fidelity, 6 particles plus 3-body interactions.

Example: two interacting bosons in two wells

Motivation: transition between the superfluid state and the Mott insulator.

$$H_0 = \frac{U}{2}\sum_{j=1}^{2} n_j(n_j - 1) - J\left(a_1 a_2^\dagger + a_1^\dagger a_2\right) \tag{40}$$

With two particles, states $|2,0\rangle$, $|1,1\rangle$, and $|0,2\rangle$:

$$H_0 = \begin{pmatrix} U & -\sqrt{2}J & 0 \\ -\sqrt{2}J & 0 & -\sqrt{2}J \\ 0 & -\sqrt{2}J & U \end{pmatrix} \tag{41}$$

Example: two interacting bosons in two wells

Parametrization:

$$U = E_0 \cos\varphi, \tag{42}$$

$$J = \frac{E_0}{4} \sin\varphi, \tag{43}$$

Hamiltonian:

$$H_0 = E_0 \begin{pmatrix} \cos\varphi & -\frac{1}{2\sqrt{2}}\sin\varphi & 0 \\ -\frac{1}{2\sqrt{2}}\sin\varphi & 0 & -\frac{1}{2\sqrt{2}}\sin\varphi \\ 0 & -\frac{1}{2\sqrt{2}}\sin\varphi & \cos\varphi \end{pmatrix} \tag{44}$$

with the eigenenergies

$$E_1 = \frac{E_0}{2}(\cos\varphi - 1), \tag{45}$$

$$E_2 = E_0 \cos\varphi, \tag{46}$$

$$E_3 = \frac{E_0}{2}(\cos\varphi + 1), \tag{47}$$

Example: two interacting bosons in two wells

Eigenvectors

$$|\phi_1\rangle = \begin{pmatrix} \frac{1}{2}\sqrt{1-\cos\varphi} \\ \frac{1}{\sqrt{2}}\sqrt{1+\cos\varphi} \\ \frac{1}{2}\sqrt{1-\cos\varphi} \end{pmatrix}, \tag{48}$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \tag{49}$$

$$|\phi_3\rangle = \begin{pmatrix} \frac{1}{2}\sqrt{1+\cos\varphi} \\ -\frac{1}{\sqrt{2}}\sqrt{1-\cos\varphi} \\ \frac{1}{2}\sqrt{1+\cos\varphi} \end{pmatrix}. \tag{50}$$

Example: two interacting bosons in two wells

Exact compensation:

$$H_B = \frac{\hbar\dot{\varphi}}{2\sqrt{2}} \begin{pmatrix} 0 & i & 0 \\ -i & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \tag{51}$$

Simple form, but challenging to realize experimentally.

Alternative approach, "Lie transforms": [S. Martinez-Garaot, E. Torrontegui, Xi Chen, and J. G. Muga, Phys. Rev. A **89,** 053408 (2014)]

Example: particle in an expanding box

Infinitely deep box, one wall moving:

$$H_0 = \frac{p^2}{2m} + U(x) \tag{52}$$

with

$$U(x) = \begin{cases} \infty & \text{for } x < 0, \\ 0 & \text{for } 0 < x < D(t), \\ \infty & \text{for } D(t) < x. \end{cases} \tag{53}$$

Berry compensation: solved by Jarzynski [arXiv:1305.4967 (2013)]

$$H_B = \frac{\dot{D}}{2D}(xp + px). \tag{54}$$

If H_B is not available, another option:

$$H_C = \frac{\dot{D}}{2} p. \tag{55}$$

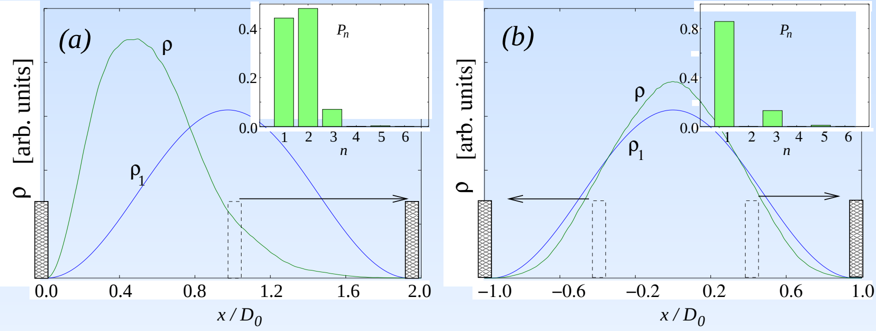
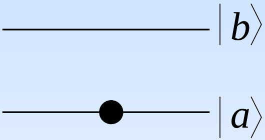


Figure: Particle in expanding box.

Spin squeezing

Example: two-level atoms



Squeezing with Rydberg blockade

Spin squeezing

Example: two-level atoms

Single-atom operators:

$$\begin{aligned} S_x &= \frac{1}{2}(|a\rangle\langle b| + |b\rangle\langle a|), \\ S_y &= \frac{i}{2}(-|a\rangle\langle b| + |b\rangle\langle a|), \\ S_z &= \frac{1}{2}(|a\rangle\langle a| - |b\rangle\langle b|). \end{aligned}$$

Spin squeezing

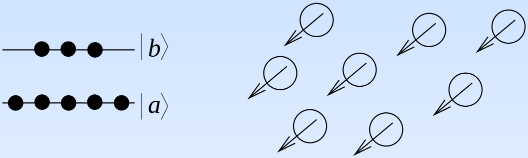
Example: two-level atoms

Many atoms:

$$
\begin{aligned}
\vec{J} &= \sum_k \vec{S}_k \\
J_x &= \frac{1}{2}(a^\dagger b + ab^\dagger), \\
J_y &= \frac{i}{2}(a^\dagger b - ab^\dagger), \\
J_z &= \frac{1}{2}(a^\dagger a - b^\dagger b), \\
[a, a^\dagger] &= [b, b^\dagger] = 1 \\
[J_x, J_y] &= -iJ_z
\end{aligned}
$$

Spin squeezing

Example: many two-level atoms

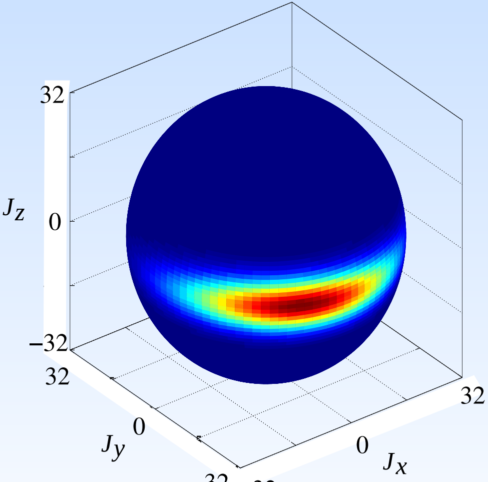


Example: squeezing with Rydberg blockade

Spin squeezing

Many two-level atoms

Poincare sphere



Example: squeezing with Rydberg blockade

Spin squeezing

Many two-level atoms

PHYSICAL REVIEW A   VOLUME 50, NUMBER 1   JULY 1994

Squeezed atomic states and projection noise in spectroscopy

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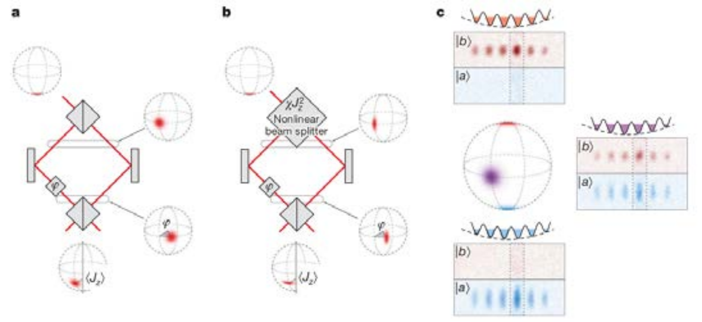
(Received 11 January 1994)

We investigate the properties of angular-momentum states which yield high sensitivity to rotation. We discuss the application of these "squeezed-spin" or correlated-particle states to spectroscopy. Transitions in an ensemble of N two-level (or, equivalently, spin-$\frac{1}{2}$) particles are assumed to be detected by observing changes in the state populations of the particles (population spectroscopy). When the particles' states are detected with 100% efficiency, the fundamental limiting noise is projection noise, the noise associated with the quantum fluctuations in the measured populations. If the particles are first prepared in particular quantum-mechanically correlated states, we find that the signal-to-noise ratio can be improved over the case of initially uncorrelated particles. We have investigated spectroscopy for a particular case of Ramsey's separated oscillatory method where the radiation pulse lengths are short compared to the time between pulses. We introduce a squeezing parameter ξ_R which is the ratio of the statistical uncertainty in the determination of the resonance frequency when using correlated states vs that when using uncorrelated states. More generally, this squeezing parameter quantifies the sensitivity of an angular-momentum state to rotation. Other squeezing parameters which are relevant for use in other contexts can be defined. We discuss certain states which exhibit squeezing parameters $\xi_R \simeq N^{-1/2}$. We investigate possible experimental schemes for generation of squeezed-spin states which might be applied to the spectroscopy of trapped atomic ions. We find that applying a Jaynes-Cummings–type coupling between the ensemble of two-level systems and a suitably prepared harmonic oscillator results in correlated states with $\xi_R < 1$.

Example: squeezing with Rydberg blockade

Spin squeezing

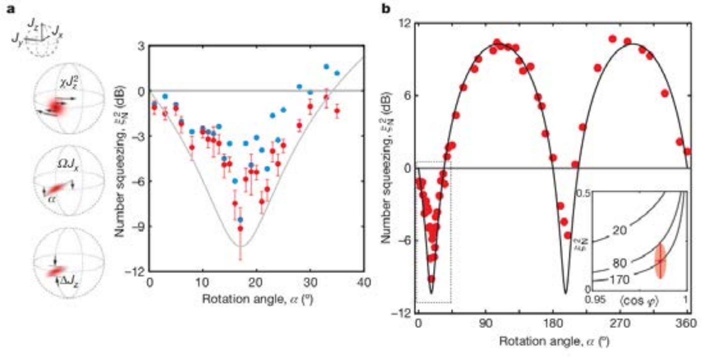
Gross et al., Nature 464, 1165 (2010)



$\sim 10^3$ atoms squeezed by $\sim$ 5 dB in $\sim$ 10 ms

Spin squeezing

Gross et al., Nature 464, 1165 (2010)

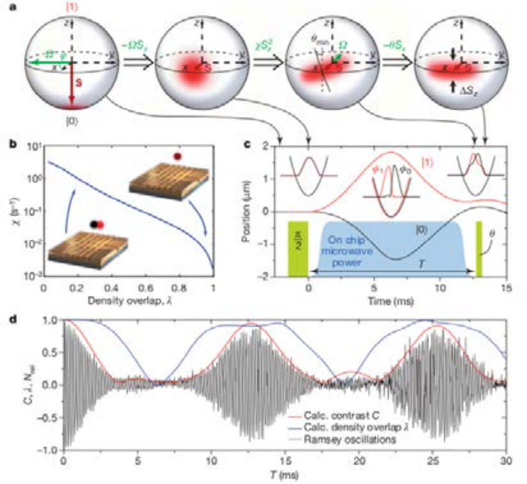


$\sim 10^3$ atoms squeezed by ~ 5 dB in ~ 10 ms

Example: squeezing with Rydberg blockade

Spin squeezing

Riedel et al., Nature 464, 1170 (2010)



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Spin squeezing
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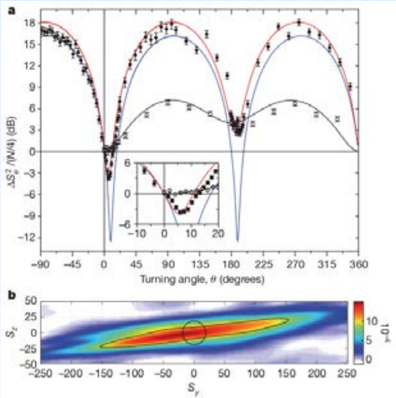


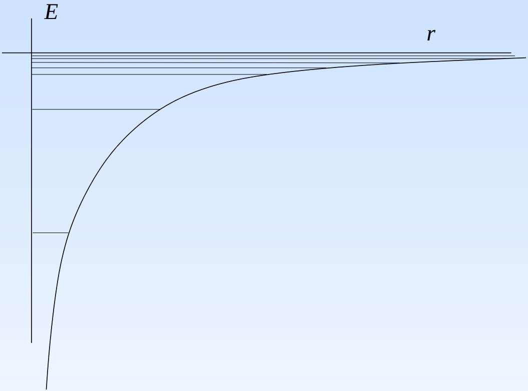
$\sim 10^3$ atoms squeezed by $\sim$ 5 dB in $\sim$ 10 ms

Example: squeezing with Rydberg blockade

Rydberg atom

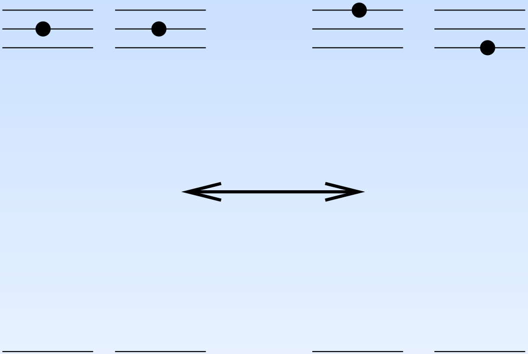
Excited atom with large principal number n

- size $\sim n^2$ ($\sim 0.3\ \mu$m for $n \approx 80$)
- lifetime $\sim n^3$–$n^{4.5}$ ($\sim 600\ \mu$s for $n \approx 80$)



Rydberg atom

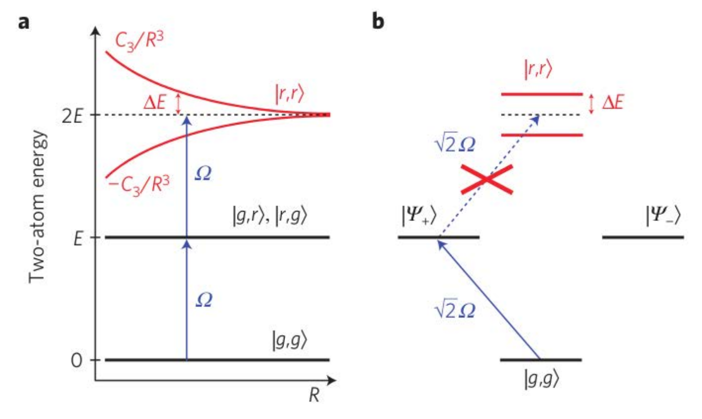
Rydberg blockade: resonance transitions



Example: squeezing with Rydberg blockade

Rydberg atom

Rydberg blockade: resonance transitions



Gaetan et al., Nature Physics 5, 115 (2009)

Example: squeezing with Rydberg blockade

Jaynes - Cummings model

A single two-level atom and a single-mode quantum field

$$
\begin{aligned}
H_{JC} &= g a^{+} \sigma_{-} + g^{*} a \sigma_{+} \\
\sigma_{+} &= |b\rangle\langle a| \\
\sigma_{-} &= |a\rangle\langle b|
\end{aligned}
$$

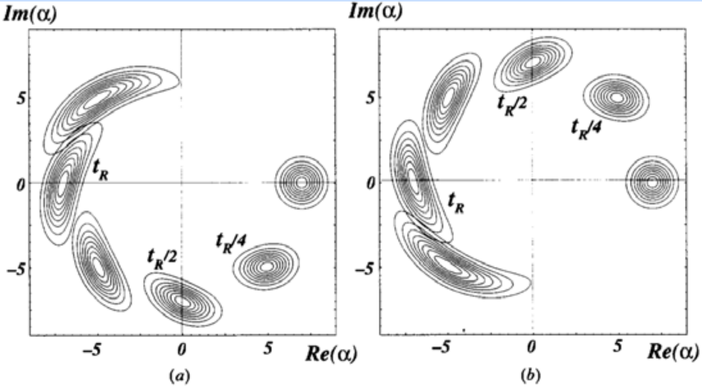
- Photon generation and atom deexcitation
- Photon absorption and atom excitation

Example: squeezing with Rydberg blockade

Jaynes - Cummings model

A single two-level atom and a single-mode quantum field

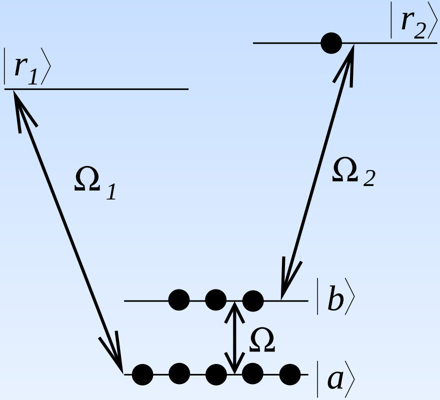
Squeezing of the field



G. Banacloche, PRL 65, 3385 (1990); picture from JMO 40, 2361 (1993).

Example: squeezing with Rydberg blockade

Spin squeezing and Schrödinger cat generation in atomic samples with Rydberg blockade



T. Opatrný and K. Mølmer, PRA 86, 023845 (2012)

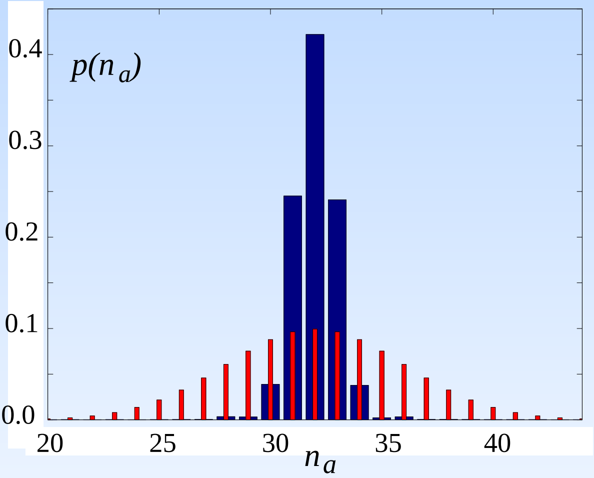
Example: squeezing with Rydberg blockade

Hamiltonian

$$
\begin{aligned}
H_{JC1} &= \Omega_1 a \sigma_+^{(1)} + \Omega_1^* a^\dagger \sigma_-^{(1)} \\
H_{JC2} &= \Omega_2 b \sigma_+^{(2)} + \Omega_2^* b^\dagger \sigma_-^{(2)}
\end{aligned}
$$

- Initialize the state
- Act with the Hamiltonian
- Rotate the state

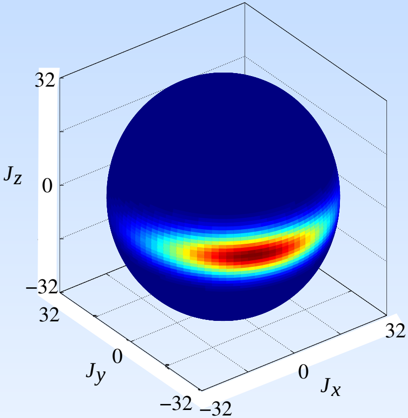
Results



Statistics of the atomic states $|a\rangle$ and $|b\rangle$ (64 atoms)

Example: squeezing with Rydberg blockade

Results



Q-function of the resulting state (64 atoms)

Example: squeezing with Rydberg blockade

Adiabatic squeezing: Hamiltonian eigenstates

$$
\begin{aligned}
|\psi_{+,+}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle + |n_a - 1, n_b, 1, 0\rangle\right. \\
&+ \left.|n_a, n_b - 1, 0, 1\rangle + |n_a - 1, n_b - 1, 1, 1\rangle\right), \\
|\psi_{+,-}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle + |n_a - 1, n_b, 1, 0\rangle\right. \\
&- \left.|n_a, n_b - 1, 0, 1\rangle - |n_a - 1, n_b - 1, 1, 1\rangle\right), \\
|\psi_{-,+}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle - |n_a - 1, n_b, 1, 0\rangle\right. \\
&+ \left.|n_a, n_b - 1, 0, 1\rangle - |n_a - 1, n_b - 1, 1, 1\rangle\right), \\
|\psi_{-,-}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle - |n_a - 1, n_b, 1, 0\rangle\right. \\
&- \left.|n_a, n_b - 1, 0, 1\rangle + |n_a - 1, n_b - 1, 1, 1\rangle\right),
\end{aligned}
$$

Example: squeezing with Rydberg blockade

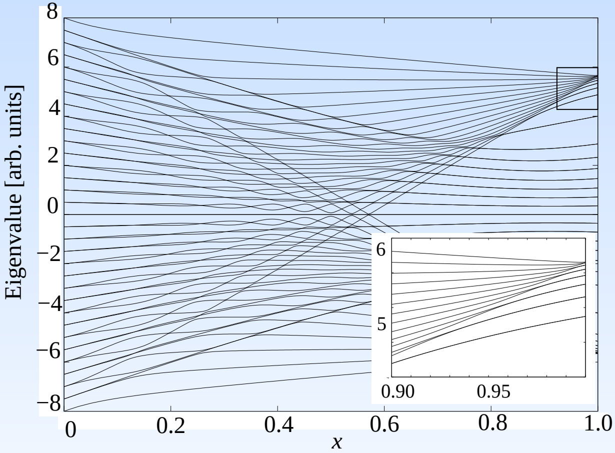
Adiabatic squeezing: Eigenenergies

$$\begin{aligned}
E_{+,+}^{(n_a,n_b)} &= \Omega_{JC}\left(\sqrt{n_a}+\sqrt{n_b}\right), \\
E_{+,-}^{(n_a,n_b)} &= \Omega_{JC}\left(\sqrt{n_a}-\sqrt{n_b}\right), \\
E_{-,+}^{(n_a,n_b)} &= \Omega_{JC}\left(-\sqrt{n_a}+\sqrt{n_b}\right), \\
E_{-,-}^{(n_a,n_b)} &= \Omega_{JC}\left(-\sqrt{n_a}-\sqrt{n_b}\right).
\end{aligned}$$

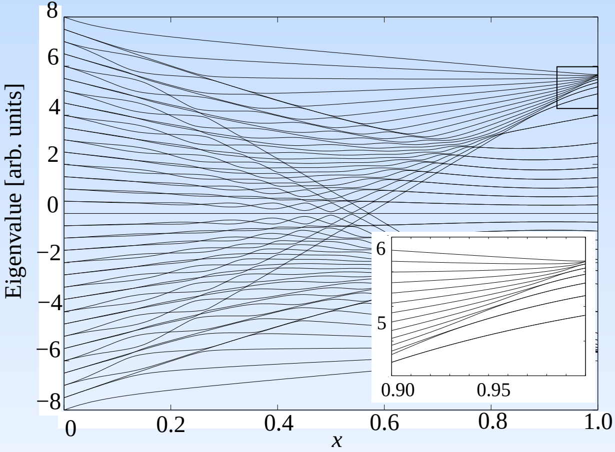
Example: squeezing with Rydberg blockade

Adiabatic squeezing: Combine Hamiltonian

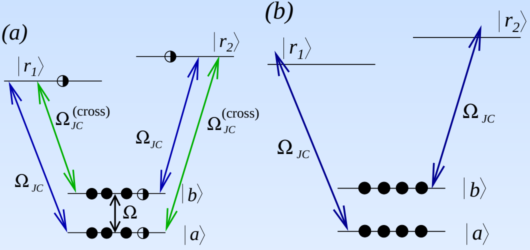
$$H = uH_{JC} + (1-u)J_x$$



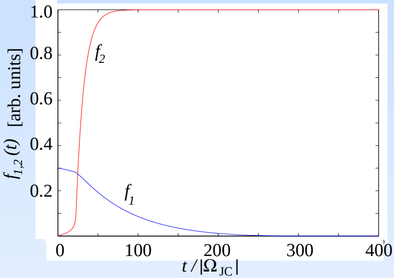
BUT PROBLEM: LINES TOO CLOSE!

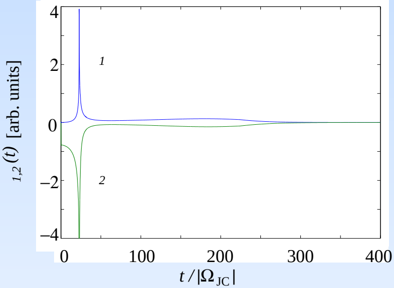


Adiabatic squeezing



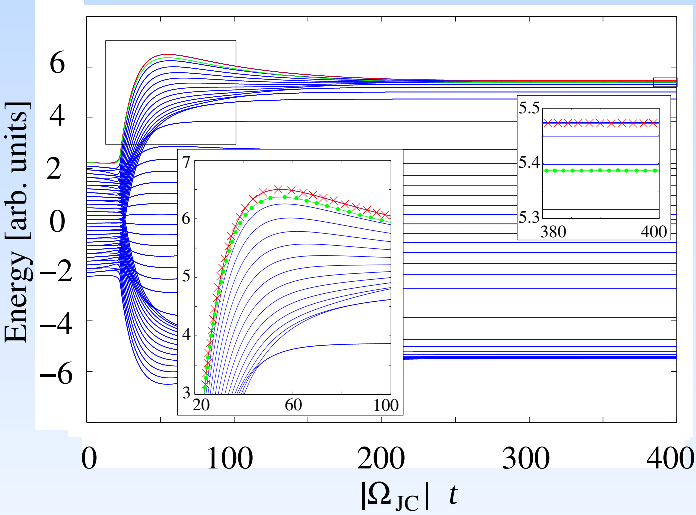
Adiabatic squeezing





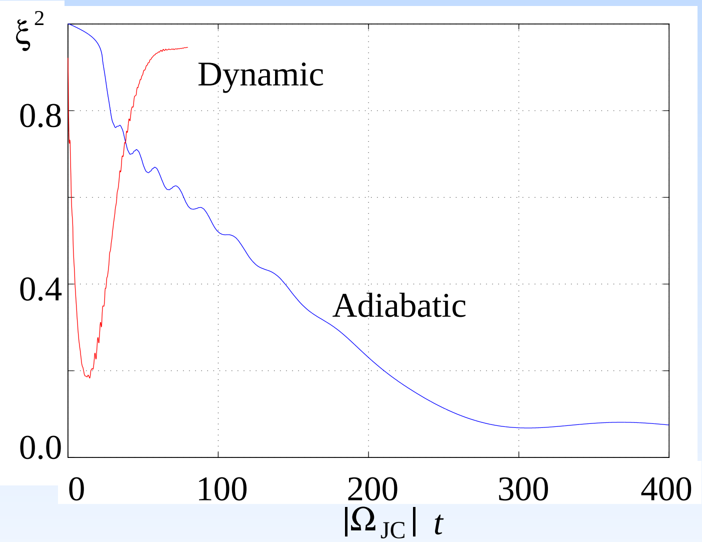
Example: squeezing with Rydberg blockade

Adiabatic squeezing



Example: squeezing with Rydberg blockade

Adiabatic squeezing



Partial suppression of nonadiabatic transitions

Discussion, open questions

- Any general rule for the spin systems?
- Applicability in trapped ion systems?
- Is there any possibility for compensation of nonadiabatic processes in superfluid — Mott insulator transitions?
- Any suitable approximative methods for large systems (Hilbert space expands, impossible to solve exactly)?
- So far optimization for a single state $|0\rangle$. Any possibility for optimization of more states? What about qubit?

Partial suppression of nonadiabatic transitions

Summary

- Adiabatic processes - robust, but slow. Speeding up means transitions to unwanted states.
- Additional Hamiltonian H_B can fully compensate nonadiabatic transitions. Easy to compute, but often impossible to produce in a lab.
- Partial compensation using available operators L_k: need to have nonzero averages $\langle L_k H_B + H_B L_k \rangle$ in the wanted state $|0\rangle$.
- Several examples: paramagnetic vs. antiferomagnetic spin interactions, expanding box, atoms with Rydberg blockade.
- Many open questions remain.

Thanks for your attention!