Optical quantum information processing with inherently stable bulk interferometer

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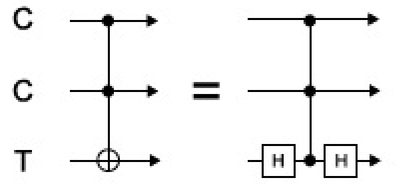
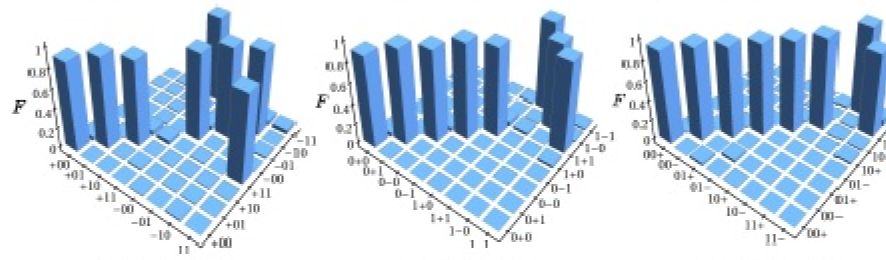
Quantum Toffoli gate

The Toffoli gate has two control qubits and one target qubit. If both control qubits are in state $|1\rangle$ the gate performs NOT operation on the target qubit. Toffoli gate is equivalent to CCZ gate up to local single-qubit Hadamard transforms on the target qubit.



Lower bound on the gate fidelity

To effectively evaluate the gate performance, we have developed a lower bound on quantum process fidelity for *N*-qubit CZ gates [4], based on the original Hofmann fidelity bound [5]. Our generalized Hofmann fidelity bound is determined by the average output state fidelities for *N* partially conjugate product bases.



$F_1 = 0.928(1)$ $F_2 = 0.947(1)$ $F_3 = 0.955(1)$

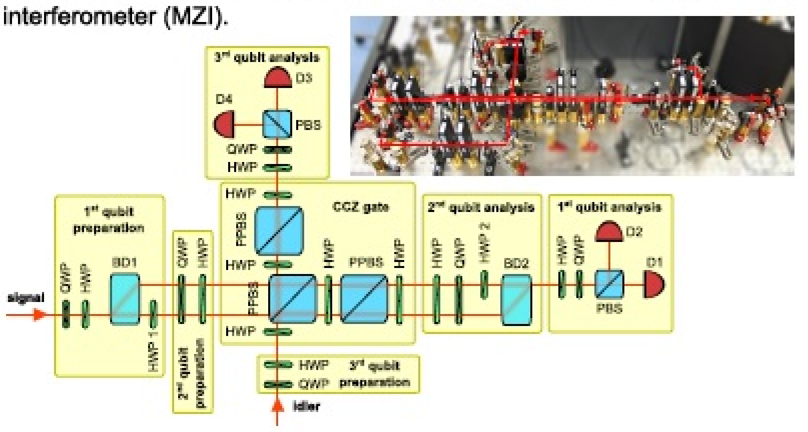
$$F_{CCZ} \geq F_1 + F_2 + F_3 - 2$$
$$\Downarrow$$
$$F_{CCZ} \geq 0.830(2)$$

Monte Carlo sampling

$F_{CCZ} = 0.89$

Experimental realization

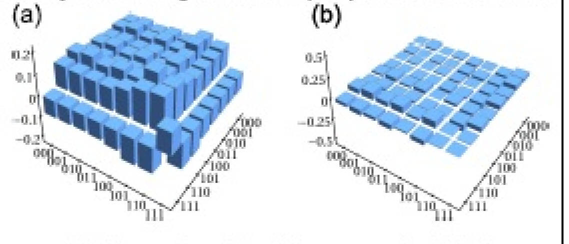
Correlated photon pairs generated by SPDC are employed. Two qubits are encoded into spatial and polarization degrees of freedom of the signal photon and the third qubit is encoded into polarization of the idler photon. The scheme is based on the two-qubit linear optical CZ gate operating in the coincidence basis [1-3] placed into Mach-Zehnder interferometer (MZI).



BD - calcite beam displacer, PPBS - partially polarizing beam splitter with transmittances TV = 1/3 and TH = 1, PBS - polarizing beam splitter, HWP - half-wave plate, QWP - quarter-wave plate, D - single-photon detector.

Entanglement generation

Real (a) and imaginary (b) parts of the reconstructed density matrix of the three-qubit state generated by the CCZ gate from input product state $|{+}{+}{+}\rangle$:

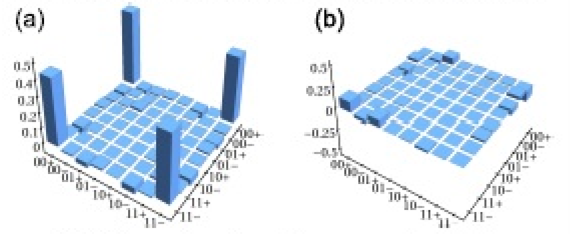


Fidelity and purity of the generated state

$F = 0.951$ **$P = 0.959$**

Preparation of GHZ state

Real (a) and imaginary (b) parts of the reconstructed density matrix of the three-qubit GHZ state generated by the CCZ gate from input state $|\Phi_2^+\rangle|+\rangle$:

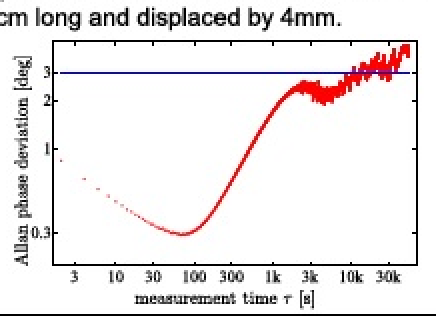


Fidelity and purity of the generated state

$F = 0.962$ **$P = 0.949$**

Phase stability of MZI

We employ the Allan variance [6] to evaluate the phase stability of the Mach-Zehnder interferometer formed by two calcite beam displacers. The interferometer arms are 65cm long and displaced by 4mm.



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Quantum orthogonalization

Perfect universal NOT (U-NOT) gate is forbiden by the laws of quantum physics:

$$|\psi\rangle \nrightarrow |\psi_\perp\rangle, \ \langle\psi_\perp|\psi\rangle = 0.$$

Imperfect U-NOT gate is possible but fundamentally limited [7]. The minimum average overlap $F(d) = 1/(d+1)$, where d is dimension of the input state. For optimal imperfect U-NOT gate the average fidelity reads 2/3 [8-11]. Perfect U-NOT is possible for partly unknown quantum states [12,13].

Orthogonalization of partly unknown states

The only information required for perfect orthogonalization is knowledge of a mean value $a = \langle\psi|A|\psi\rangle$ of some operator A. By applying a quantum filter $(A-aI)$ to input state $|\psi\rangle$ we conditionally prepare orthogonal state:

$$|\psi_\perp\rangle \propto (A-aI)\,|\psi\rangle.$$

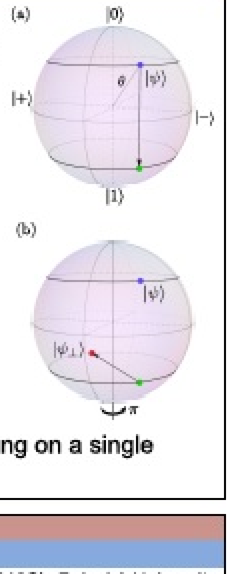
The success probability $p_\perp$ of the procedure is proportional to the variance of operator $\Delta A=(A-aI)$: $p_\perp \leq \langle(\Delta A)^2\rangle/\lambda^2$,

where $\lambda = \max_j|\Delta A_j|$ and ΔA_j denotes eigenvalues of ΔA.

Orthogonalization protocol

Input state: $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
Operator A: $\sigma_Z = |0\rangle\langle 0| - |1\rangle\langle 1| \Rightarrow \langle\sigma_Z\rangle = \cos\theta$
Filter ΔA: $\propto \sigma_Z - I\cos\theta \propto \tan^2\frac{\theta}{2}\,|0\rangle\langle 0| - |1\rangle\langle 1|$
Success probability: $p_\perp = \tan^2(\theta/2)$

The orthogonalization steps:
a) state selective amplitude attenuation
b) unitary π phase shift



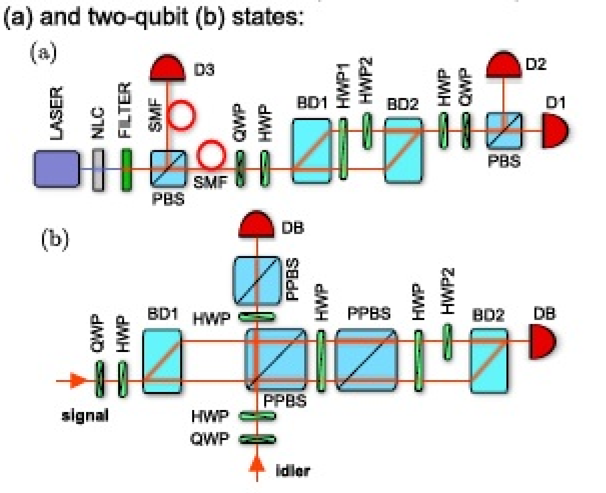
This protocol can be generalized to multipartite systems. Consider bipartite pure state $|\Psi\rangle_{AB}$ with known mean value of an operator A acting on subsystem A, $a = \langle\Psi|\,A_A \otimes I_B\,|\Psi\rangle$. The orthogonal state is:

$$|\Psi_\perp\rangle_{AB} \propto (A-aI)_A \otimes I_B\,|\Psi\rangle_{AB}.$$

The orthogonalization can be performed by local filtering on a single subsystem.

Experimental realization

Experimental setup for orthogonalization of single-qubit (a) and two-qubit (b) states:

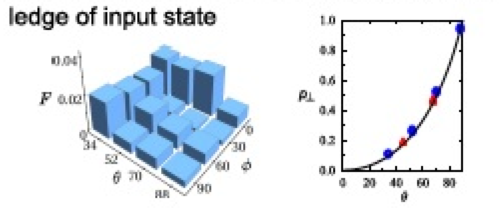


NLC - nonlinear crystal, SMF - single-mode fiber, BD - calcite beam displacer, PPBS - partially polarizing beam splitter, PBS - polarizing beam splitter, HWP - half-wave plate, QWP - quarter-wave pate, D - single-photon detector, DB - detection block consisting of a HWP, QWP, PBS and two single photon detectors.

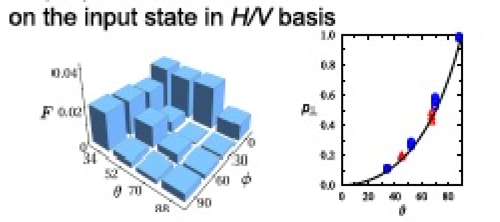
Results - single-qubit states

Overlap F between the input and orthogonalized states and success probability $p_\perp$:

a) $\langle\sigma_Z\rangle$ is determined from theoretical knowledge of input state



b) $\langle\sigma_Z\rangle$ is determined from measurements on the input state in *H/V* basis



Fidelity and purity for all measured cases

$F < 0.0254$ **$P > 0.986$**

Results - two-qubit states

Overlap F between the input (I) and orthogonalized (O) two-qubit states and purity P and entanglement of formation E_f of the input and orthogonalized states. The data are presented for orthogonalization using the knowledge of $\langle\sigma_Z\rangle$ from state preparation (F, P_O, $E_{f,O}$) and for orthogonalization where $\langle\sigma_Z\rangle$ is determined from measurements on the first qubit (F', P_O, $E_{f,O}$).

θ_1	ϕ_1	θ_2	ϕ_2	F	F'	P_I	P_O	P'_O	$E_{f,I}$	$E_{f,O}$	$E'_{f,O}$
45°	0°	90°	0°	0.040	0.044	0.964	0.890	0.909	0.547	0.616	0.622
67.5°	0°	90°	0°	0.031	0.037	0.961	0.891	0.907	0.819	0.807	0.781
45°	0°	45°	0°	0.021	0.029	0.936	0.944	0.942	0.286	0.334	0.361
67.5°	0°	45°	0°	0.008	0.008	0.975	0.952	0.941	0.523	0.482	0.496
67.5°	90°	45°	90°	0.041	0.035	0.971	0.946	0.935	0.497	0.518	0.468

Acknowledgments

This work was supported by the Czech Science Foundation (13-20319S), Palacký University (Project No. PrF-2013-008) and M.S. acknowledges support by the Operational Program Education for Competitiveness - European Social Fund (project No. CZ.1.07/2.3.00/30.0004) of the Ministry of Education, Youth and Sports of the Czech Republic.