Extracting work from quantum states of radiation

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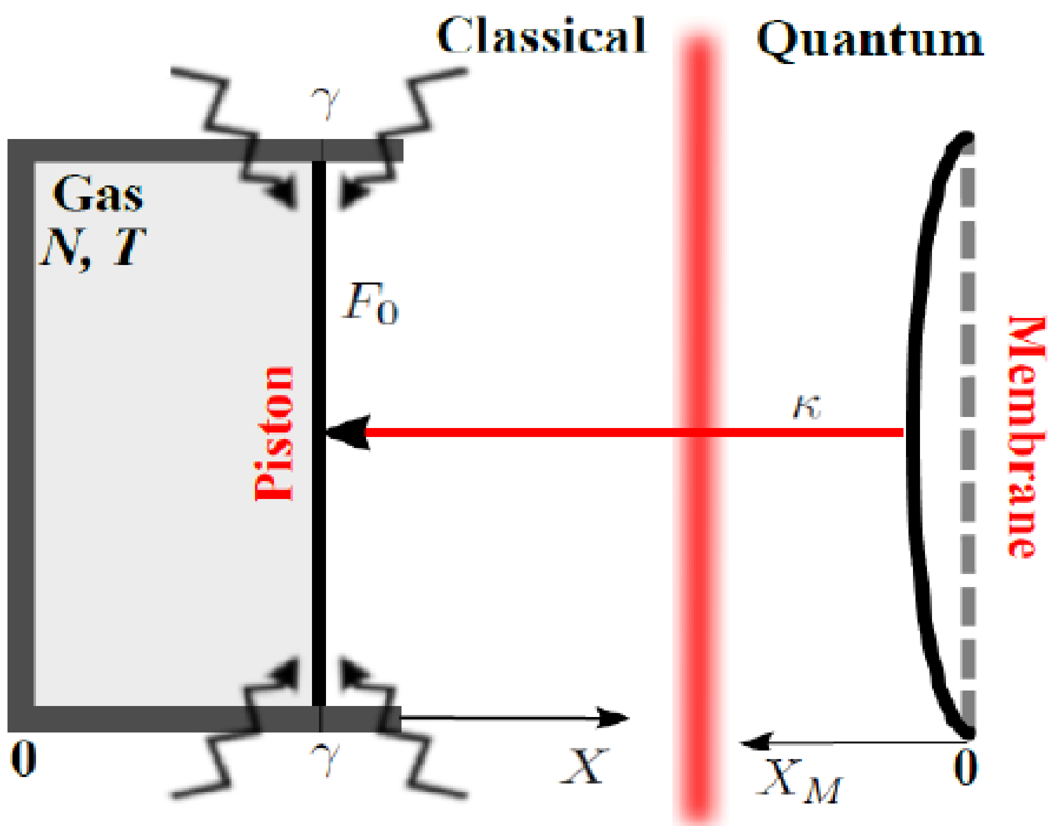
Thermodynamics ⟷ optomechanics ⟷ quantum optics

Energy transfer?

Thermodynamics ⟵ optomechanics ⟵ quantum optics

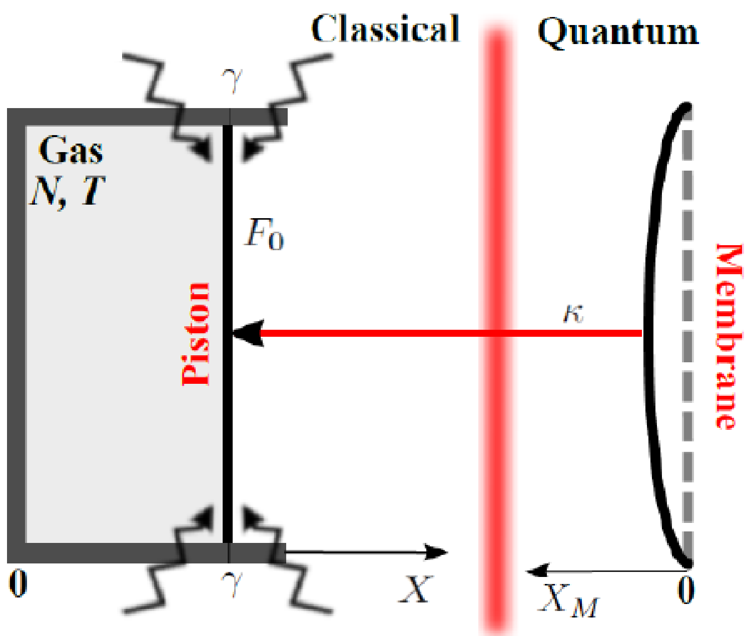


Initially, the quantum state of the light is *ideally* transferred to the state of the mechanical membrane. This assumption causes the virtual absence of the light in our chain. The membrane position (together with its uncertainty) is transferred to the classical piston sealing the container with a classical ideal gas.

The Model

The ideal gas:

$$PS = \frac{Nk_BT}{X}$$



The membrane:

$$\hat{H}_M = \frac{\hat{P}_M^2}{2m_M} + \frac{m_M\omega^2}{2}\hat{X}_M^2$$

The piston:

$$H_{MP} = \kappa X_M X.$$

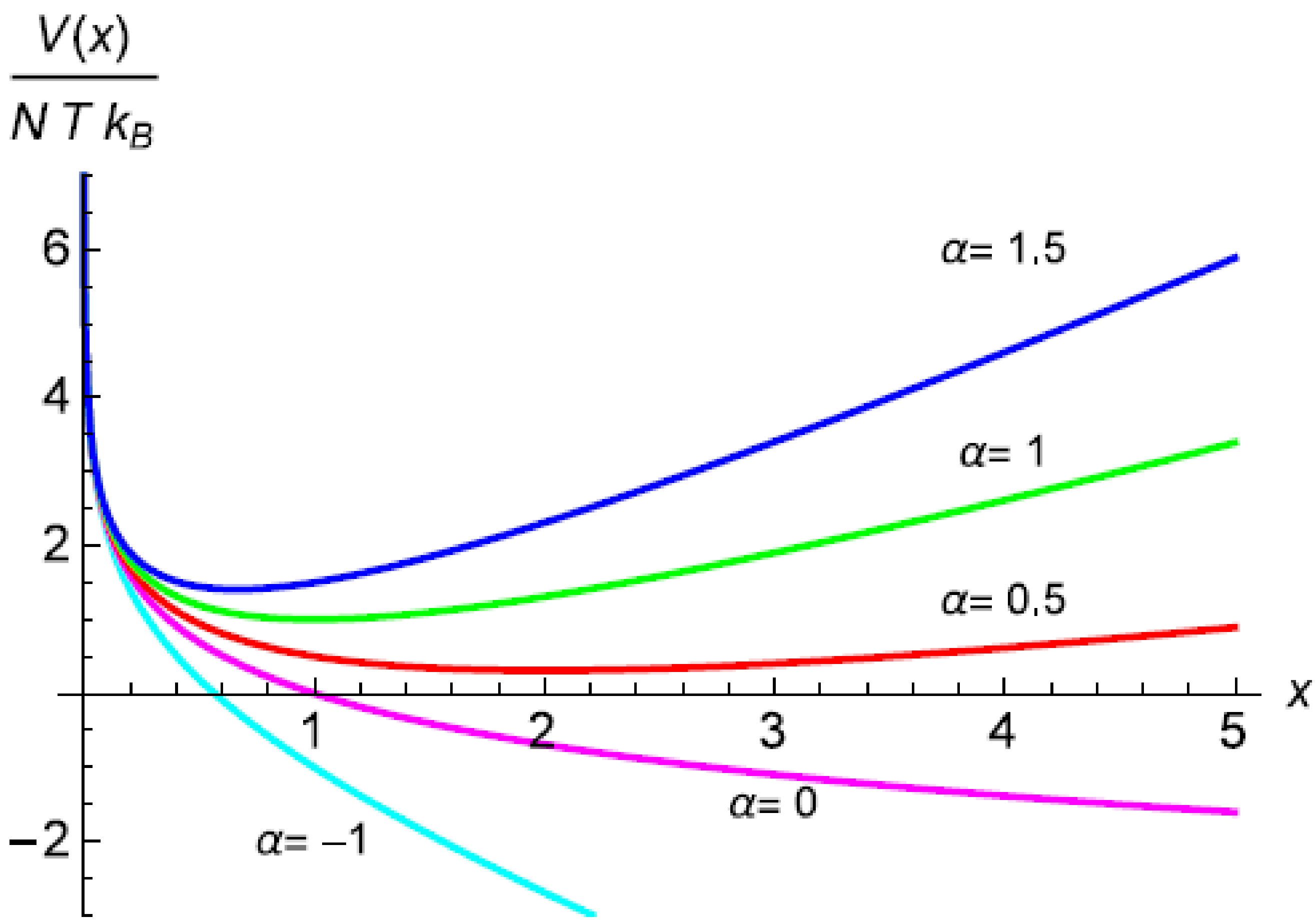
$$\gamma\dot{X} = F(X,t) + \sqrt{2\gamma k_B T}\,\xi(t)$$

$$F(X,t) = -\kappa X_M - F_0 + \frac{Nk_BT}{X}$$

The piston potential:

$$F(X,t) = -\kappa X_M - F_0 + \frac{Nk_BT}{X} \qquad V(X) = (\kappa X_M + F_0)\,X - Nk_BT\ln(X/L)$$

$$L \equiv Nk_BT/F_0 \qquad x \equiv X/L$$



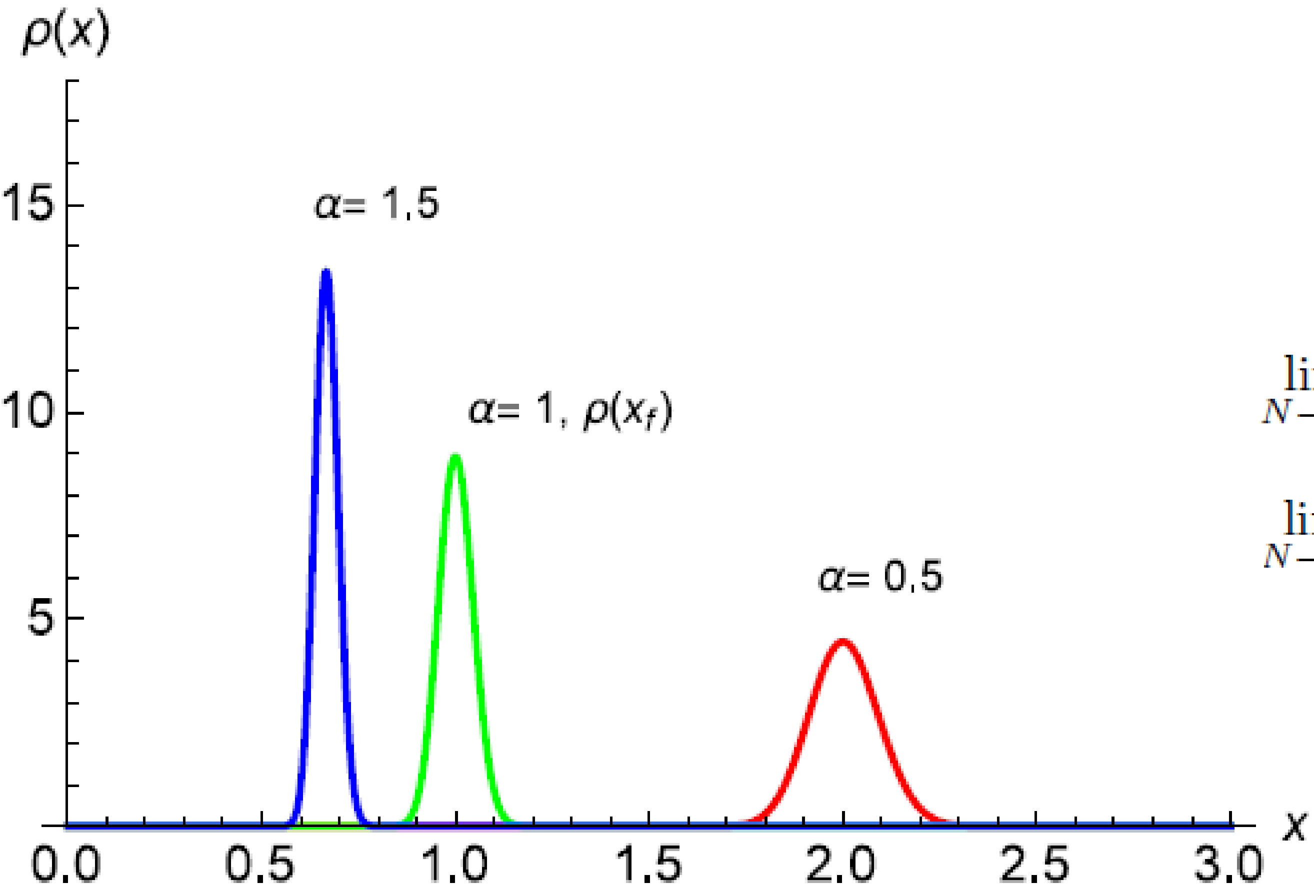
$$V(x) = Nk_BT\,(\alpha x - \ln x)$$

$$\alpha \equiv 1 + \frac{\kappa X_M}{F_0}$$

The piston equilibrium position distribution:

$$\rho(x) = \frac{1}{Z} x^N \exp\left[-\alpha N x\right] \qquad x \geq 0 \quad \alpha > 0$$

$$Z = \frac{\Gamma(N+1)}{(\alpha N)^{(N+1)}}$$



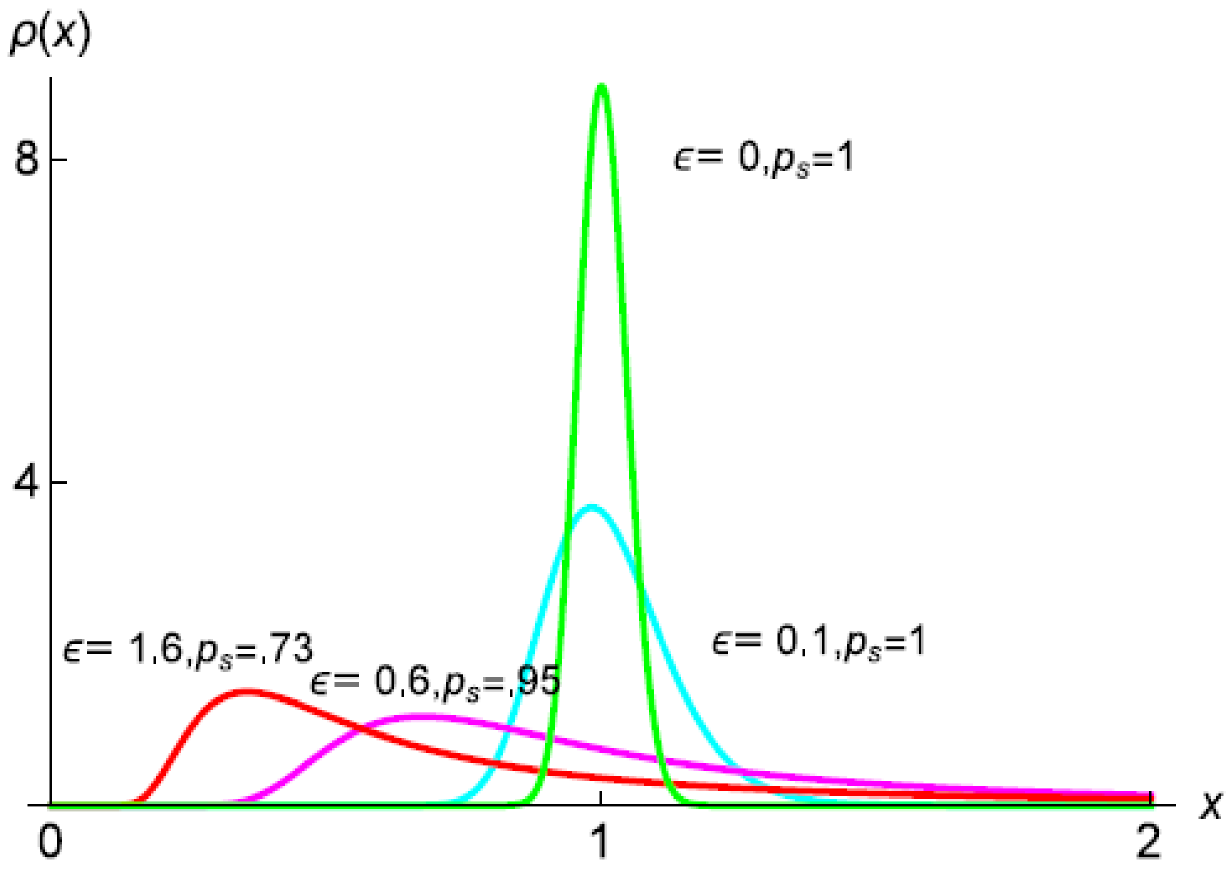
$$\lim_{N\to\infty} \rho(x_i) = \delta(x_i - 1/\alpha)$$

$$\lim_{N\to\infty} \rho(x_f) = \delta(x_f - 1)$$

The membrane with uncertainty:

$$\rho(x_i) = \frac{\int_0^\infty p(\alpha) Z_i^{-1} x_i^N \exp\left[-\alpha N x_i\right] \mathrm{d}\alpha}{\int_0^\infty p(\alpha)\mathrm{d}\alpha} \qquad p_s \equiv \int_0^\infty p(\alpha)\mathrm{d}\alpha$$

Gaussian family: $$p(\alpha) = \frac{1}{\sqrt{2\pi\epsilon}} \exp\left[-\frac{(\alpha-\alpha_0)^2}{2\epsilon^2}\right]$$

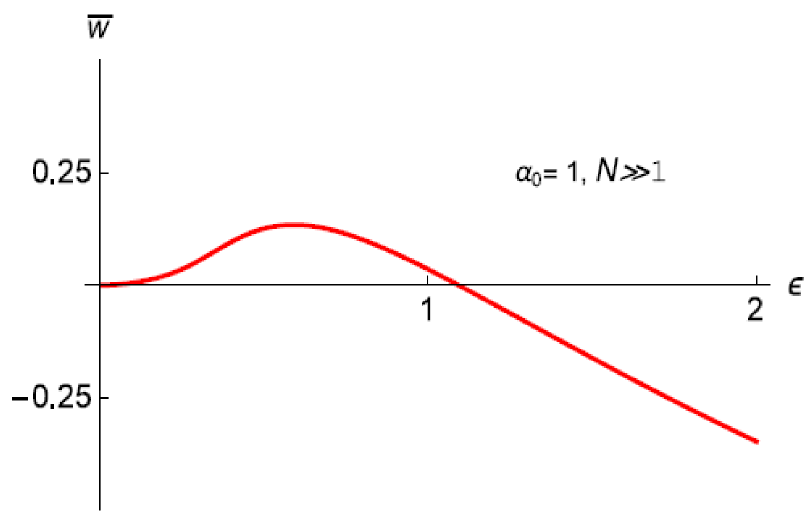


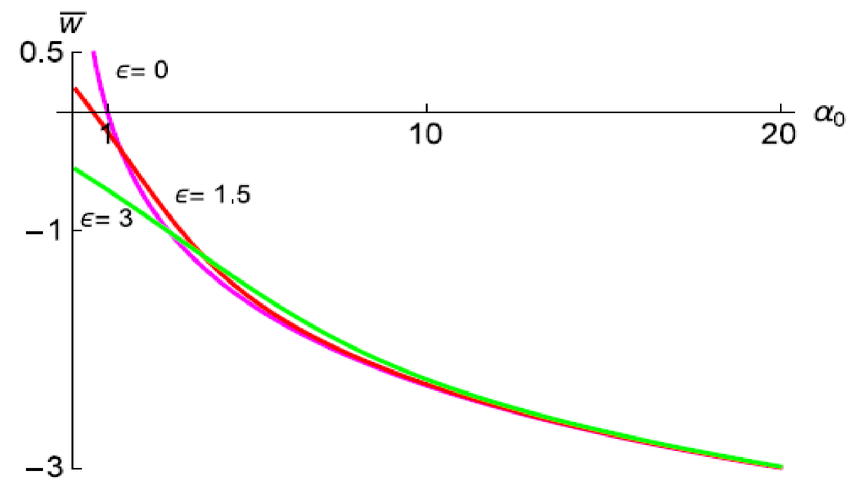
Work of the gas and piston:

$$W \equiv -\int_{x_i}^{x_f} PS\,\mathrm{d}x = -Nk_BT\ln\frac{x_f}{x_i} \qquad N \gg 1$$

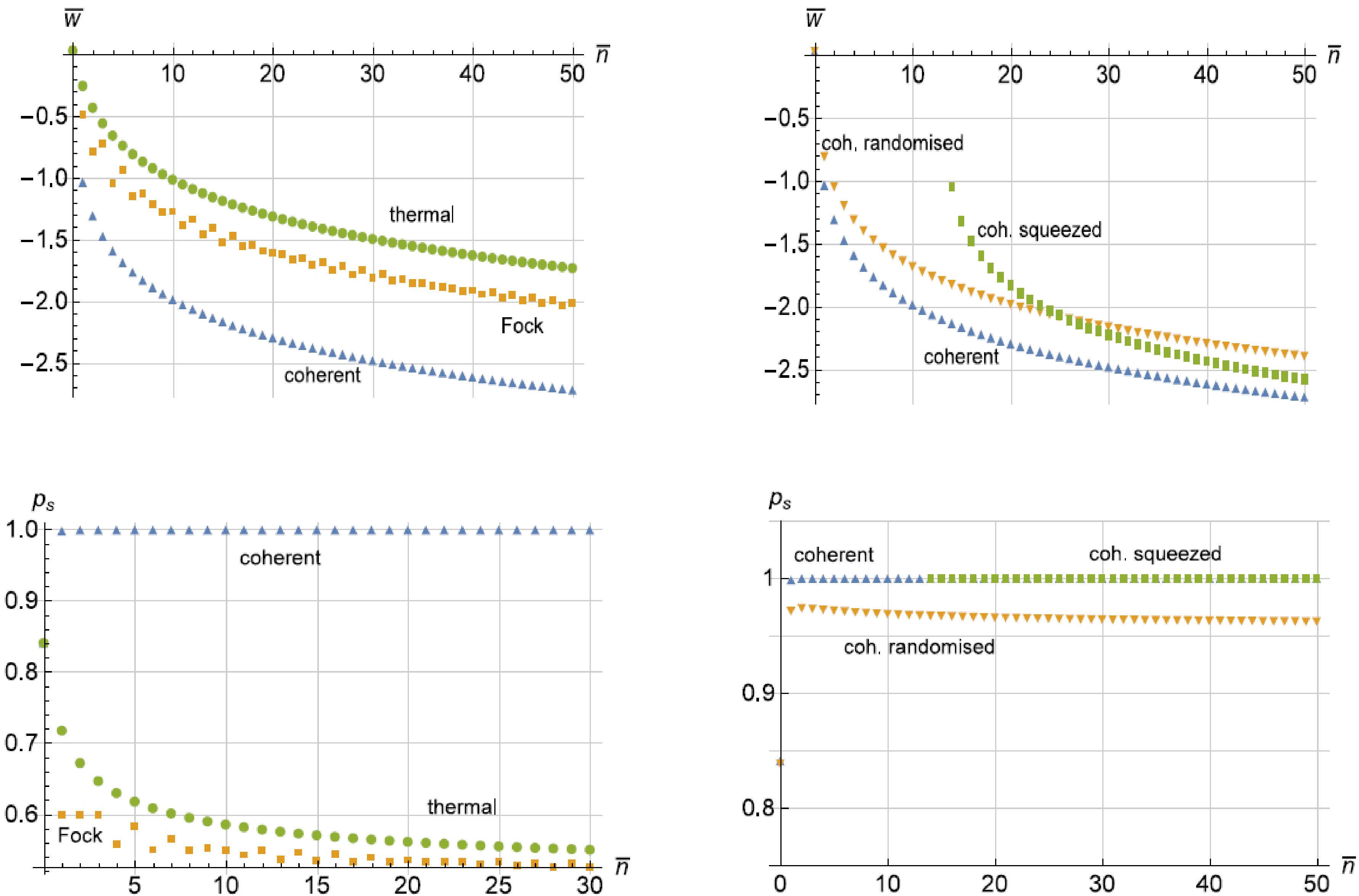
$$w(\alpha) \equiv \frac{W}{Nk_BT} = -\ln\alpha,\ \alpha = 1+\frac{\kappa X_M}{F_0}$$

$$\overline{w} = -\overline{\ln\alpha} = -\int_{-\infty}^{\infty}\ln(\alpha)\overline{p}(\alpha)\mathrm{d}\alpha \qquad \overline{p}(\alpha) \equiv \frac{\theta(\alpha)p(\alpha)}{\int_{-\infty}^{\infty}\theta(\alpha)p(\alpha)\,\mathrm{d}\alpha}$$





Work of the gas and piston:



Optomechanical oscillator controlled by variation in its heat bath temperature

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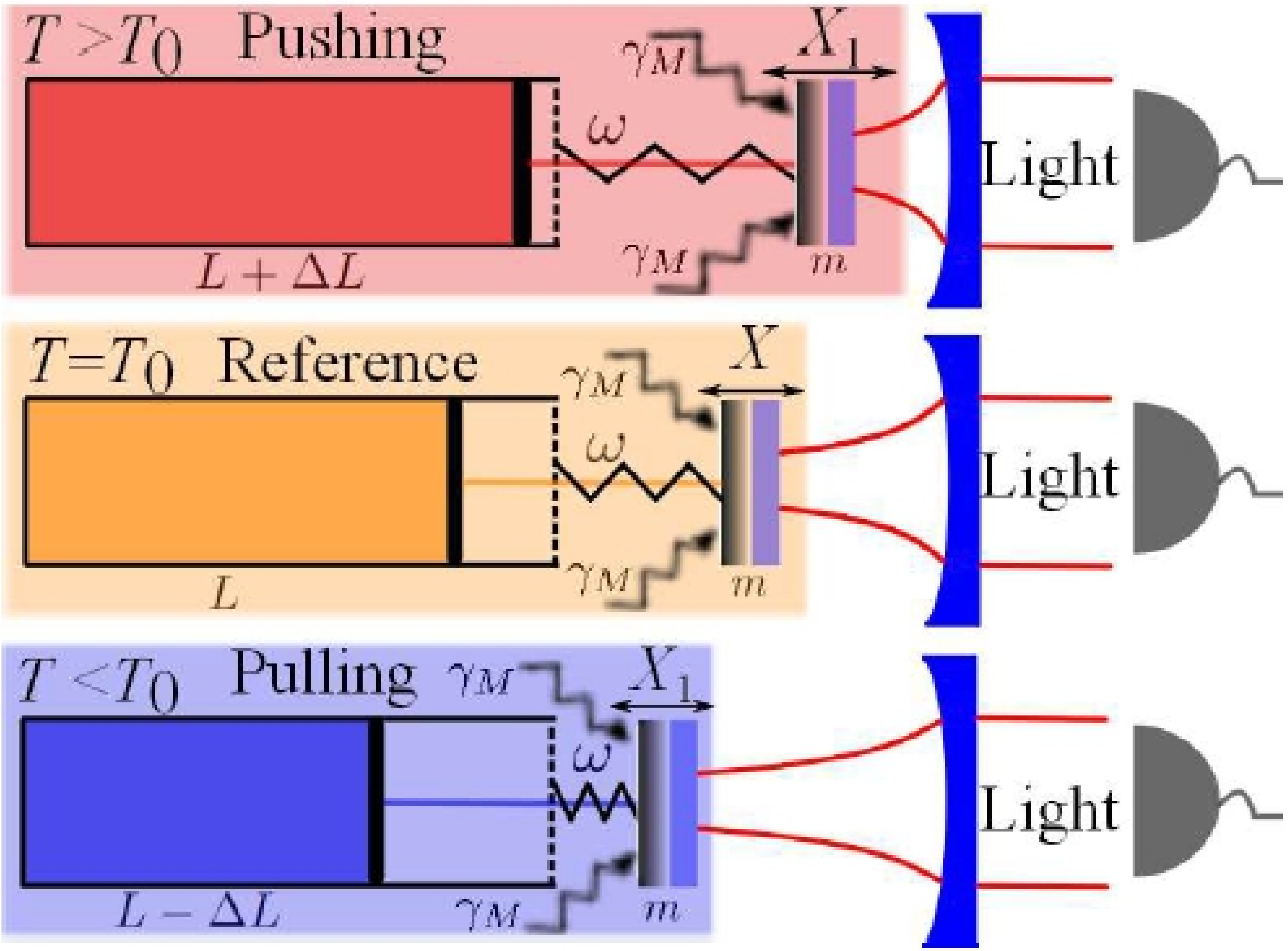
Low noise mechanical states prepared by incoherent control?

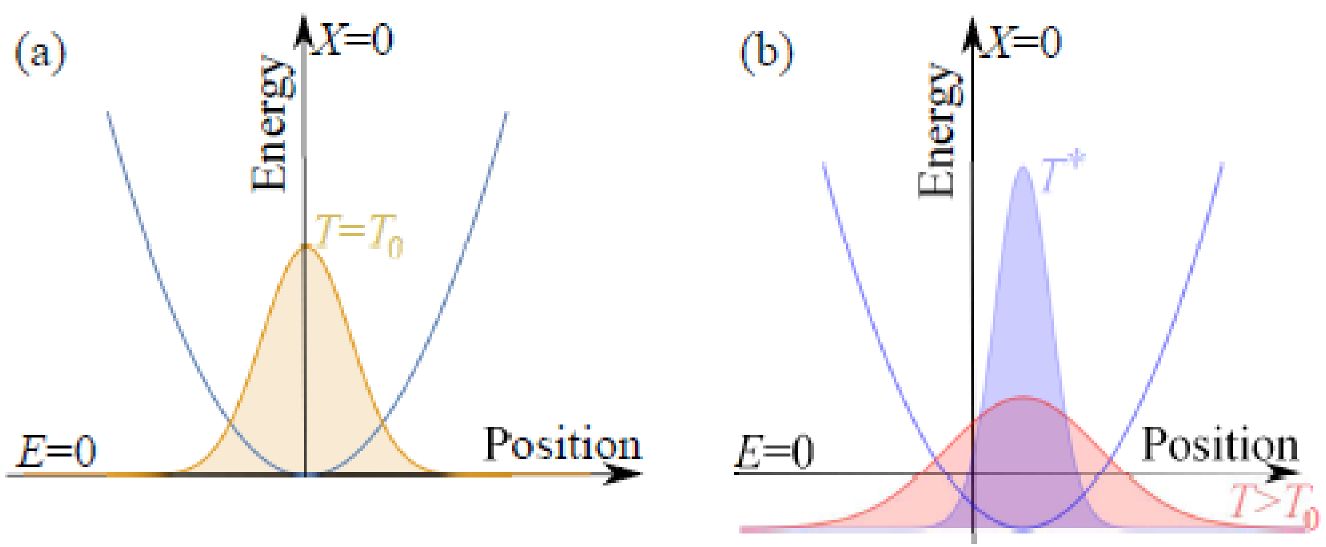


Schematic of the analyzed opto-thermo-mechanical setup. The optomechanical membrane and the thermomechanical piston are both embedded in a heat bath with some coupling strength at temperature T. At some reference temperature T_0 the thermomechanical piston has a length L, setting some reference equilibrium position $X = 0$ of the membrane (with mass m and angular frequency). By changing the bath temperature T we shift the equilibrium position of the membrane to $X_1 = 0$ due to the piston thermal expansion.

The model:

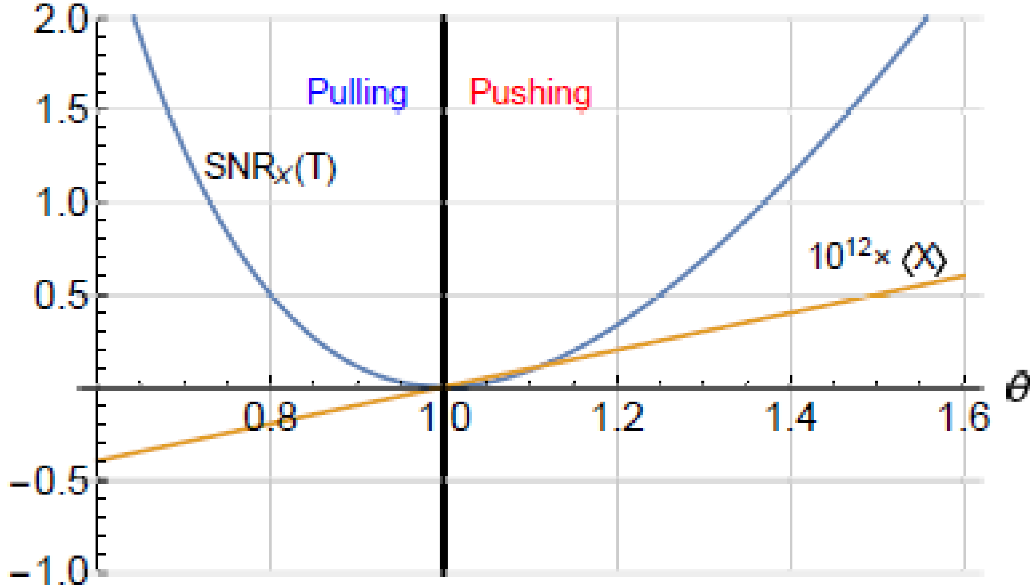
$$\hat{H}_0 = \frac{\hat{P}^2}{2m} + \frac{m\omega^2}{2}\hat{X}^2 \qquad \hat{H}_I(T) = -f(T)\hat{X}$$

$$\hat{H}(T) = \frac{\hat{P}^2}{2m} + \frac{m\omega^2}{2}\left[\hat{X} - \frac{f(T)}{m\omega^2}\right]^2 - \frac{f(T)^2}{2m\omega^2}$$



The mechanical aspect of the equilibrium state:

$$\mathrm{SNR}_X(T) = \frac{\langle X \rangle^2}{\mathrm{Var}(X)} = \frac{f(T)^2}{k_B T m \omega^2}$$



$$f(T) = \kappa\alpha(T - T_0)$$

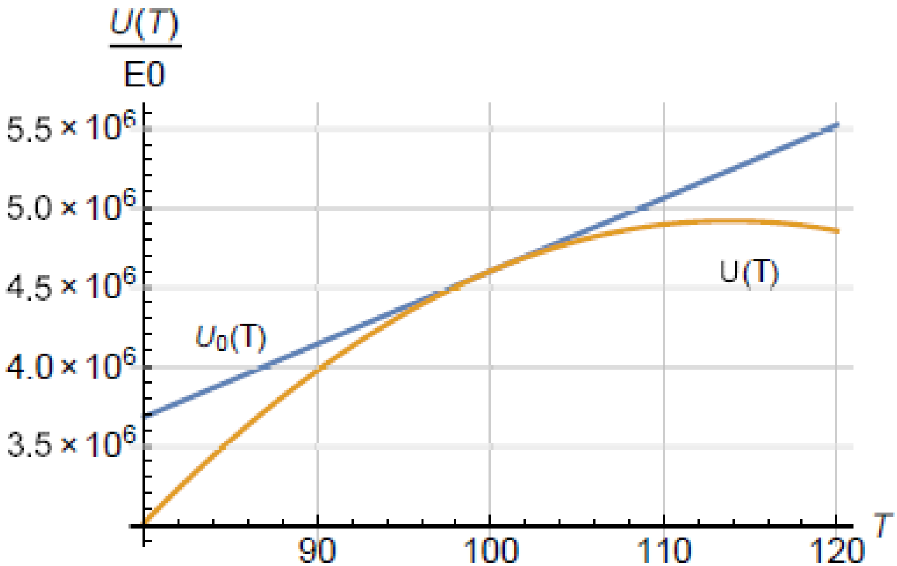
$$\langle H(T) \rangle = k_B T \left[1 - \frac{\mathrm{SNR}_X(T)}{2} \right]$$

The thermodynamic aspects of the equilibrium state:

$$U = U_0(T) - \frac{f(T)^2}{2m\omega^2} \qquad U = \langle H(T) \rangle \qquad U_0(T) = \langle \hat{H}_0 \rangle_{f=0}$$

$$F = F_0(T) - \frac{f(T)^2}{2m\omega^2} \qquad F_0(T) = -k_B T \ln Z_0$$

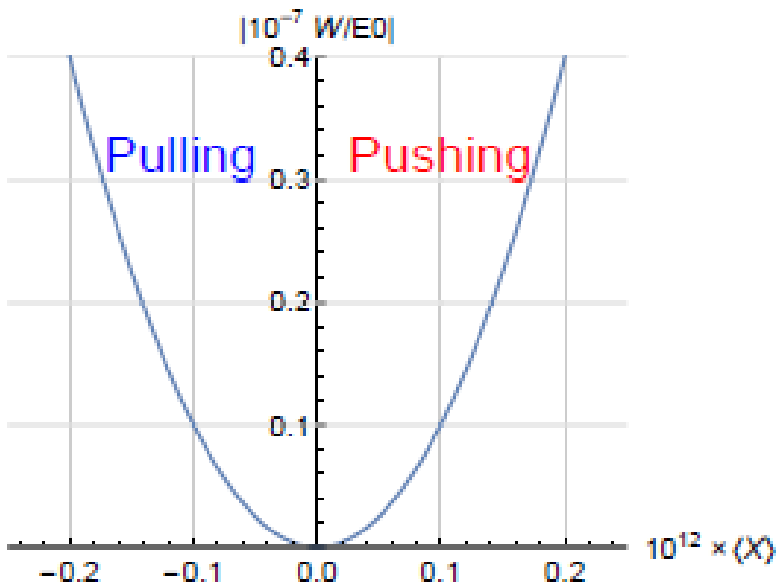
$$S = -\frac{\partial F_0}{\partial T}$$

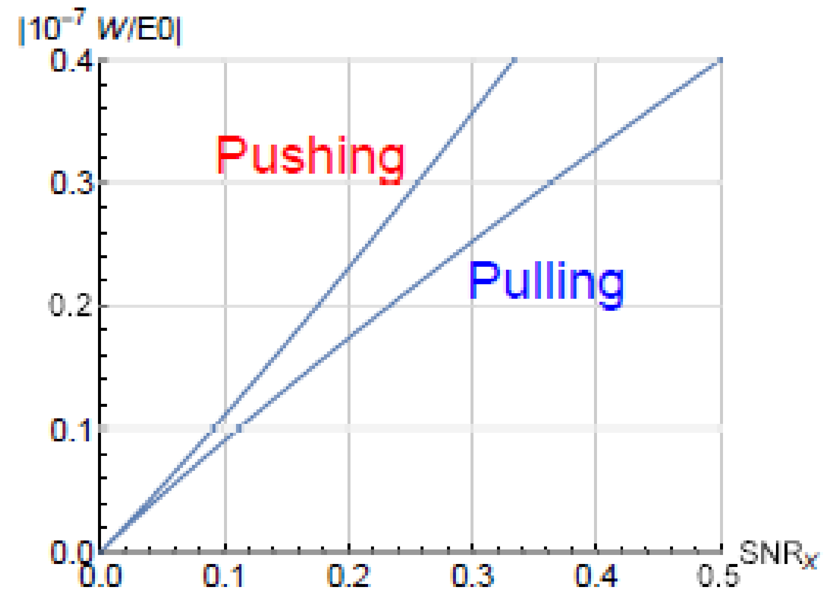


$$c_0(T) = \frac{\delta Q_\mathrm{M}}{\mathrm{d}T} = \frac{\mathrm{d}U_0}{\mathrm{d}T} = T\frac{\partial S}{\partial T}$$

$$c(T) = \frac{\delta W + \delta Q_\mathrm{M}}{\mathrm{d}T} = \frac{\mathrm{d}U}{\mathrm{d}T} = T\frac{\partial S}{\partial T} - \frac{1}{2m\omega^2}\frac{\mathrm{d}f^2}{\mathrm{d}T}$$

The thermodynamic aspects of the equilibrium state:





$$|W(T_0 \to T)| = \frac{f(T)^2}{2m\omega^2} = k_B T \frac{\mathrm{SNR}_X(T)}{2}$$