Achieving quantum-limited optical resolution

Jaroslav Rehacek, Zdenek Hradil,
Martin Paur, and Bohumil Stoklasa

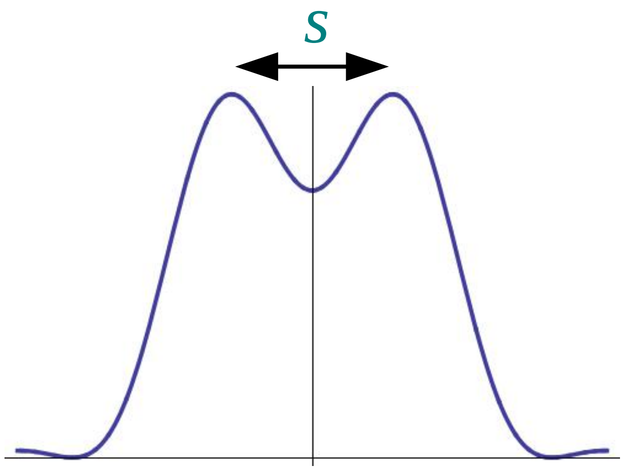
Palacky University, Olomouc, Czech Republic

Luis L. Sánchez-Soto

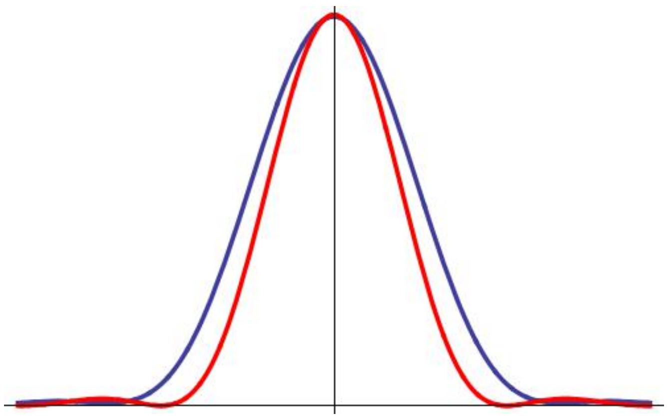
Universidad Complutense, Madrid, Spain

two-point resolution

- two mutually incoherent point sources
- imaging – linear system
- PSF – point spread function



standard resolution

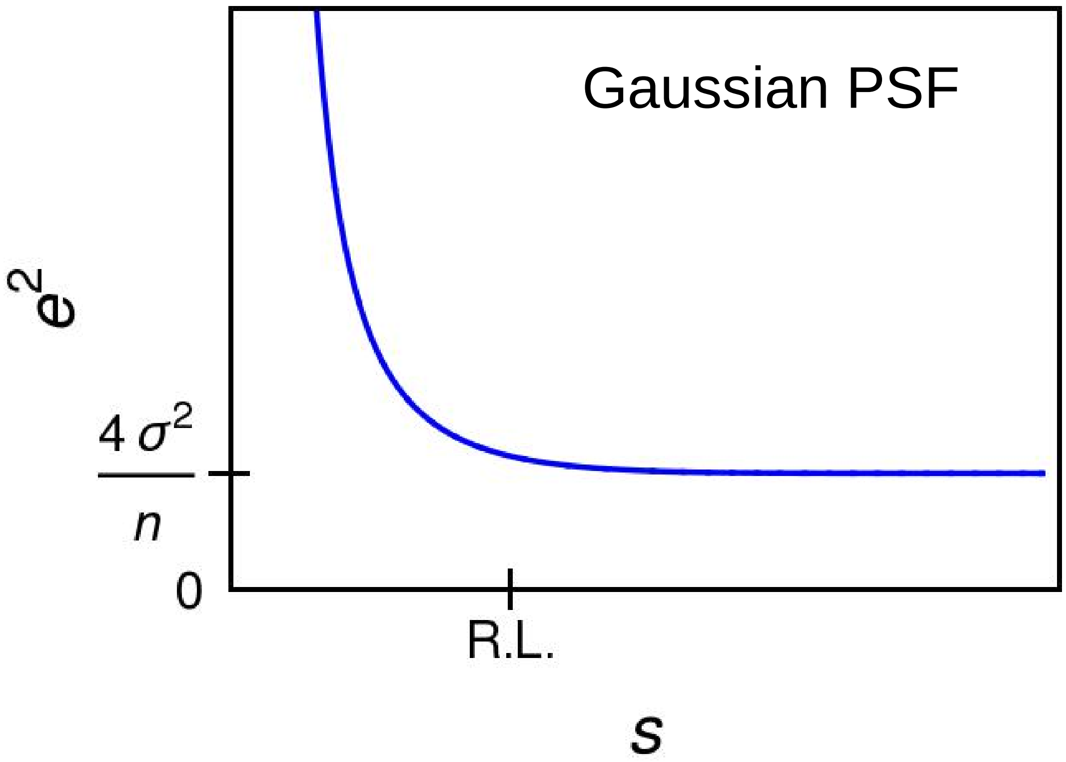


super-resolution

- unlimited resources = unlimited resolution for any PSF
- what can be achieved with limited resources?

efficient unbiased estimators

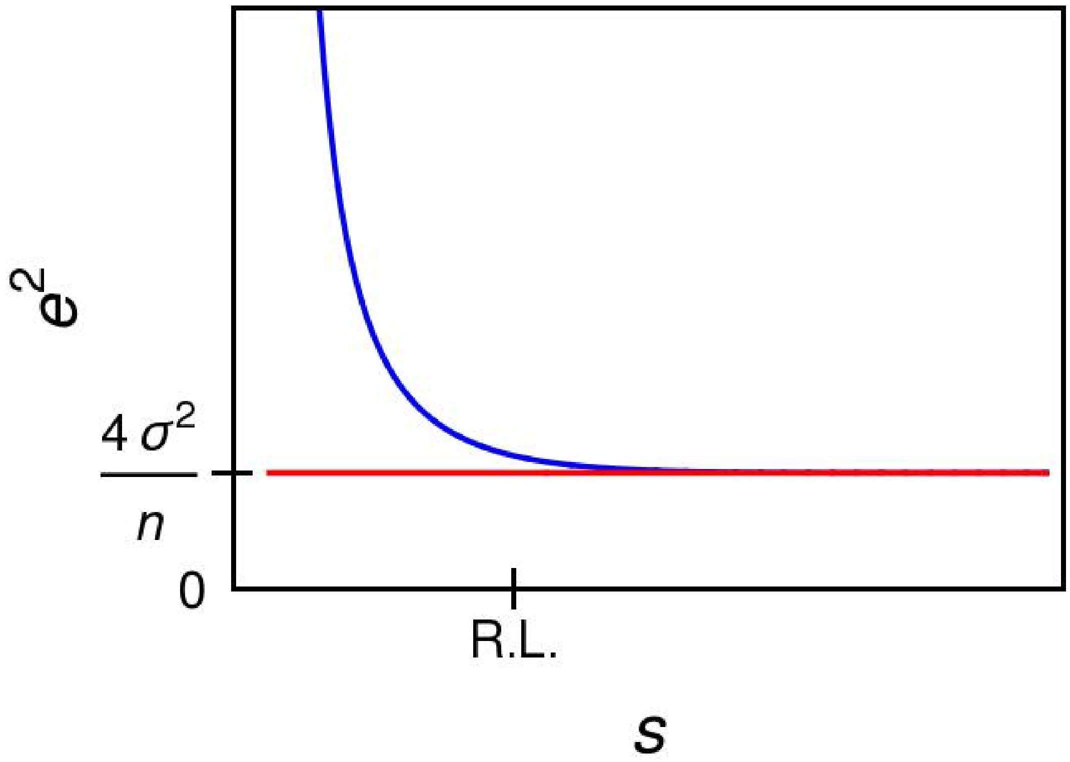
CCD detection – CRLB (classical limit)



- separations well below the R.L. are difficult to estimate
- super-resolution is expensive

efficient unbiased estimators ...

optimal detection – ”quantum” CRLB (quantum limit)

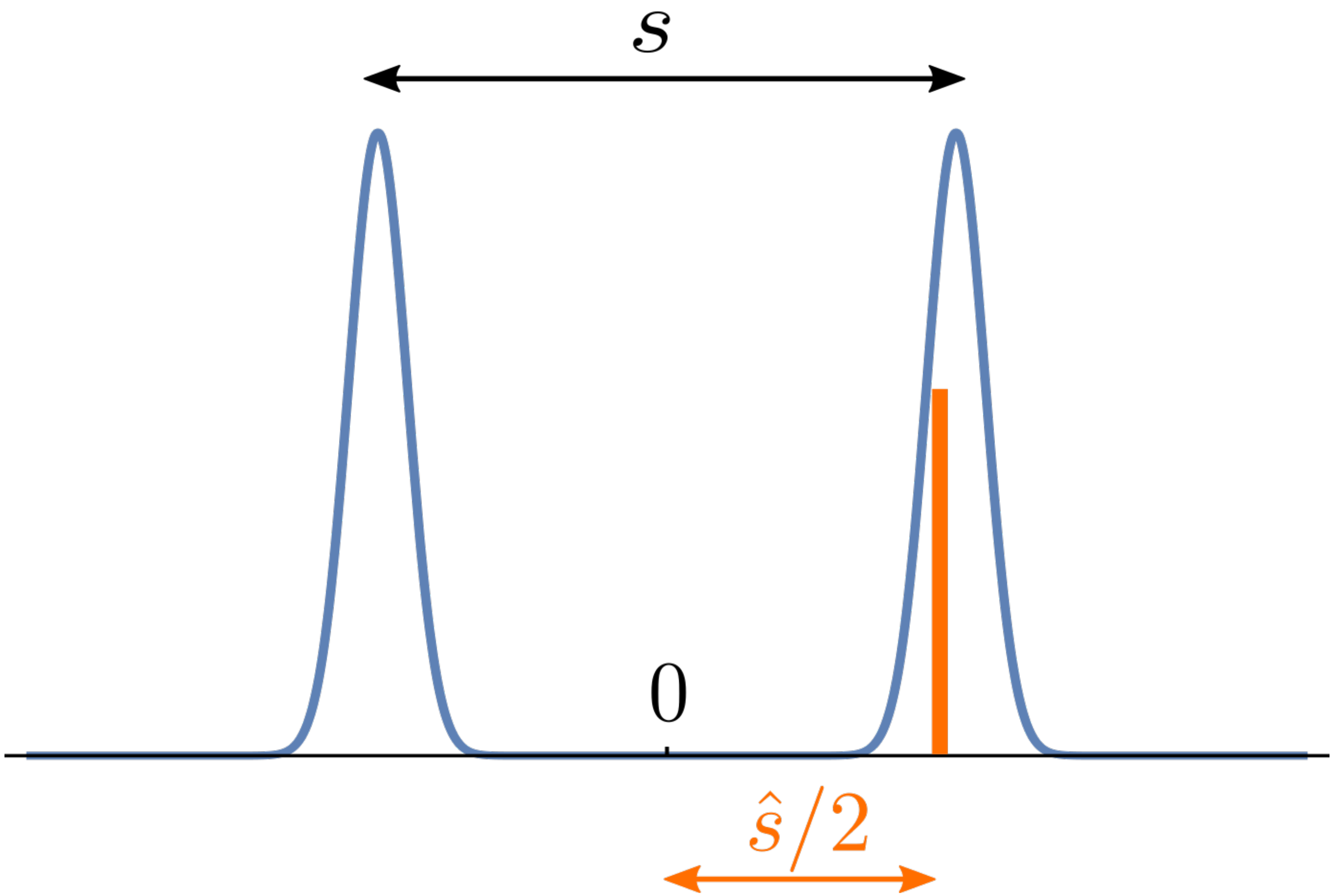


- two-point resolution is reduced to localization (PALM, astrometry)
- super-resolution is easy, at least in theory ...

M. Tsang et al. PRX 2016

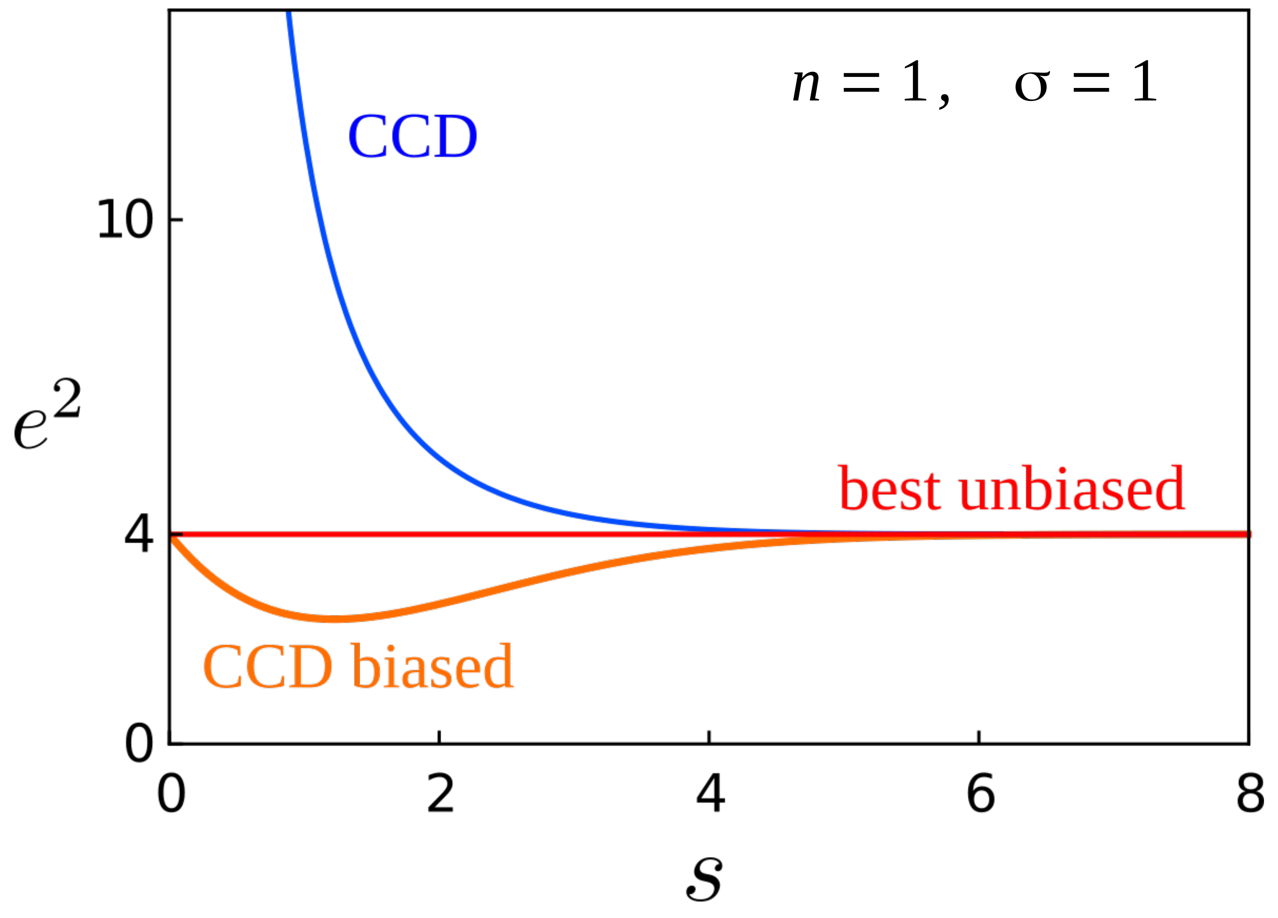
biased estimation

- CRLB applies to unbiased estimators only
- biased estimators can (significantly) violate the CRLB
- consider estimating the separation from a single detection event



biased estimation...

- this produces biased estimates
- quantum limit is violated for almost all separations!



biased estimation...

- meaningful estimators have finite error at $s = 0$
- biased estimators should be considered

- no simple bounds on biased estimators available
- many copies = negligible bias (except for $s \approx 0$)
- hence analysis based on the CRLB makes sense in this limit

optimal measurements

impulse response $\psi(x) = \langle x|\psi\rangle$

momentum

true state $\rho \propto |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|, \quad |\psi_{1,2}\rangle = e^{\pm i P s/2}|\psi\rangle$

small separations $s \ll 1$ (for generalization, see Zdenek's talk)

eigenbasis

$$|\psi_+\rangle \propto |\psi_1\rangle + |\psi_2\rangle \approx |\psi\rangle$$

$$|\psi_-\rangle \propto |\psi_1\rangle - |\psi_2\rangle \approx \frac{P|\psi\rangle}{\sqrt{\mathcal{F}}}$$

$$\mathcal{F} \approx \langle\psi|P^2|\psi\rangle$$

Helstrom: Quantum detection and estimation theory 1976

optimal measurements...

optimal measurement: 2-member POVM

projections on

- impulse response

$$\langle x | \psi_+ \rangle = \psi(x)$$

- response derivative

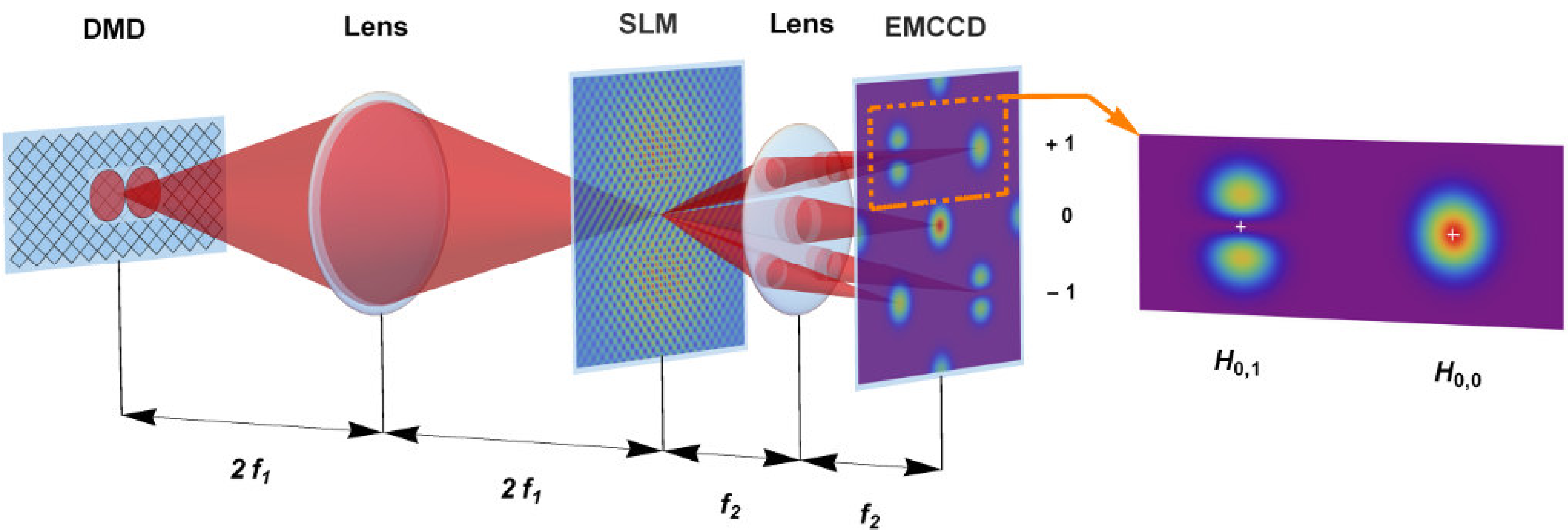
$$\langle x | \psi_- \rangle = \frac{\psi(x)'}{\sqrt{\mathcal{F}}}$$

example: Gaussian PSF

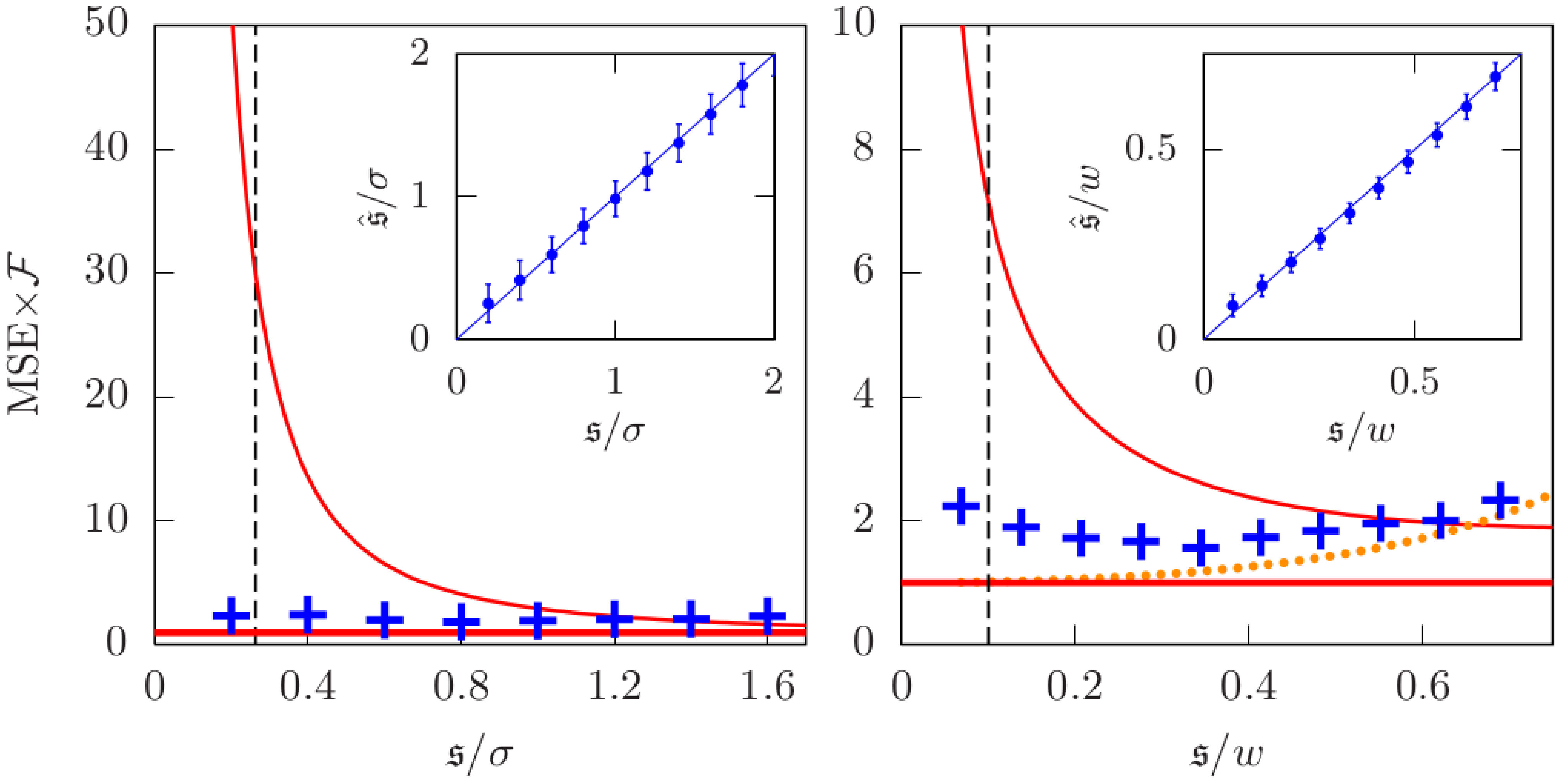
- impulse response $\psi(x) \propto H_0(x)$
- response derivative $\psi'(x) \propto H_1(x)$

experimental setup

- signal preparation – DMD
- projection – cross-correlation
- detection – EMCCD
- data – read from two pixels in the Fourier plane



results: Gaussian and sinc PSFs



results...

Physics

VIEWPOINT

Unlocking the Hidden Information in Starlight

Quantum metrology shows that it is always possible to estimate the separation of two stars, no matter how close together they are.

by Gabriel Durkin*

Regarding impact on the field, the authors' study produced a flurry of generalizations and other experimental proposals. During the past six months there have been four proof-of-principle experiments, first in Singapore by Tsang's colleague Alex Ling and collaborators [6], and then elsewhere in Canada and Europe [7–9]. A subsequent theory

[7] F. Yang, A. Taschilina, E. S. Moiseev, C. Simon, and A. I. Lvovsky, "Far-Field Linear Optical Superresolution via Heterodyne Detection in a Higher-Order Local Oscillator Mode," arXiv:1606.02662.

[8] W. K. Tham, H. Ferretti, and A. M. Steinberg, "Beating Rayleigh's Curse by Imaging Using Phase Information," arXiv:1606.02666.

[9] M. Paur, B. Stoklasa, Z. Hradil, L. L. Sanchez-Soto, and J. Rehacek, "Achieving Quantum-Limited Optical Resolution," arXiv:1606.08332.

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Achieving the ultimate optical resolution

MARTIN PAÚR,[1] BOHUMIL STOKLASA,[1] ZDENEK HRADIL,[1] LUIS L. SÁNCHEZ-SOTO,[2,3,*] AND JAROSLAV REHACEK[1]

[1]*Department of Optics, Palacký University, 17. listopadu 12, 771 46 Olomouc, Czech Republic*
[2]*Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain*
[3]*Max-Planck-Institut für die Physik des Lichts, Günther-Scharowsky-Straße 1, Bau 24, 91058 Erlangen, Germany*
**Corresponding author: lsanchez@fis.ucm.es*

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conclusions

- resolution of two mutually incoherent point sources was discussed
- optimal 2-channel measurement attaining the quantum CRLB in the super-resolution regime was derived
- this measurement was experimentally realized with a digital holography setup
- estimator variances 12dB below the classical limit were observed