Quantum nonlinearity from individual photons

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with many thanks to

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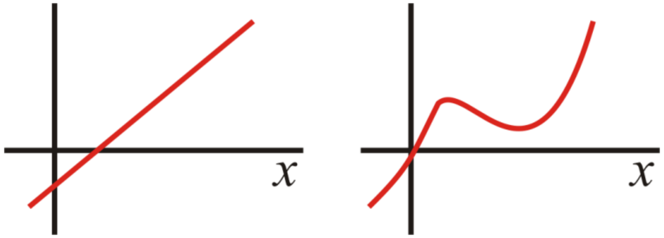
Nonlinearity: what do you mean?



- Mathematics:

  $f(x) = Ax + b$ is linear. Everything else is not

- Optics:
  - Certain materials have nonlinear response to electric fields

$$\mathbf{D} = \varepsilon_0(\mathbf{E} + \chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}^2 + \chi^{(3)}\mathbf{E}^3 + \cdots)$$

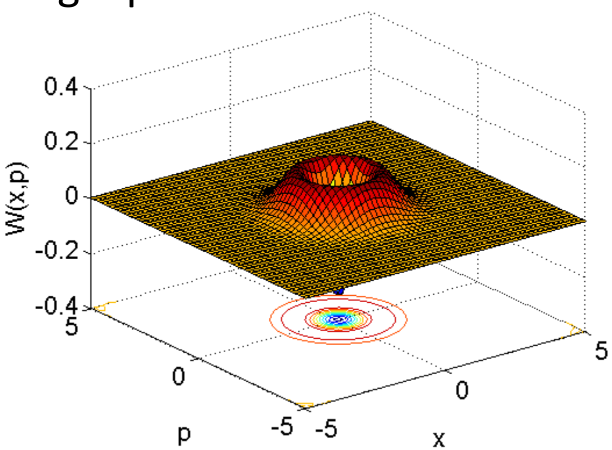
the nonlinear part

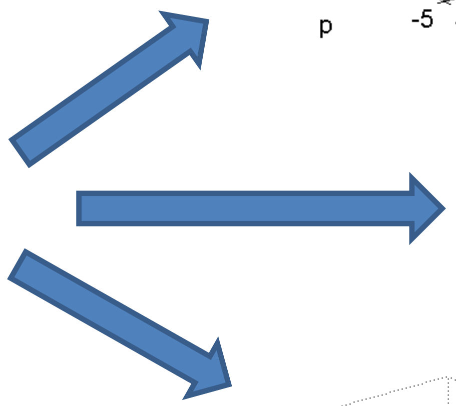
Nonlinearity in quantum optics

- Nonlinearity is quantum optics
- Parametric down-conversion
  - Generates individual photons for discrete quantum optics
  - Provides squeezed and quantum correlated light for CV quantum optics

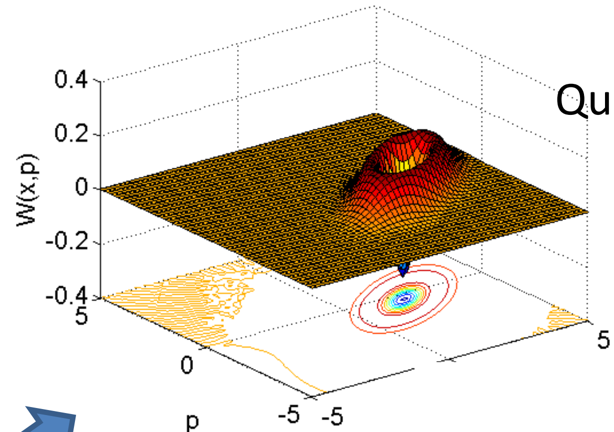
There are different kinds of nonlinearity

Single photon state

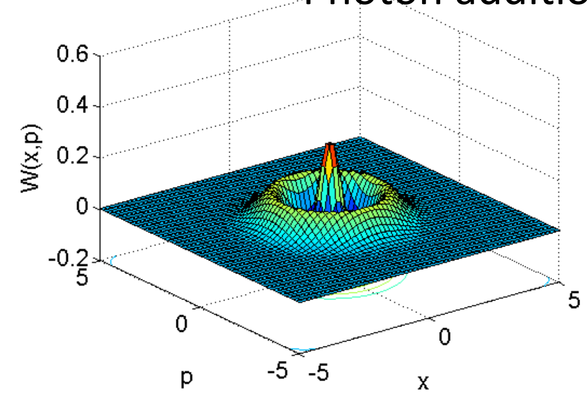




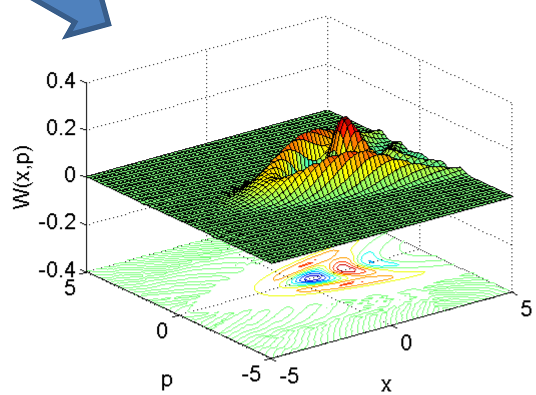
Quadratic nonlinearity



Photon addition



Cubic nonlinearity

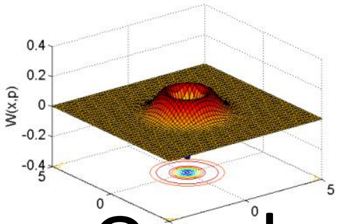


Wigner function:

-Phase space representation of a quantum state

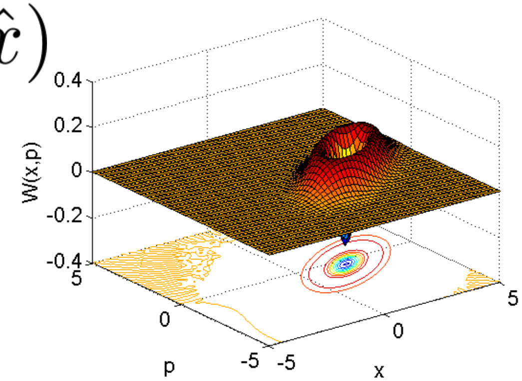
- Related to simultaneous properties in x and p

There are different kinds of nonlinearity



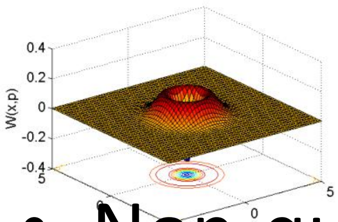
- Quadratic nonlinearity – the Gaussian one
  - Parametric down-conversion, quadratic Hamiltonians

$$\hat{H} = c_1\hat{x}^2 + c_2\hat{p}^2 + c_3(\hat{x}\hat{p} + \hat{p}\hat{x})$$

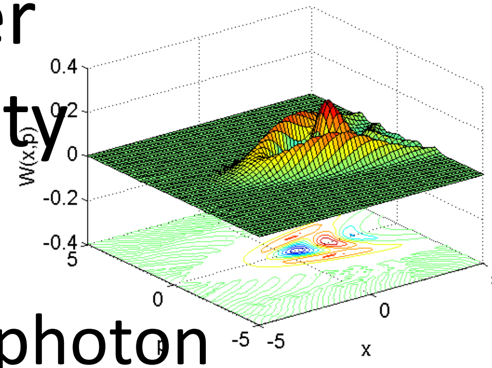


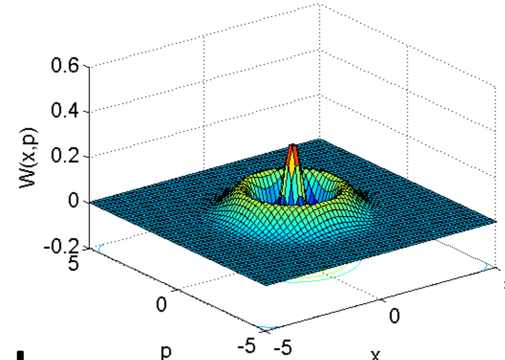
  - Linear in quadrature operators
  - Preserves "shape" of quantum states
  - Quantum teleportation, Quantum key distribution, Noise reduction in interferometry

There are different kinds of nonlinearity



- Non-quadratic nonlinearity, high order nonlinearity, non-Gaussian nonlinearity
  - Unitary (*i.e.* kerr or cubic operations)
  - Resulting from non-CPTP dynamics (*i.e.* photon addition)
  - Alter the “shapes” of quantum states





- Quantum computation, entanglement distillation, noiseless amplification

Achieving high order nonlinearity

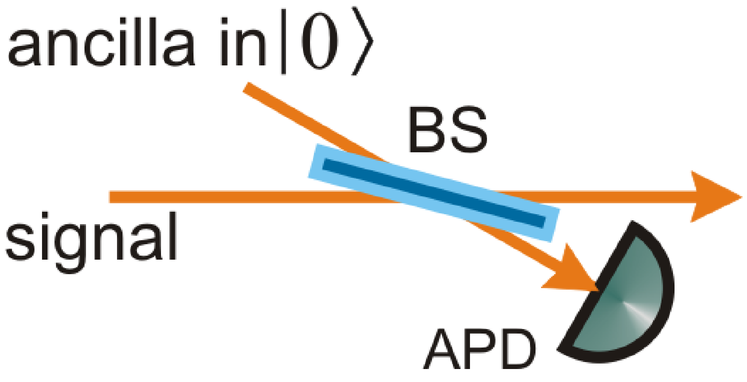
- Directly in nonlinear media
  - Currently not feasible
  - The nonlinearity is either too weak or too vulnerable to noise and imperfections
- Measurement induced way

Measurement induced high order nonlinearity

- Basic example: Photon subtraction

$$\sum_{n=0}^{\infty} c_n |n\rangle \rightarrow \sum_{n=1}^{\infty} c_n \sqrt{n} |n-1\rangle$$

  - Approximates action of annihilation operator $\hat{a}$
  - Probabilistic operation
  - Single photon projective measurement



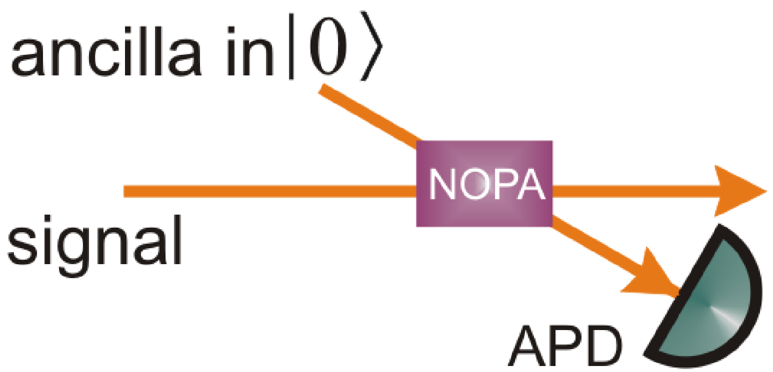
PRA **59**, 1658 (1999), Science **312**, 83 (2006)

Measurement induced high order nonlinearity

- Photon addition

$$\sum_{n=0}^{\infty} c_n |n\rangle \rightarrow \sum_{n=0}^{\infty} c_n \sqrt{n+1} |n+1\rangle$$

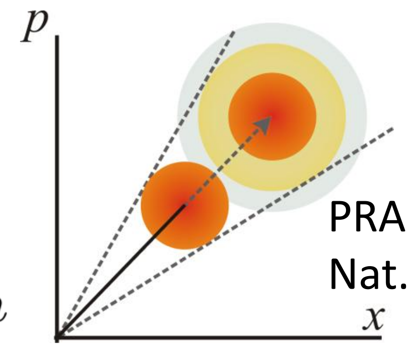
  - Approximates action of creation operator $\hat{a}^\dagger$
  - Probabilistic operation
  - seeded parametric downconversion



Science **306**, 660 (2004)

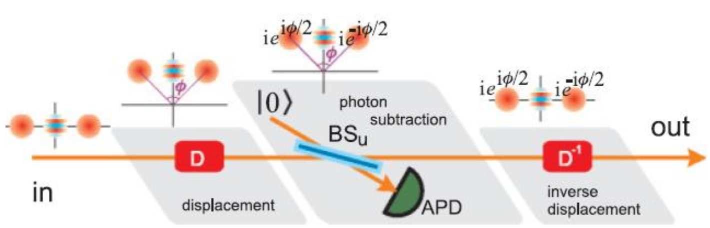
Some uses of probabilistic high order nonlinearity

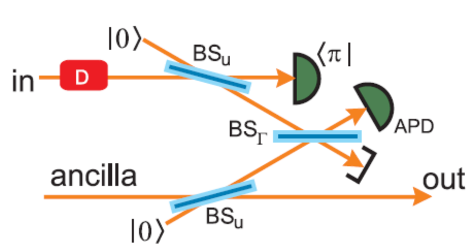
- Noiseless amplification
  - Approximates operator $g^{\hat{n}}$

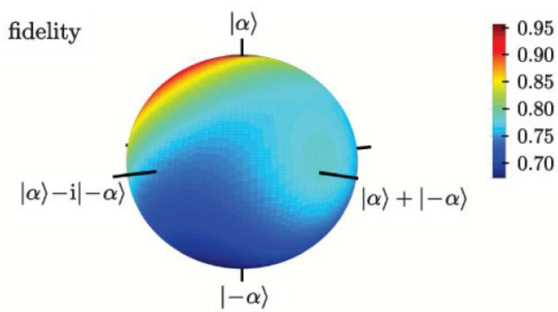


PRA 81, 022302 (2010); Nat. Phys. 6, 767 (2010)

- Proof-of-principle tests of quantum processing circuits
  - Elementary gates for superposed coherent state qubits







PRA 82, 014304 (2010); PRA 84, 050301(R) (2011)

Pros and cons of probabilistic operations

- Pro:
  - Feasible
  - Allows operations which can not exist deterministically (noiseless amplification)
- Con:
  - Exponentially vanishing probability of success
  - Not scalable

How to make a deterministic nonlinear operation?

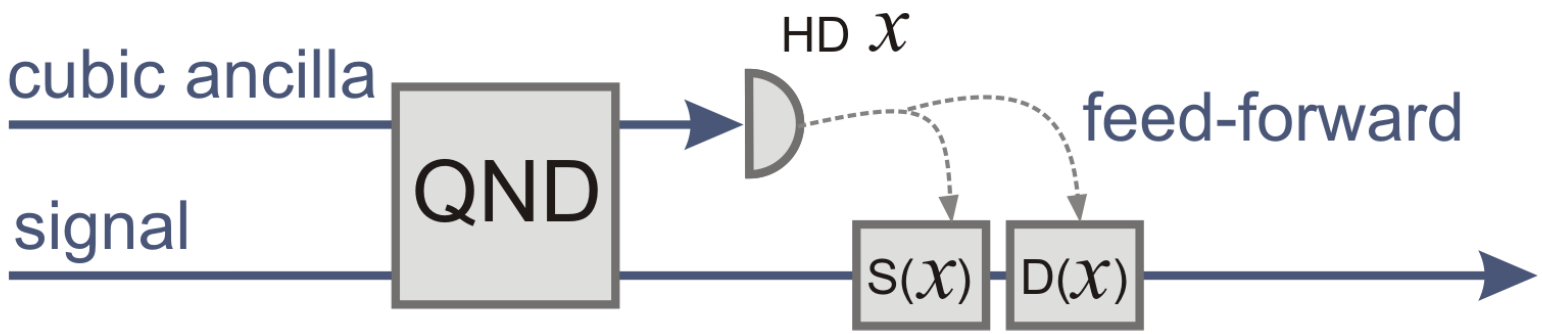
- Sources of nonlinearity:
  - Detection
  - Ancillary state
- Added constraint:
  - the feed-forward needs to be feasible!

Cubic nonlinearity

$$\hat{H} = \chi \hat{x}^3$$

- The lowest high-order nonlinearity
- In principle sufficient for realization of arbitrary quantum unitary operation

PRL **82**, 1784 (1999)

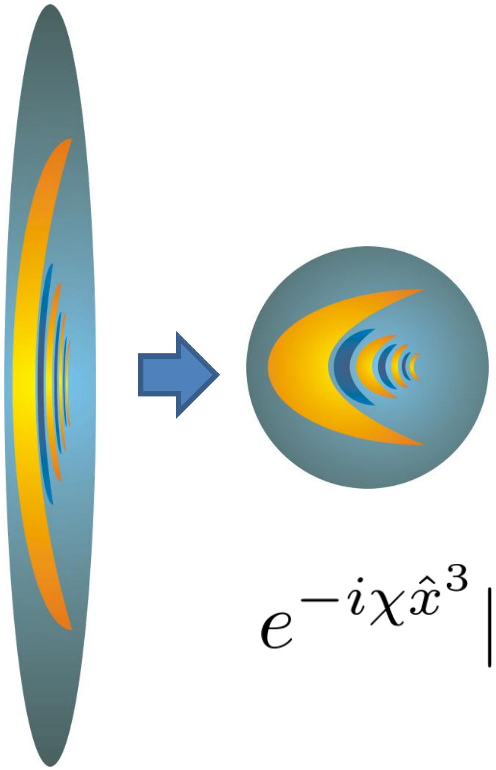


PRA **64**, 012310 (2001); PRA **84**, 053802 (2011)

Issues with the ancillary cubic state

$$e^{-i\chi\hat{x}^3}|p=0\rangle = \int_{-\infty}^{\infty} e^{-i\chi x^3}|x\rangle dx$$

- Requires infinite energy
  - Could be adjusted by squeezing
- Requires cubic nonlinearity
  - or does it?



$$e^{-i\chi\hat{x}^3}|0\rangle$$

Cubic nonlinearity constructed from photons

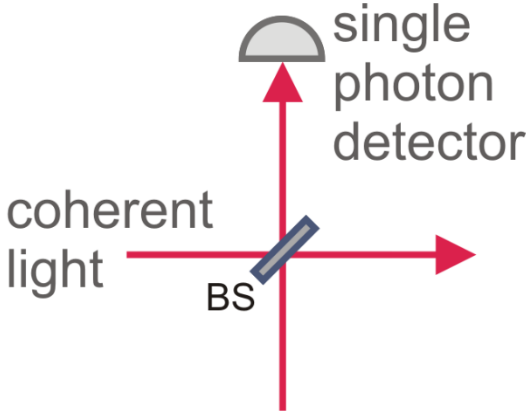
- What if the nonlinearity is weak?

$$e^{-i\chi\hat{x}^3}|0\rangle \approx (1 - i\chi\hat{x}^3)|0\rangle$$

$$= |0\rangle - i\chi\frac{\sqrt{3}}{2\sqrt{2}}\left(\sqrt{3}|1\rangle + \sqrt{2}|3\rangle\right)$$

  - The required state can be constructed as a superposition of zero, one, and three photons
  - It can be prepared in a probabilistic fashion

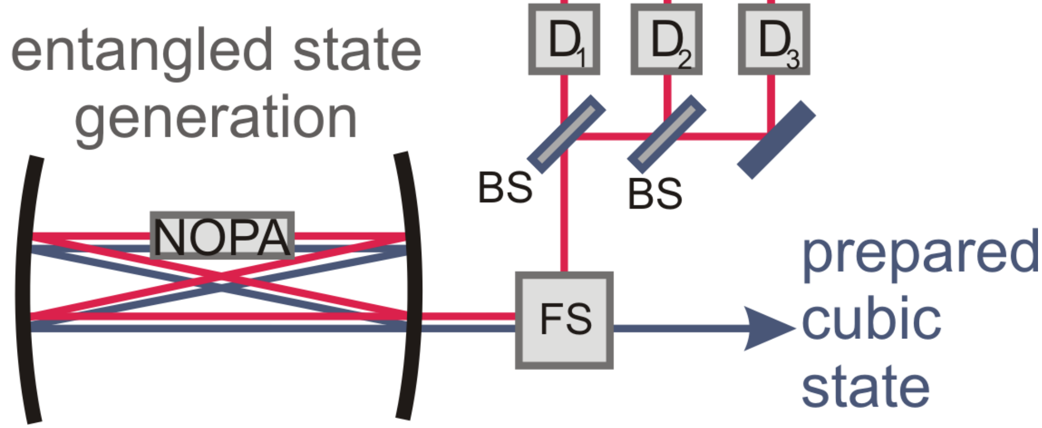
Superposition up to three photons



Projects on

$$\alpha|0\rangle + |1\rangle$$

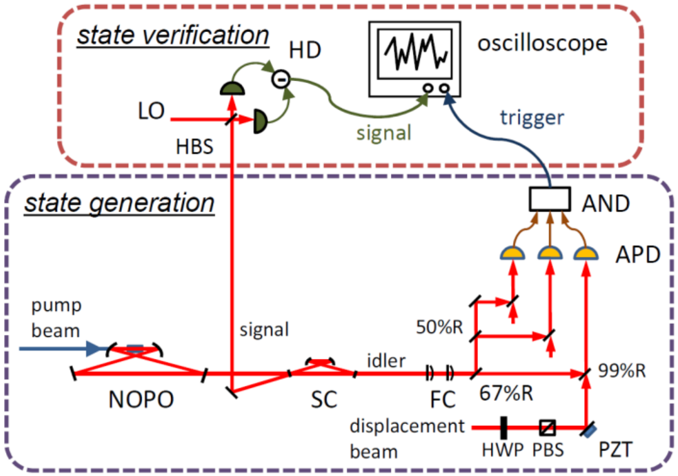
$$\sum_{n=0}^{\infty} \lambda^n |n, n\rangle$$



Three detectors with three coherent displacements project on

$$\alpha\beta\gamma|0\rangle + \frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}|1\rangle + \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}|2\rangle + |3\rangle$$

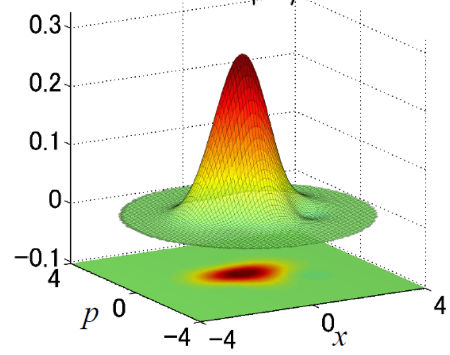
Experimentally realized in the Furusawa Lab, Tokyo

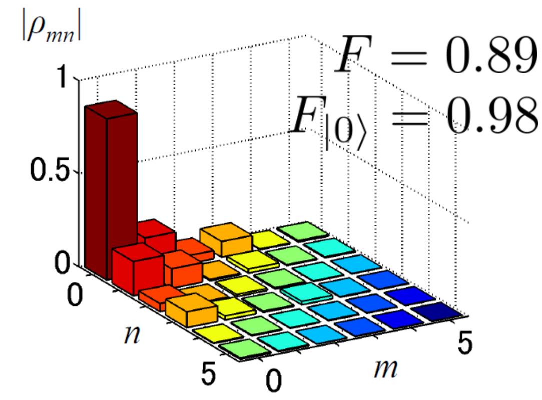


PRA 88, 053816 (2013)

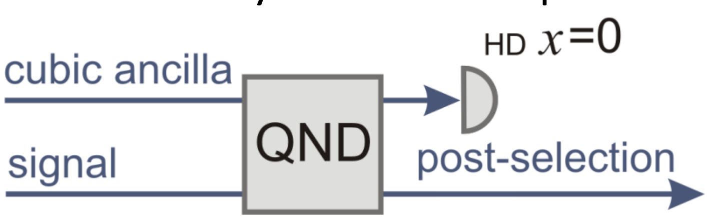
Weak cubic state

$$e^{-i\chi\hat{x}^3}|0\rangle$$





Verification by conditional operation:

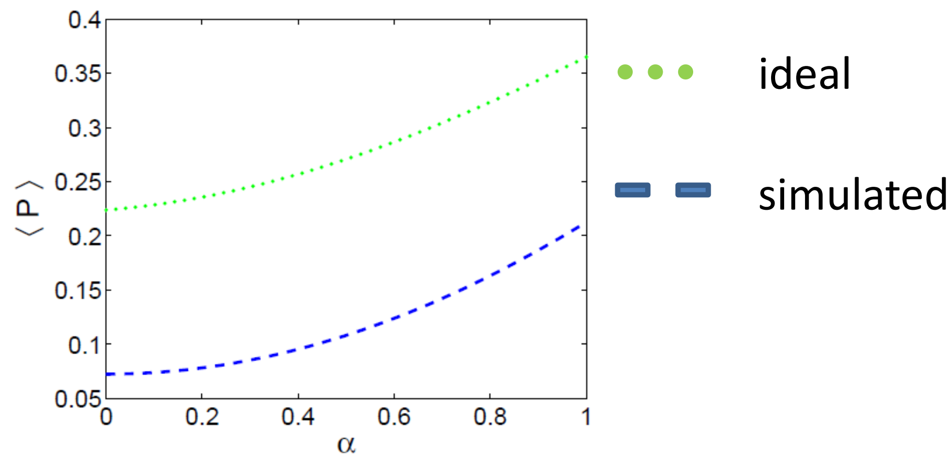


Expected behavior:

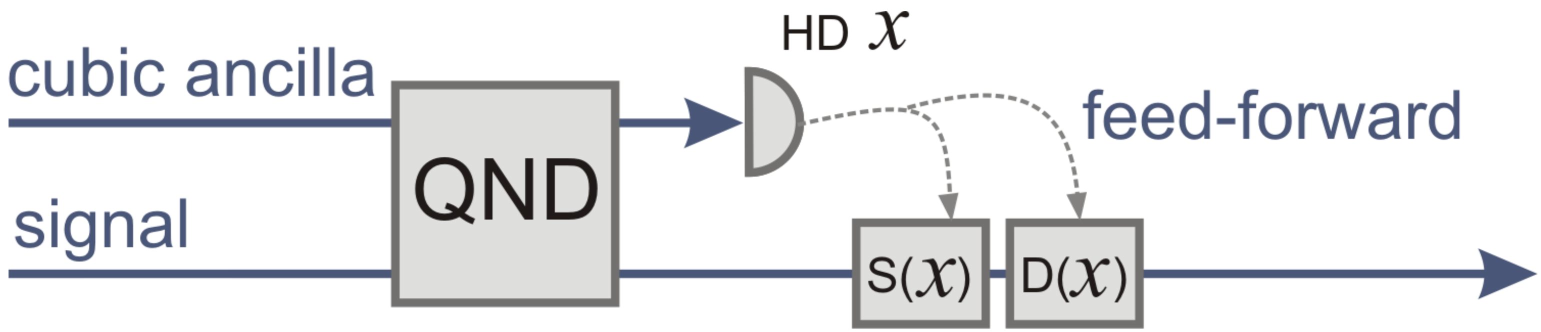
$$\hat{x} \to \hat{x}$$

$$\hat{p} \to \hat{p} + \chi\hat{x}^2$$

Transformation of coherent states:



Different angle: nonlinear squeezing

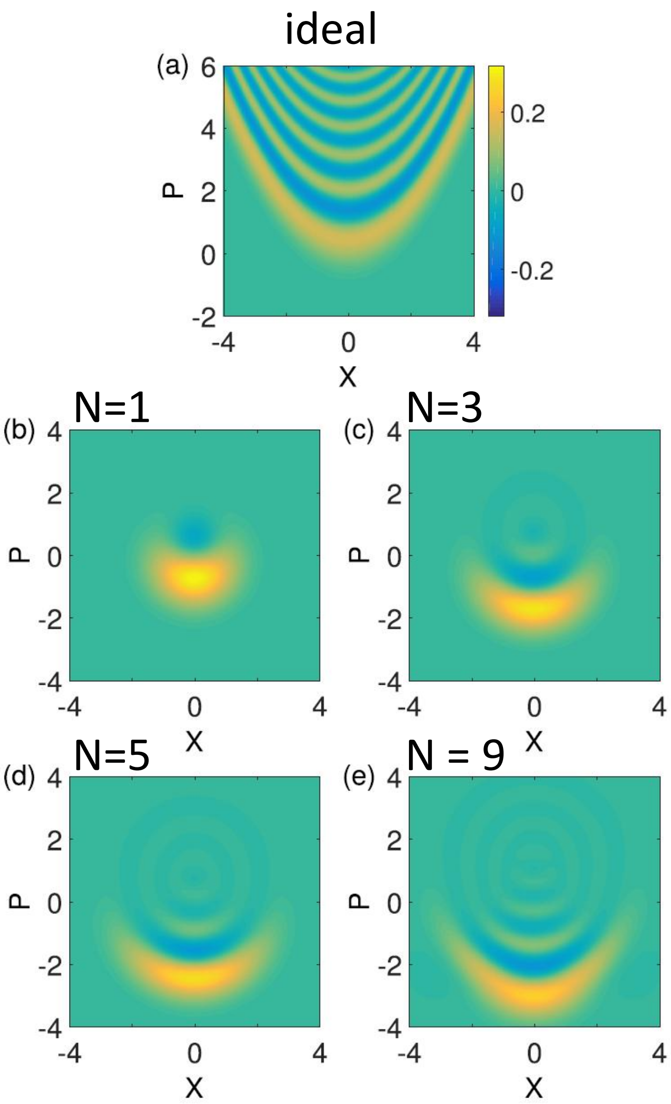


- The role of the ancilla is to reduce the noise

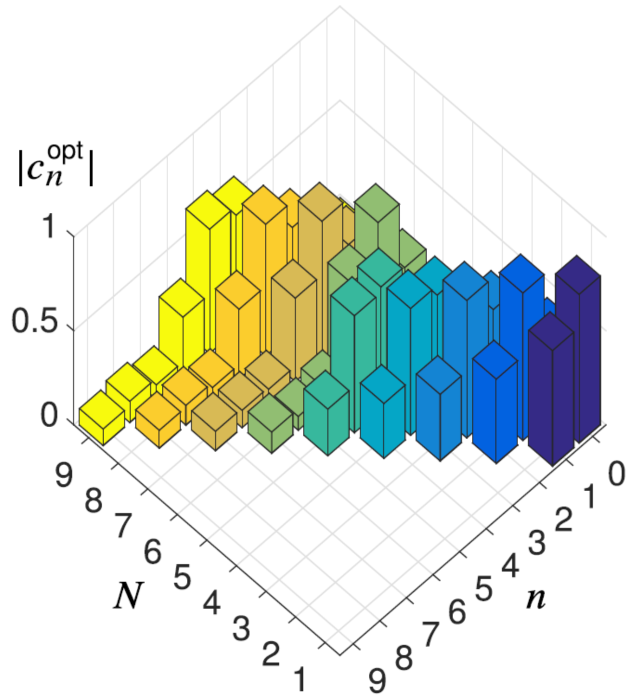
$$\langle[\Delta(\hat{p}-\chi\hat{x}^2)]^2\rangle \rightarrow 0$$

- For any given dimension of the Hilbert space we can look for states that minimize this variance

Nonlinear squeezed states in different dimensions



Diagonal elements of the cubic squeezed states:



The actual preparation of the states employs the same measurement based techniques

PRA 93, 022301 (2016)

TLDL

- High order nonlinearity is crucial for quantum information processing with CV systems
- I can’t be realized directly
- It can be constructed from individual photons and then imprinted on the target states

Thank you for the attention!