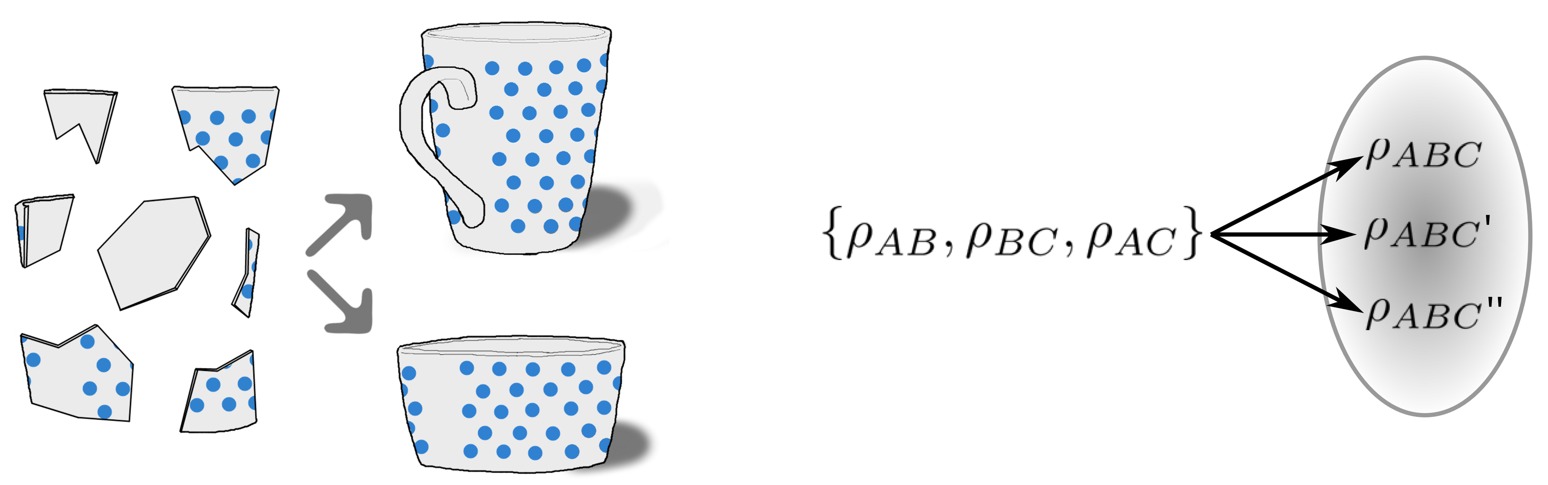
Verifying genuine multipartite entanglement of the whole from its separable parts

Ladislav Mišta, Jr., Michal Mičuda, Robert Stárek, Jan Provazník, and Olga Leskovjanová

Department of Optics, Palacký University, 17. listopadu 12, 771 46 Olomouc, Czech Republic

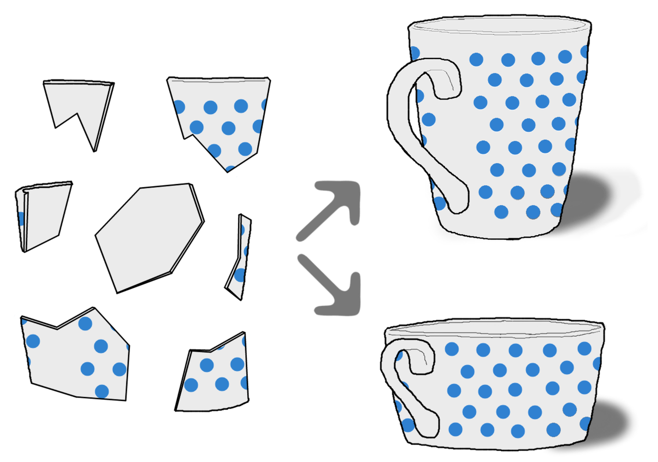
Introduction

Inference of a global property of the whole from its parts.



Separability criteria, entanglement measures, quantum marginal problem.

When parts lack the property – emergent property.



Local marginal correlations compatible only with multipartite nonlocal correlations [1].

Separable marginals compatible only with multipartite entanglement [2].

Emergent property – genuine multipartite entanglement (GME):

$$\rho \neq p_1\rho^{\text{sep}}_{A|BC} + p_2\rho^{\text{sep}}_{B|AC} + p_3\rho^{\text{sep}}_{C|AB} \text{ (biseparable state).}$$

GME verifiable from separable marginals [3].

Theory

More robust GME verifiable from separable marginals [4]:

$$\begin{aligned}\rho &= \frac{2}{3}|\xi\rangle\langle\xi| + \frac{1}{3}|\bar{W}\rangle\langle\bar{W}|,\\ |\xi\rangle &= \frac{1}{3}(e^{i\frac{\pi}{3}}|001\rangle + e^{-i\frac{\pi}{3}}|010\rangle - |100\rangle) + \sqrt{\frac{2}{3}}|111\rangle,\\ |\bar{W}\rangle &= \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle).\end{aligned}$$

Separability of marginals – PPT criterion:

$$\min[\text{eig}(\rho_{jk}^{T_j})] \doteq 0.37 \cdot 10^{-2},\ jk = AB, BC, AC$$

GME witness:

$$\begin{aligned}\mathcal{W} &= \mathcal{W}^\dagger,\\ \text{Tr}(\rho\mathcal{W}) &\geq 0 \quad \text{for all biseparable } \rho,\\ \text{Tr}(\rho\mathcal{W}) &< 0 \quad \text{for some } \rho.\end{aligned}$$

GME witness acting on marginals – SDP [4, 5]:

$$\underset{\mathcal{W},P_M,Q_M}{\text{minimize}} \quad \text{Tr}(\rho\mathcal{W})$$

subject to $\text{Tr}(\mathcal{W}) = 1$, where

$$\mathcal{W} = \sum_{i,j=0}^{3} w_{ij}^{(AB)}\sigma_i^{(A)} \otimes \sigma_j^{(B)} \otimes \mathbb{1}^{(C)} + \text{permutations},$$

and for all bipartitions $M|\overline{M}$,

$$\mathcal{W} = P_M + Q_M^{T_M}, \quad P_M \geq 0, \quad Q_M \geq 0.$$

$$\text{Tr}(\rho\mathcal{W}) = -1.98 \cdot 10^{-2}$$

White noise tolerance of 13.7%.

References

[1] L. E. Würflinger, J.-D. Bancal, A. Acín, N. Gisin, and T. Vértesi, Phys. Rev. A **86**, 032117 (2012).

[2] G. Tóth, C. Knapp, O. Gühne, and H.-J. Briegel, Phys. Rev. Lett. **99**, 250405 (2007).

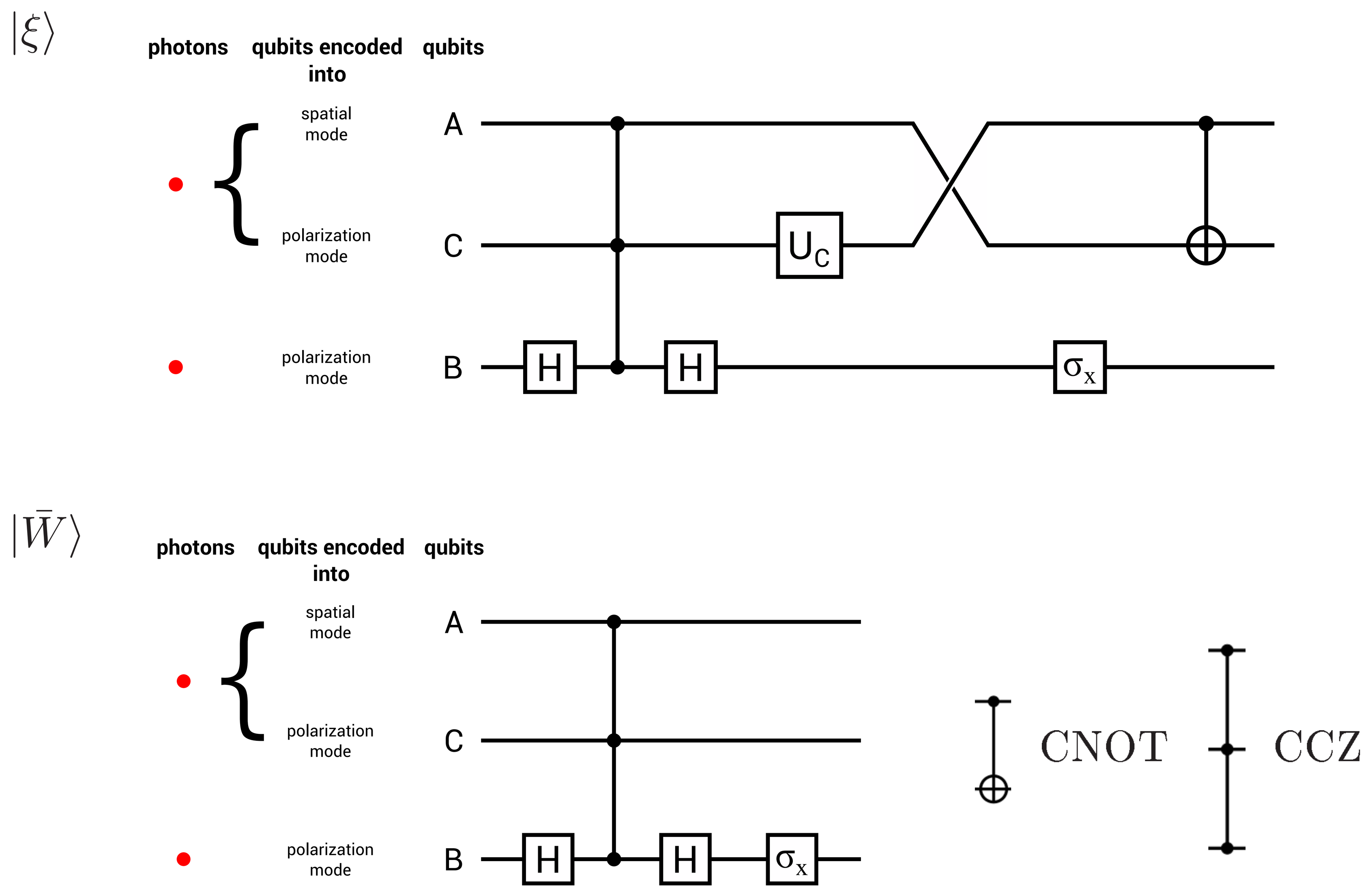
[3] L. Chen, O. Gittsovich, K. Modi, and M. Piani, Phys. Rev. A **90**, 042314 (2014).

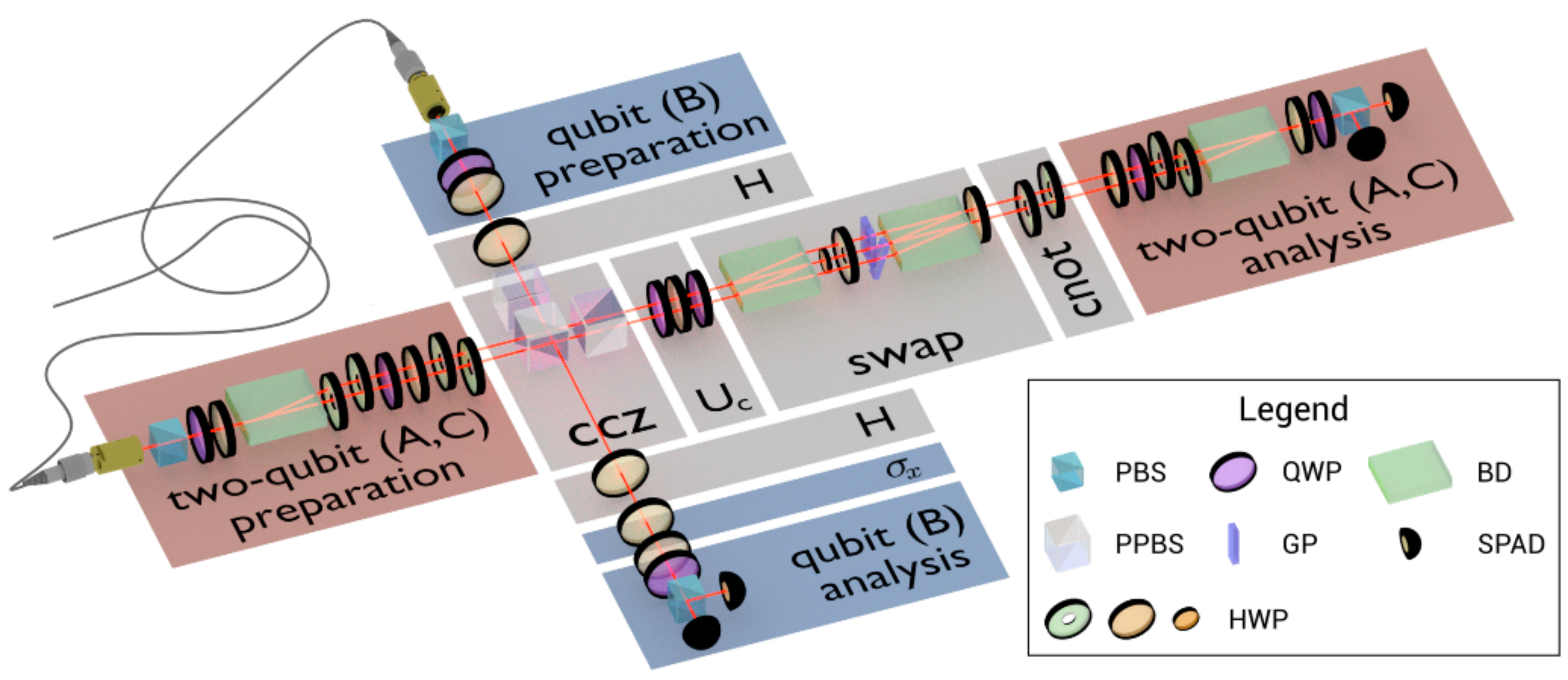
[4] N. Miklin, T. Moroder, and O. Gühne, Phys. Rev. A **93**, 020104(R) (2016).

[5] `https://ch.mathworks.com/matlabcentral/fileexchange/30968`.

Experimental scheme

Logical circuits

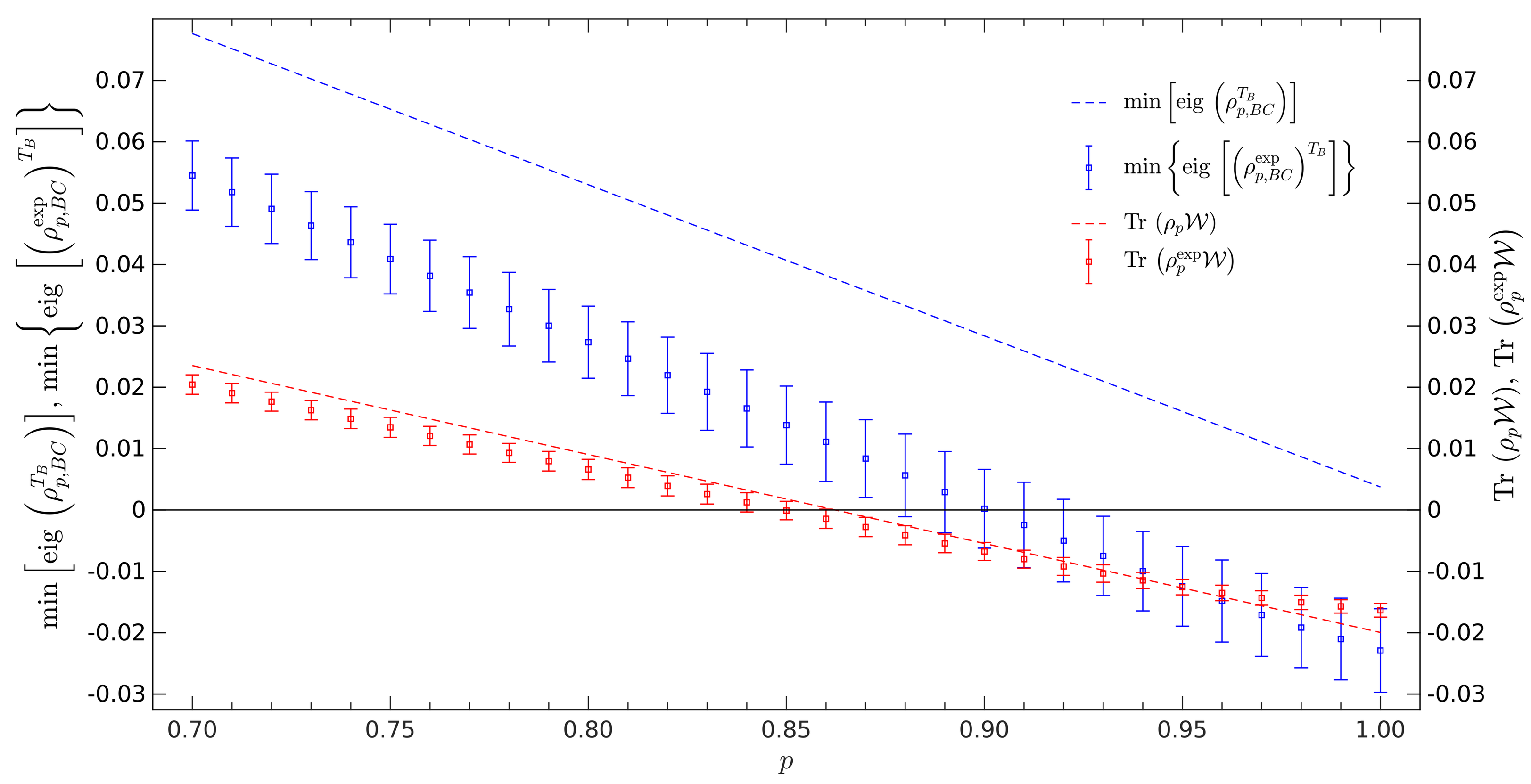




Results

Marginals of ρ close to entangled states:

$$\rho_p = p\rho + \frac{(1-p)}{8}\mathbb{1}, \quad 0 \leq p \leq 1.$$



$p_{\text{opt}} = 0.868$ – separability and GME bounds beaten by $\sim 1.5\sigma$.

Separability of marginals:

jk	AB	BC	AC
$\min\{\text{eig}[(\rho^{\text{exp}}_{p_{\text{opt}},jk})^{T_j}]\} \cdot 10^2$	2.4 ± 0.7	0.9 ± 0.6	2.1 ± 0.6

Genuine multipartite entanglement:

$$\text{Tr}(\rho^{\text{exp}}_{p_{\text{opt}}}\mathcal{W}) = (-3 \pm 2) \cdot 10^{-3}$$

Fidelity:

$$\mathcal{F}(\rho^{\text{exp}}_{p_{\text{opt}}}, \rho_{p_{\text{opt}}}) \equiv [\text{Tr}(\sqrt{\sqrt{\rho^{\text{exp}}_{p_{\text{opt}}}}\rho_{p_{\text{opt}}}\sqrt{\rho^{\text{exp}}_{p_{\text{opt}}}}})]^2 = 0.939 \pm 0.008$$

M. Mičuda, R. Stárek, J. Provazník, O. Leskovjanová, and L. Mišta (submitted).