Experimental demonstration of superresolution using signum phase mask

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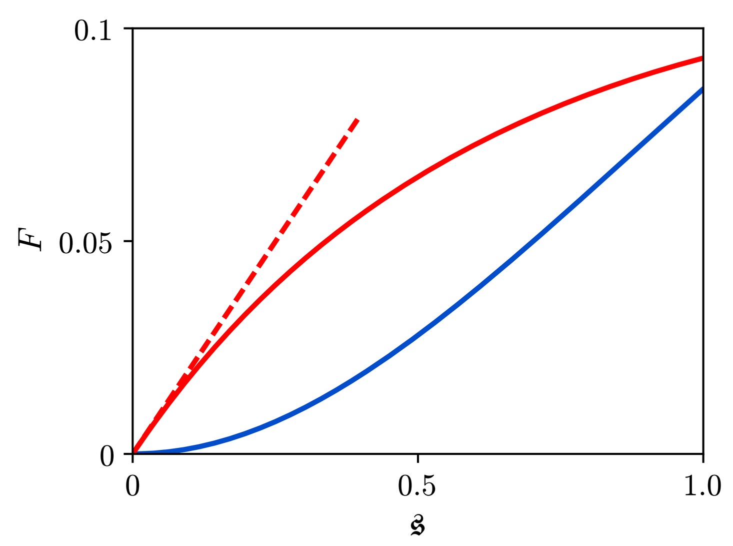
Motivation

It has been argued that, for a spatially invariant imaging system, the information one can gain about the separation of two incoherent point sources decays quadratically to zero with decreasing separation, an effect termed the Rayleigh's curse. Contrary to this belief, we identify a class of point spread functions with a linear information decrease. Moreover, we show that any well-behaved symmetric point spread function can be converted into such a form with a simple nonabsorbing signum filter. We experimentally demonstrate significant superresolution capabilities based on this idea.

Two incoherent points resolution as a statistical problem

We work with a spatially invariant imaging system and two equally bright incoherent point sources separated by $\mathfrak{s}$. If $I(x)$ is the spatial distribution of the intensity in the image from a point source (PSF), the direct image can be written as

$$p(x,\mathfrak{s}) = \frac{1}{2}[I(x-\mathfrak{s}/2)+I(x+\mathfrak{s}/2)] \qquad (1)$$



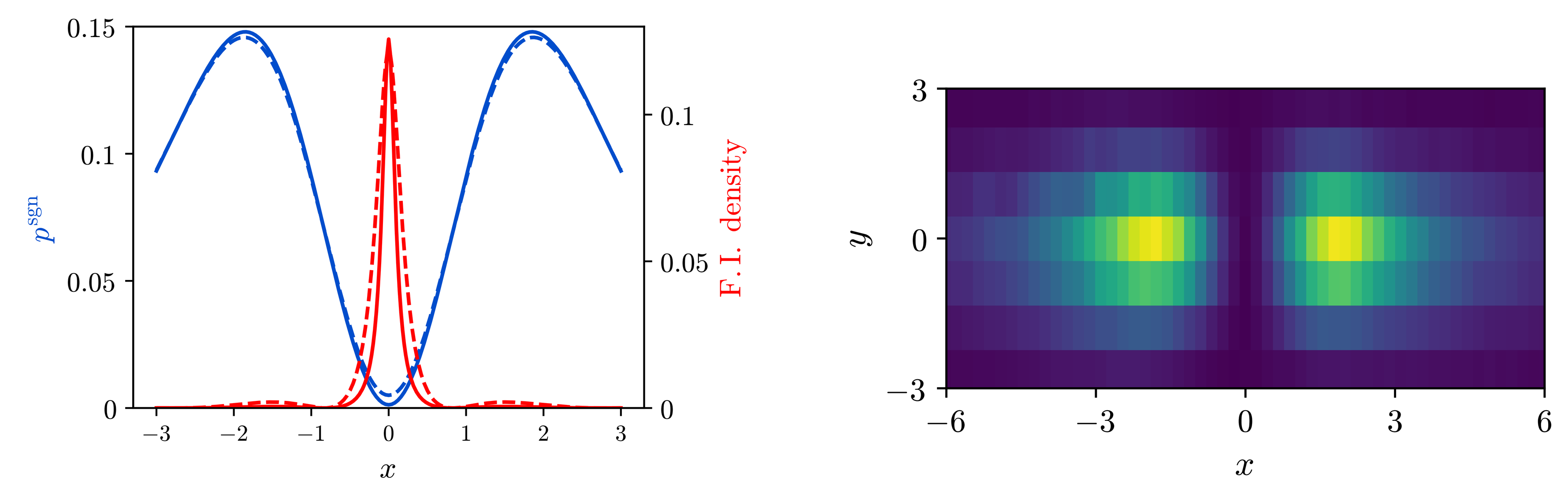
The precision in estimating $\mathfrak{s}$ is governed by the Fisher information

$$F(\mathfrak{s}) = N\int \frac{[\partial_{\mathfrak{s}} p(x,\mathfrak{s})]^2}{p(x,\mathfrak{s})}dx \qquad (2)$$

The CRLB ensures that the variance of any unbiased estimator $\hat{\mathfrak{s}}$ of the quantity $\mathfrak{s}$ is bounded by the reciprocal of the Fisher information; viz,

$$(\Delta\hat{\mathfrak{s}})^2 \geq \frac{1}{F(\mathfrak{s})}. \qquad (3)$$

PSF reshaping and Fisher information



Since we are chiefly interested in the case of small separations, the associated Fisher information becomes

$$F(\mathfrak{s}) = \frac{\mathfrak{s}^2}{16}\int\left\{\frac{[I''(x)]^2}{I(x)}+O(\mathfrak{s}^2)\right\}dx. \qquad (4)$$

Let us assume that our PSF is well approximated by a parabolic profile near the origin; i.e., $I(x)\simeq \alpha x^2$, which implies

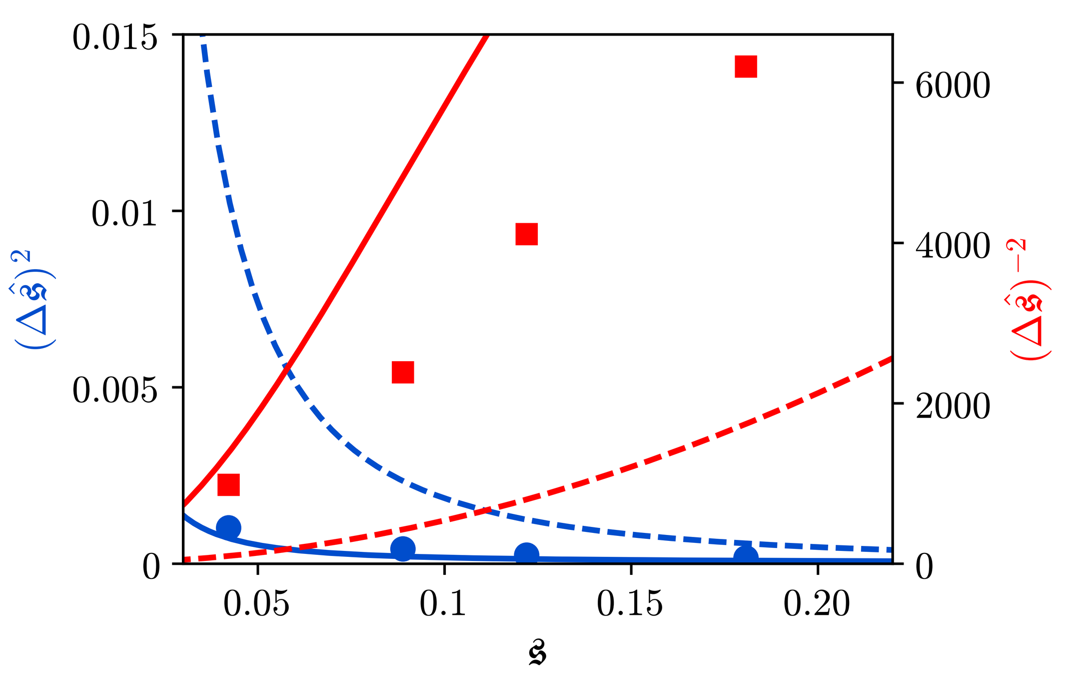
$$p(x,\mathfrak{s}) \simeq \alpha(x^2+\mathfrak{s}^2/4), \qquad x,\mathfrak{s}\ll 1. \qquad (5)$$

As a result, we get

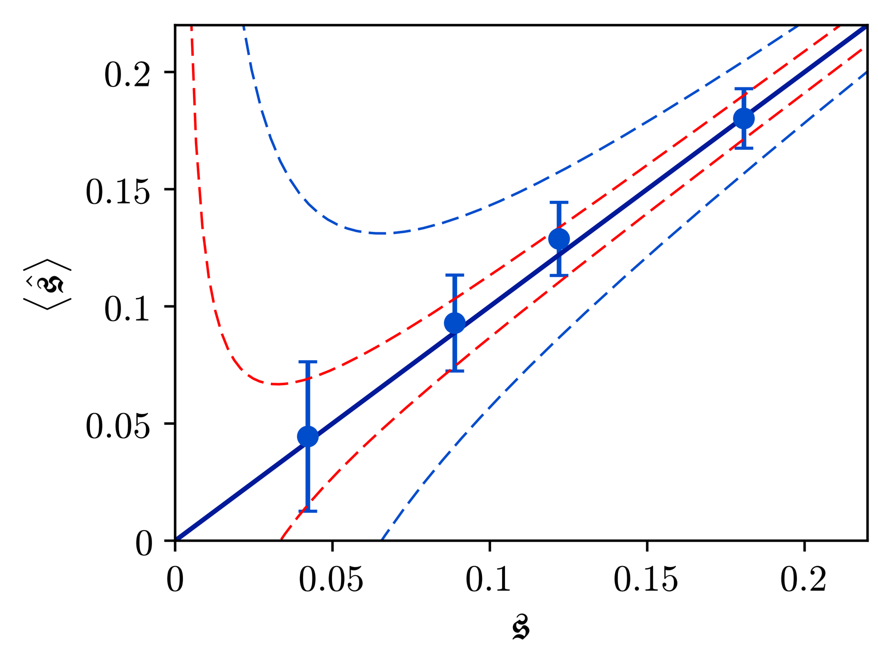
$$F(\mathfrak{s}) \simeq \lambda\,\mathfrak{s}, \qquad (6)$$

with $\lambda = \pi\alpha/2$, and the information is indeed linear rather than quadratic at small separations.

Experimental results

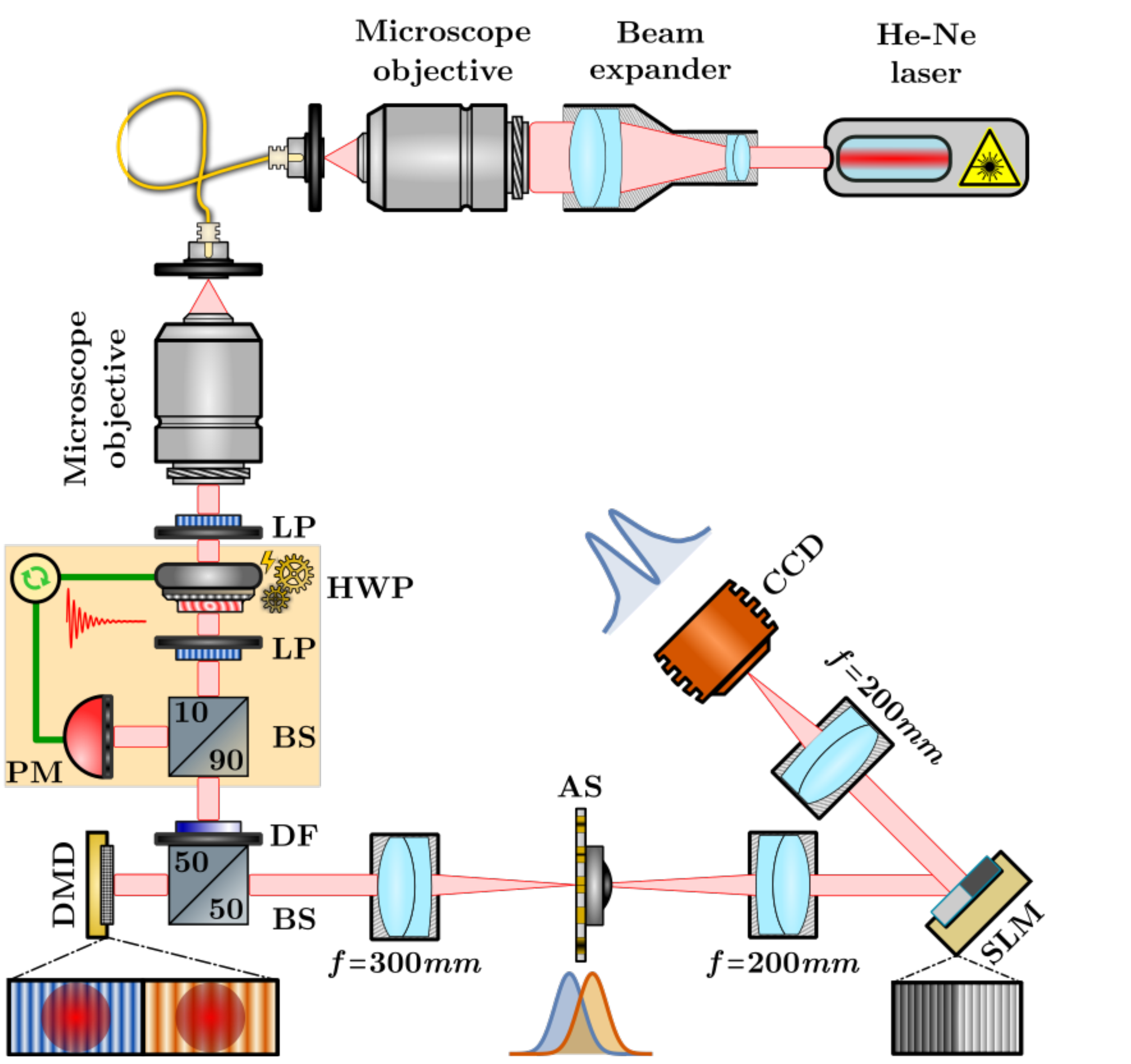


Experimental variances of the separation estimator (blue dots) compared with the direct detection (blue broken line) and the signum-enhanced limit (solid blue line). The latter is corrected for the finite pixel size of 7.6 μm. For completeness, the reciprocal of the variances, called the precisions, are shown in red.



Estimation of the separation from signum-enhanced imaging. Estimator means (dots) and standard deviations (error bars) are shown. Same statistics is provided for the best unbiased estimators from direct (blue lines) and signum-enhanced (red lines) imaging as given by the CRLB. The latter takes in account the finite pixel size used in the experiment.

Experimental setup



DMD:digital micromirror chip, AS: aperture stop at the Fourier plane of the lens, SLM: spatial light modulator, LP: linear polarizer, HWP: half-wave plate, PM: power meter, DF: density filter and BS: beam-splitter.

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