Preparing genuine multipartite entanglement verifiable from separable marginals

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Introduction

Inference of global properties of a composite system from its parts.

Theory

- Representability (marginal) problem**:** when a pth-order density matrix is a reduction of some Nth -order density matrix (N>p)?

  (A. J. Coleman, Rev. Mod. Phys. **5**, 668 (1963)).

- Extensibility of ρ_{AB} : $\exists \quad \rho_{ABE}, \quad \mathrm{Tr}_E(\rho_{ABE}) = \rho_{AB}.$

Separability and security criteria, calculation of entanglement measures.

Experiment

- Analysis of composite systems involving many subsystems.

Examples:

1. Almost all $|\psi\rangle_{ABC}$ are uniquely determined by ρ_{AB} and ρ_{BC}.
2. Many different $|\psi\rangle_{AB}$ are compatible with given ρ_A and ρ_B.

Mixed ρ_A and ρ_B → $|\psi\rangle_{AB} \neq |\chi\rangle_A|\phi\rangle_B$ (entanglement).

Mixed states: $\rho_A = \rho_B = \frac{1}{2}\mathbb{1}$ compatible with

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $\frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$

$\mathcal{E} = \{\rho_A, \rho_B\}$: **compatibility set** $\mathcal{C}(\mathcal{E}) = \{\sigma_{AB} | \sigma_A = \rho_A, \quad \sigma_B = \rho_B\}$

If $\mathcal{C}(\mathcal{E})$ contains only σ_{AB} with a certain property the property can be inferred from σ_A and σ_B .

Can a property be inferred from marginals that do not possess the property?

Entanglement from separable marginals in multiqubit systems ($\mathrm{dim}\mathcal{H} = 2$)
(G. Tóth et. al., Phys. Rev. Lett. **99**, 250405 (2007)).

Three qubits A, B, C
Genuine multipartite entanglement (GME):

$$\rho \neq p_1\rho^{\rm sep}_{A|BC} + p_2\rho^{\rm sep}_{\rm B|AC} + p_3\rho^{\rm sep}_{C|AB} \quad \text{(biseparable state)},$$

$$\rho^{\rm sep}_{X|Y} = \sum_i p_i \rho^{(i)}_X \otimes \rho^{(i)}_Y \quad \text{(separable state)}.$$

GHZ, W, cluster states etc.

Can we infer GME from separable marginals?

∃ separable $\{\rho_{AB}, \rho_{BC}, \rho_{AC}\}$ compatible only with GME ρ_{ABC}.
(L. Chen et al., PRA **90**, 042314 (2014))

Theory

State:

$$\rho = \frac{2}{3}|\xi\rangle\langle\xi| + \frac{1}{3}|\bar{W}\rangle\langle\bar{W}|,$$

$$|\xi\rangle = \frac{1}{3}(e^{i\frac{\pi}{3}}|001\rangle + e^{-i\frac{\pi}{3}}|010\rangle - |100\rangle) + \sqrt{\frac{2}{3}}|111\rangle,$$

$$|\bar{W}\rangle = \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle).$$

(N. Miklin et al., PRA **93**, 020104 (2016))

Separable marginals:

$$\min[\mathrm{eig}(\rho_{jk}^{T_j})] = 0.0037.$$

GME witness:

$$\begin{aligned} W &= W^\dagger, \\ \mathrm{Tr}(\rho W) &\geq 0 \quad \text{for all biseparable } \rho, \\ \mathrm{Tr}(\rho W) &< 0 \quad \text{for some } \rho. \end{aligned}$$

SDP:

$$\begin{aligned} \underset{W, P_M, Q_M}{\text{minimize}} \quad & \mathrm{Tr}(\rho W) \\ \text{subject to} \quad & \mathrm{Tr}(W) = 1, \\ & W = \sum_{i,j=0}^{3} w_{ij}^{(AB)} \sigma_i^{(A)} \otimes \sigma_j^{(B)} \otimes \mathbb{1}^{(C)} + \text{permutations}, \\ & \text{and for all bipartitions } M|\overline{M}, \\ & W = P_M + Q_M^{T_M}, \quad P_M \geq 0, \quad Q_M \geq 0. \end{aligned}$$

$$\mathrm{Tr}(\rho W) = -0.0502$$

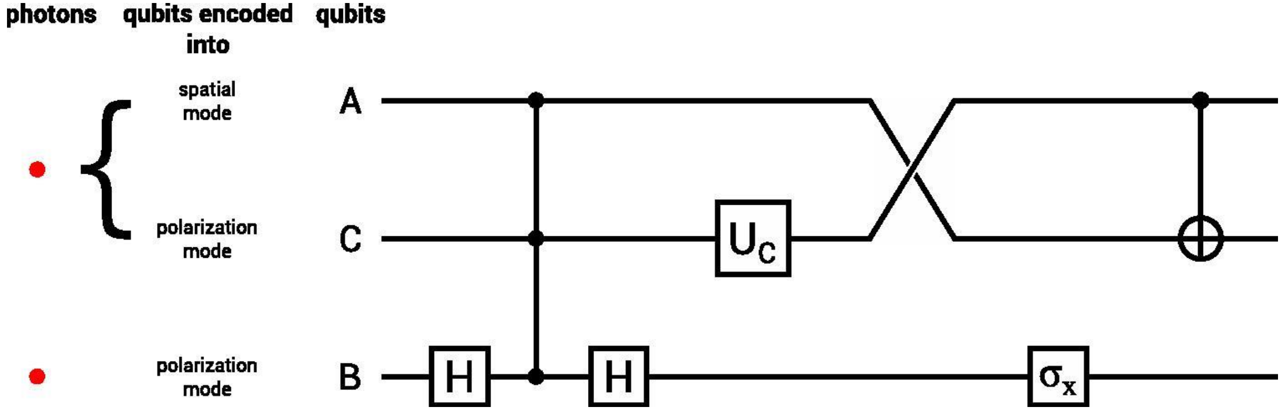
Robustness:

$$p\rho + \frac{(1-p)}{8}\mathbb{1}$$

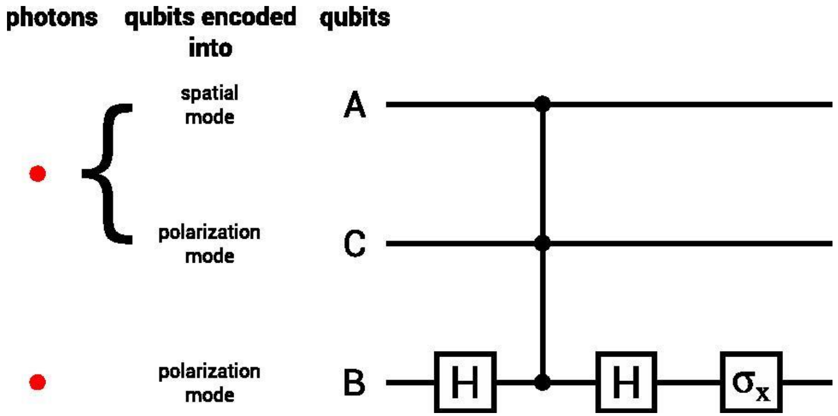
exhibits the effect for up to 13.7% of white noise.

Logical circuit

$|\xi\rangle$



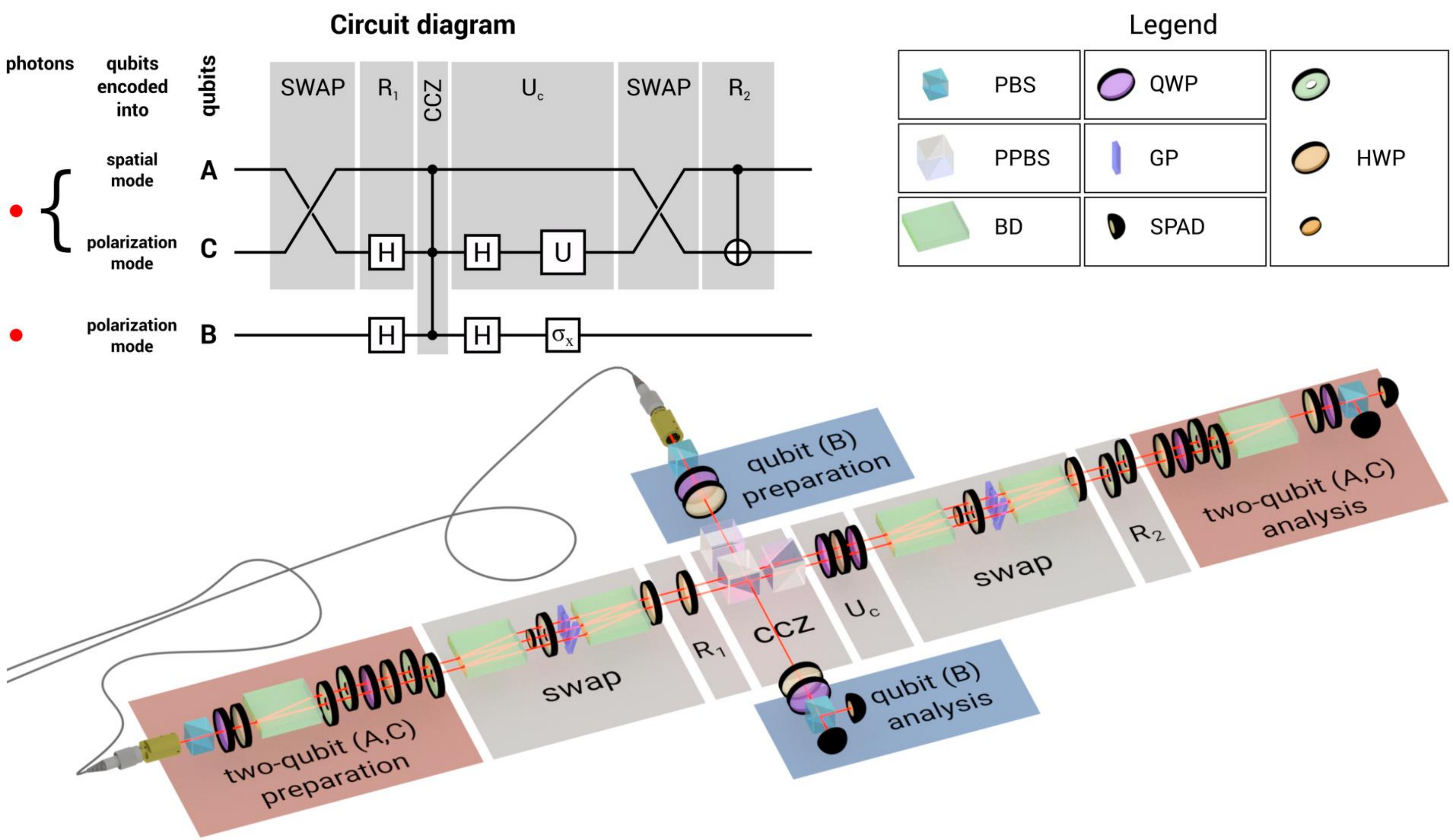
$|\bar{W}\rangle$



CNOT gate:
Bit flip on the 2nd qubit when the 1st qubit is $|1\rangle$.

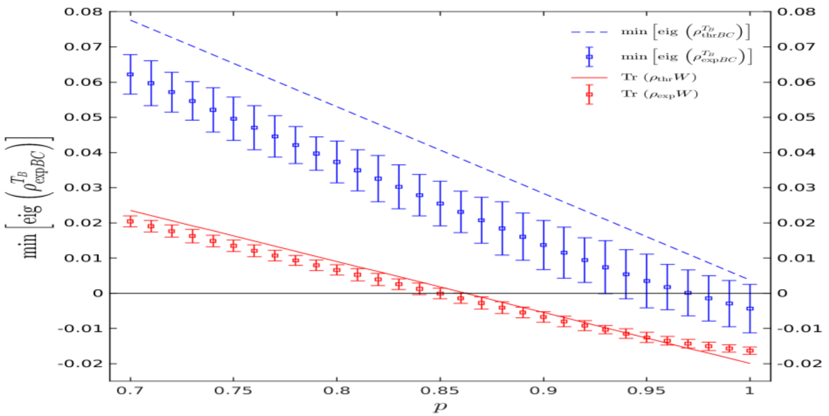
Toffoli gate:
Bit flip on the 3rd qubit when the 1st and 2nd qubit is $|1\rangle$.

Experiment

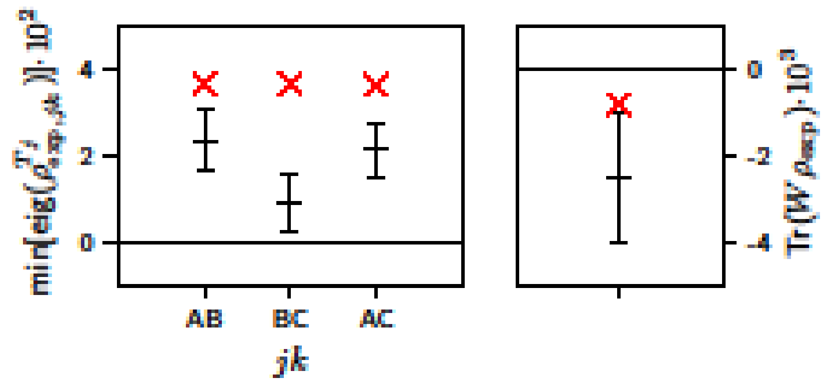


Circuit diagram
photons
qubits encoded into
qubits
spatial mode
polarization mode
polarization mode
A
C
B
SWAP
R1
CCZ
Uc
SWAP
R2
H
H
H
H
U
σx
Legend
PBS
PPBS
BD
QWP
GP
SPAD
HWP
qubit (B) preparation
two-qubit (A,C) preparation
swap
R1
CCZ
Uc
swap
R2
two-qubit (A,C) analysis
qubit (B) analysis

Results



p=0.868



Separable marginals

jk	AB	BC	AC
$\min[\mathrm{eig}(\rho_{\exp,jk}^{T_j})] \cdot 10^2$	2.4 ± 0.7	0.9 ± 0.6	2.1 ± 0.6

Genuine ME

$$\mathrm{Tr}(\rho_{\exp} W) = (-3 \pm 2) \cdot 10^{-3}$$

Fidelity

$$\mathcal{F}(\rho_{\exp}, \rho) \equiv \{\mathrm{Tr}[(\rho_{\exp}^{1/2} \rho \rho_{\exp}^{1/2})^{1/2}]\}^2 = 0.939 \pm 0.008.$$

Conclusion

- Experimental verification of genuine multipartite entanglement of a global state from its separable marginals.
- Is there a Gaussian version of this phenomenon?